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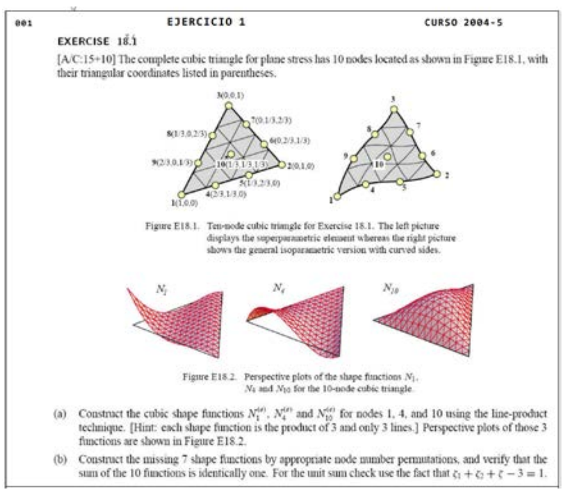
Estas ACTIVIDADES DE CLASE deberá realizarse descargando los documentos *NB* disponibles en las páginas web, completandolos adecuadamente, denominandolos de la forma especificada y subiendolos a tu cuenta de entrega personal. En este documento *PDF* habrá que contestar a las *PREGUNTAS* que planteo a lo largo de la grabación en video correspondiente a la clase.

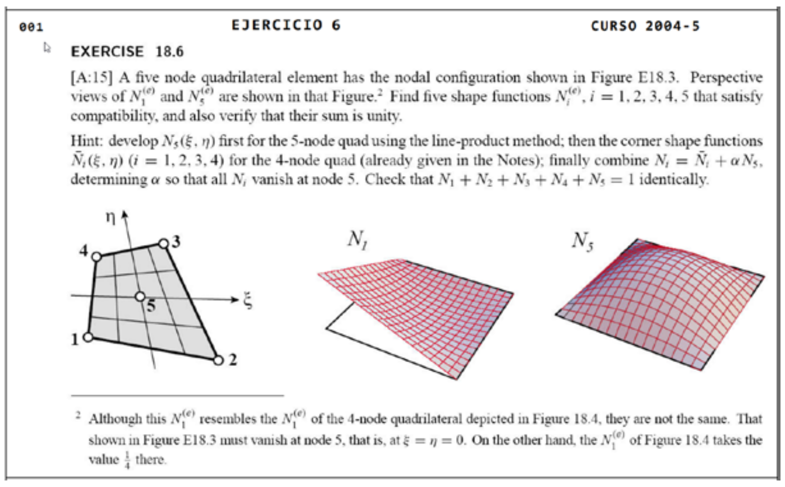
Para familiarizarnos con las ***Funciones de Forma***, el ***concepto de Convergencia***, el ***método de Solucionar las Ecuaciones*** y la ***Implementación del Elemento Cuadrilátero***, su definición, su terminología y su planteamiento; durante las explicaciones en clase habrá que completar este documento PDF.

Estas son imágenes de algunos de los ejercicios considerados en las ACTIVIDADES de esta CLASE:

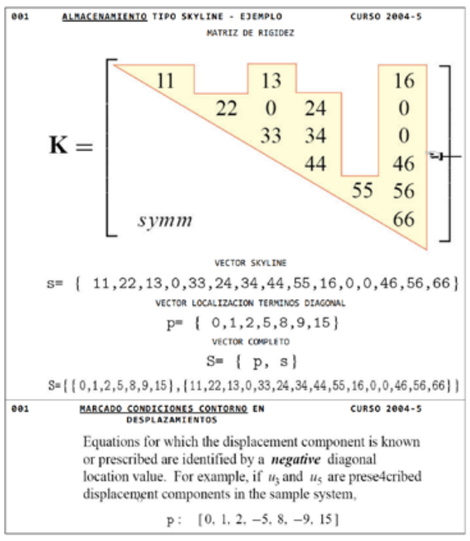
18-CP-C6-Mathematica-C1



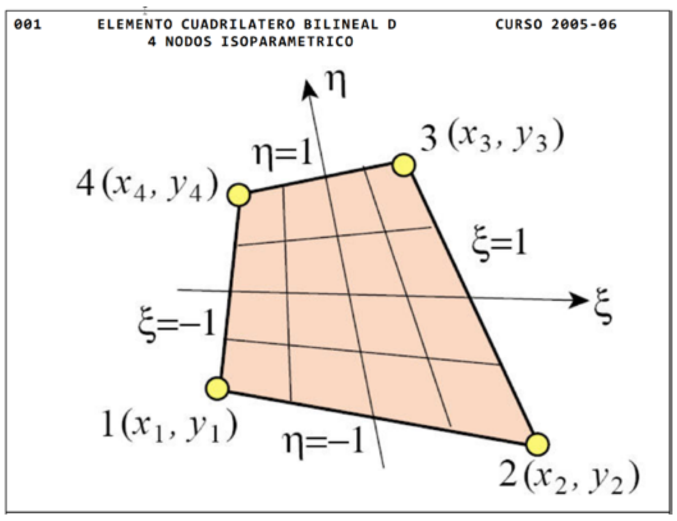
18-CP-C6-Mathematica-C2



18-CP-C7-Mathematica-C



18-CP-C8-Mathematica-C



PREGUNTAS Y TUS CONTESTACIONES:

DOCUMENTO PDF A COMPLETAR:

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FUNCIONES DE FORMA

001 SOBRE LAS FUNCIONES DE FORMA CURSO 2004-5

Means *Direct*

Do in 15 minutes what took smart people several months (and less gifted,)

But ... it looks like magic to the uninitiated

001 CONDICIONES A SATISFACER POR LAS FUNCIONES DE FORMA ISOPARAMETRICAS CURSO 2004-5

Shape Function Requirements

(A)

(B)

(C)

(D)

(A) Takes a unit value at node i, and is zero at all other nodes.

(B) Vanishes along any element boundary (a side in 2D, a face in 3D) that does not include node i.

(C) . Satisfies C^0 continuity between adjacent elements on any element boundary that includes node i.

(D) The interpolation is able to represent exactly any displacement field which is a linear polynomial in x and y; in particular, a constant value.

A statement equivalent to (C) is that the value of the shape function along a side common to two elements must uniquely depend only on its nodal values on that side.

Completeness is a property of *all* element isoparametric shape functions taken together, rather than of an individual one. If the element satisfies (B) and (C), in view of the discussion in §16.6 it is sufficient to check that the *sum of shape functions is identically one*.

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001 CONSTRUCCION DIRECTA DE FUNCIONES DE FORMA CURSO 2004-5

Direct Construction of Shape Functions
as

$$N_i^{(e)} \overset{\text{guess}}{=} c_i \; \ldots \; L_m$$

where $L_k = 0$ are equations of "lines" expressed in natural coordinates, that cross except i

For two-dimensional isoparametric elements, the ingredients in (18.1) are chosen according to the following rules.

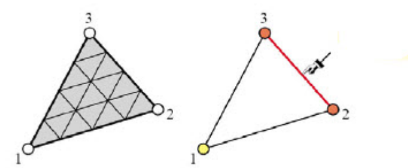
1. Select the L_j as the minimal number of lines or curves linear in the natural coordinates that cross all nodes except the i^{th} node. Primary choices are the and . The examples below illustrate how this is done.
2. Set coefficient c_i so that $N_i^{(e)}$ has the value at the node.
3. Check the polynomial order variation over each interelement boundary that contains node i. If this order is n, there must be exactly $n+1$ nodes on the boundary for the compatibility condition to hold.
4. If compatibility is satisfied, check that the of shape functions is identically

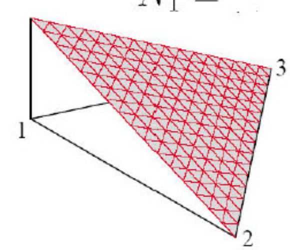
Specific two-dimensional examples in the following subsections show these rules in action. Essentially the same technique is applicable to one- and three-dimensional elements.

001 APLICACIÓN AL TRIANGULO DE TRES NODOS LINEAL CURSO 2004-5

The Three Node Linear Triangle

$N_1 =$





$$N_1 \overset{\text{guess}}{=} c_1 L_1 = c_1$$

At node 1, $N_1 = 1$ whence $c_1 =$
and $N_1 = \zeta_1$ Likewise for N_2 and N_3

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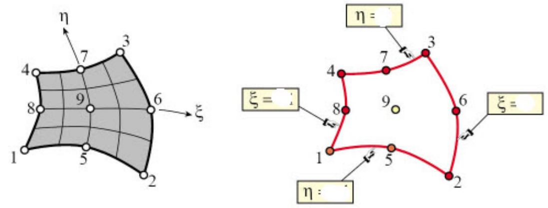
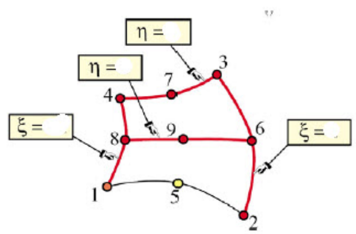
001 **CUADRILATERO BICUADRATICO DE NUEVE NODOS - NODO INTERNO** CURSO 2004-5



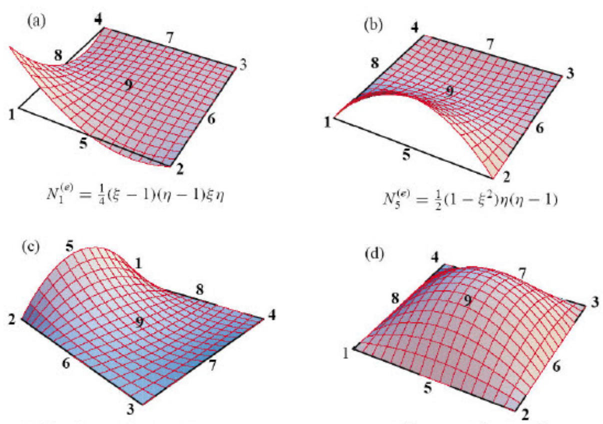
$$N_9^{(e)} = c_9\, L_{1\text{-}2} I \quad {}_3L_{3\text{-}4} I \quad = c_9 \,(\quad -1)(\eta-1)(\quad +1)(\eta+1)$$

001 **CUADRILATERO BICUADRATICO DE NUEVE NODOS - NODO MEDIO** CURSO 2004-5



$$N_5^{(e)} = c_5 L_{2\text{-}3} I \quad {}_4L_{6\text{-}8} I \quad = c_5 \quad -1)(\xi+1) \quad (\eta-1) = c_5\,(1-\xi^2)\eta(1-\eta)$$

001 **GRAFICO FUNCIONES FORMA** CURSO 2004-5



(a) $N_1^{(e)} = \frac{1}{4}(\xi-1)(\eta-1)\xi\eta$

(b) $N_5^{(e)} = \frac{1}{2}(1-\xi^2)\eta(\eta-1)$

(c) $N_5^{(e)} = \frac{1}{2}(1-\xi^2)\eta(\eta-1)$ (back view)

(d) $N_9^{(e)} = (1-\xi^2)(1-\eta^2)$

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CONVERGENCIA

001 **SIGNIFICADO DEL TERMINO "CONVERGENCIA"** CURSO 2004-5

Convergence: discrete (FEM) solution approaches the analytical (math model) solution in some sense

Convergence = +

(Lax-Wendroff)

001 **SIGNIFICADO DE LOS TERMINOS "CONSISTENCIA" Y "ESTABILIDAD"** CURSO 2004-5

- **Consistency**

  individual elements

  element patches

- **Stability**

  individual elements

  individual elements

Completeness. The elements must have enough *approximation power* to capture the analytical solution in the limit of a mesh refinement process.

Compatibility. The shape functions must provide *displacement continuity* between elements.

Completeness and compatibility are two aspects of the so-called **consistency** condition between the discrete and mathematical models.

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001 1º REQUERIMIENTO A CUMPLIR POR LA MATRIZ DE RIGIDEZ DEL ELEMENTO PARA ASEGURAR LA "ESTABILIDAD" "SUFICIENCIA DE RANGO" **CURSO 2004-5**

The element stiffness matrix must not possess any zero-energy kinematic mode other than rigid body modes.

This can be mathematically expressed as follows. Let be the number of element degrees of freedom, and be the number of independent rigid body modes. Let r denote the rank of $\mathbf{K}^{(e)}$. The element is called *rank sufficient* if $r = n_F - n_R$ and *rank deficient* if $r < n_F - n_R$. In the latter case,

$$d = (n_F - n_R) - r \qquad (19.5)$$

is called the rank deficiency.

If an isoparametric element is numerically integrated, let be the number of Gauss points, while denotes the order of the stress-strain matrix $\mathbf{E}$. Two additional assumptions are made:

(i) The element shape functions satisfy completeness in the sense that the rigid body modes are exactly captured by them.

(ii) Matrix $\mathbf{E}$ is of full rank.

Then each Gauss point adds n_E to the rank of $\mathbf{K}^{(e)}$, up to a maximum of $n_F - n_R$. Hence the rank of $\mathbf{K}^{(e)}$ will be

$$r = \min(n_F - n_R, n_E\, n_G) \qquad (19.6)$$

To attain rank sufficiency, $n_E n_G$ must equal or exceed $n_F - n_R$:

(19.7)

from which the appropriate Gauss integration rule can be selected.

In the plane stress problem, $n_E = 3$ because $\mathbf{E}$ is a 3×3 matrix of elastic moduli; see Chapter 14. Also $n_R = 3$. Consequently $r = \min(n_F - 3, 3n_G)$ and $3n_G \geq n_F - 3$.

EXAMPLE 19.5

Consider a plane stress 6-node quadratic triangle. Then $n_F = 2 \times 6 = 12$. To attain the proper rank of $12 - n_R = 12 - 3 = 9$, $n_G \geq 3$. A 3-point Gauss rule, such as the midpoint rule defined in §24.2, makes the element rank sufficient.

EXAMPLE 19.6

Consider a plane stress 9-node biquadratic quadrilateral. Then $n_F = 2 \times 9 = 18$. To attain the proper rank of $18 - n_R = 18 - 3 = 15$, $n_G \geq 5$. The 2×2 product Gauss rule is insufficient because $n_G = 4$. Hence a 3×3 rule, which yields $n_G = 9$, is required to attain rank sufficiency.

Table 19.1 collects rank-sufficient Gauss integration rules for some widely used plane stress elements with n nodes and $n_F = 2n$ freedoms.

Element	n	n_F	$n_F - 3$	Min n_G	Recommended rule
3-node triangle	3	6	3		centroid*
6-node triangle	6	12	9		3-midpoint rule*
10-node triangle	10	20	17		7-point rule*
4-node quadrilateral	4	8	5		2 x 2
8-node quadrilateral	8	16	13		3 x 3
9-node quadrilateral	9	18	15		3 x 3
16-node quadrilateral	16	32	29		4 x 4

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001 2º REQUERIMIENTO A CUMPLIR POR LA GEOMETRIA DEL ELEMENTO PARA ASEGURAR LA "ESTABILIDAD" "JACOBIANO POSITIVO" **CURSO 2004-5**

The geometry of the element must be such that the determinant $J = \det \mathbf{J}$ of the Jacobian matrix defined[4] in §17.2, is positive everywhere. As illustrated in Equation (17.20), J characterizes the local metric of the element natural coordinates.

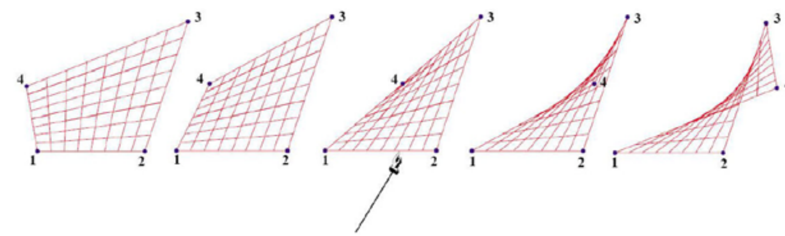
001 TRIANGULO DE 3 NODOS **CURSO 2004-5**

For a three-node triangle J is constant and in fact equal to $2A$. The requirement $J > 0$ is equivalent

This is called a *convexity condition*. It is easily checked by a finite element program.

001 CUADRILATERO DE 4 NODOS **CURSO 2004-5**

But for 2D elements with more than 3 nodes distortions may render *portions* of the element metric negative. This is illustrated in Figure 19.2 for a 4-node quadrilateral in which node 4 is gradually moved to the right. The quadrilateral morphs from a convex figure into a nonconvex one. The center figure is a triangle; note that the metric near node 4 is badly distorted (in fact $J = 0$ there) rendering the element unacceptable. This contradicts the (erroneous) advise of some FE books, which state that quadrilaterals can be reduced to triangles as special cases, thereby rendering triangular elements unnecessary.



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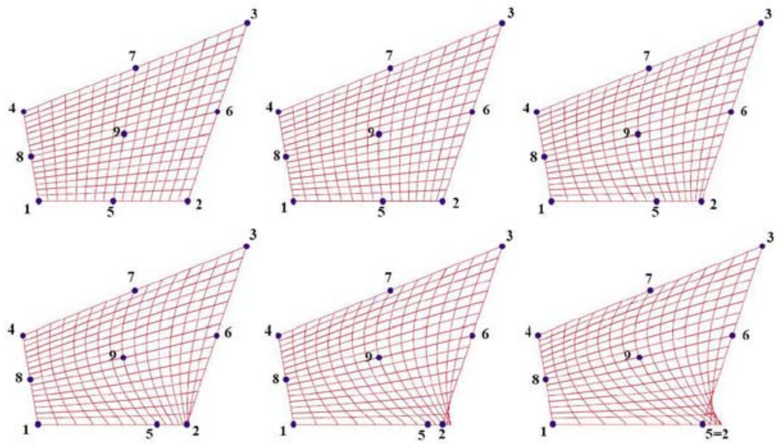
001 CUADRILATERO DE 9 NODOS CURSO 2004-5

For higher order elements proper location of corner nodes is not enough.

The effect of midpoint motions in quadratic elements is illustrated in Figures 19.3 and 19.4.

Figure 19.3 depicts the effect of moving midside node 5 tangentially in a 9-node quadrilateral element while keeping all other 8 nodes fixed. When the location of 5 reaches the quarter-point of side 1-2, the metric at corner 2 becomes singular in the sense that $J = 0$ there. Although this is disastrous in ordinary FE work, it has applications in the construction of special "crack" elements for linear fracture mechanics.



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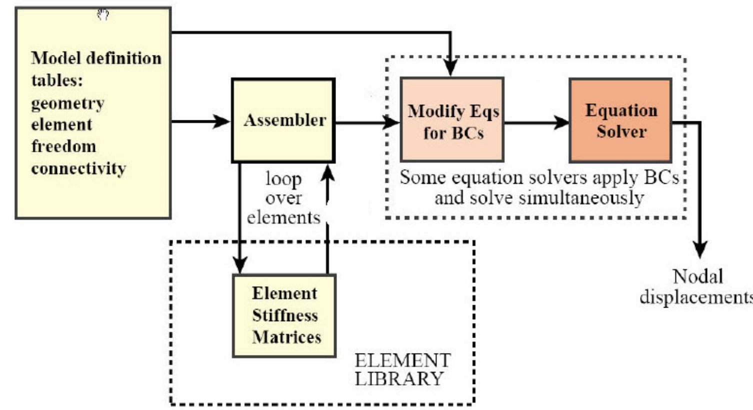
SOLUCION ECUACIONES

001 PROBLEMA MATEMATICA A RESOLVER CURSO 2004-5

001 VISUALIZACION DEL PROCESO GENERAL DE ANALISIS POR EF CURSO 2004-5



001 RECURSOS COMPUTACIONALES NECESARIOS SOLUCION ECUACIONES – MATRIZ COMPLETA CURSO 2004-5

Storage and Solution Times for a Fully Stored Stiffness Matrix

Matrix order N	Storage (double prec)	Factor op. units	Factor time workstation/PC	Factor time supercomputer
10^4		$10^{12}/6$	3 hrs	2 min
10^5	80 GB	$10^{15}/6$	4 mos	30 hrs
10^6	8 TB	$10^{18}/6$	300 yrs	3 yrs

time numbers last adjusted in 1998
to get current times divide by 10-20

As regards memory needs, a full square matrix stored without taking advantage of symmetry, requires storage for N^2 entries. If each entry is an 8-byte, double precision floating-point number, the required storage is $8N^2$ bytes. Thus, a matrix of order $N = 10^4$ would require 8×10^8 bytes or 800 MegaBytes (MB) for storage.

For large N the solution of (26.1) is dominated by the factorization of **K**, an operation discussed in §26.2. This operation requires approximately $N^3/6$ floating point operation units. [A floating-point operation unit is conventionally defined as a (multiply,add) pair plus associated indexing and data movement operations.] Now a fast workstation can typically do 10^7 of these operations per second, whereas a supercomputer may be able to sustain 10^9 or more.

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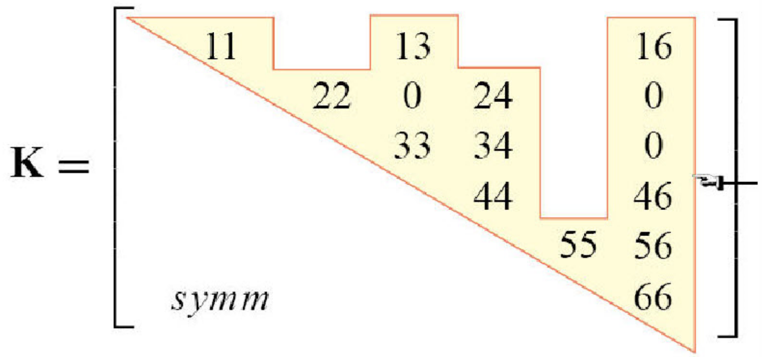
001 **ALMACENAMIENTO TIPO SKYLINE - EJEMPLO** **CURSO 2004-5**

MATRIZ DE RIGIDEZ



VECTOR SKYLINE

s= { }

VECTOR LOCALIZACION TERMINOS DIAGONAL

p= { . , , }

VECTOR COMPLETO

S= { p, s }

S={ { 0,1,2,5,8,9,15 }, { 11,22,13,0,33,24,34,44,55,16,0,0,46,56,66 } }

001 **MARCADO CONDICIONES CONTORNO EN DESPLAZAMIENTOS** **CURSO 2004-5**

Equations for which the displacement component is known or prescribed are identified by a ***negative*** diagonal location value. For example, if u_3 and u_5 are prese4cribed displacement components in the sample system,

p : [0, 1, 2, , 8, , 15]

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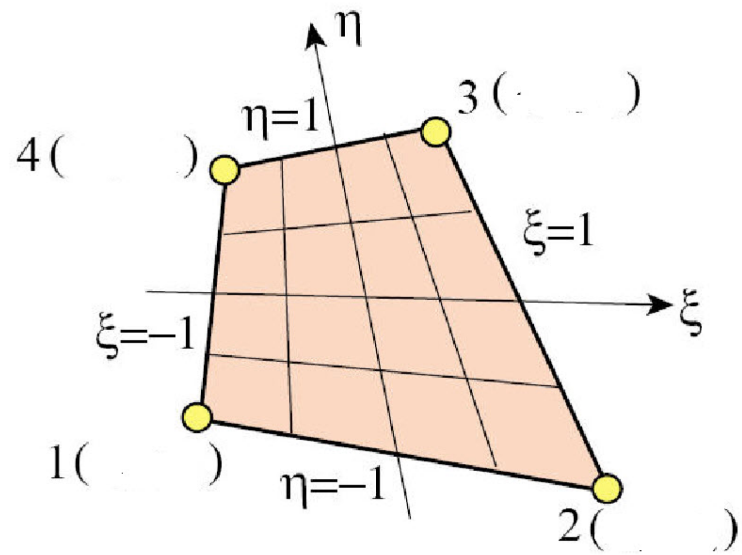
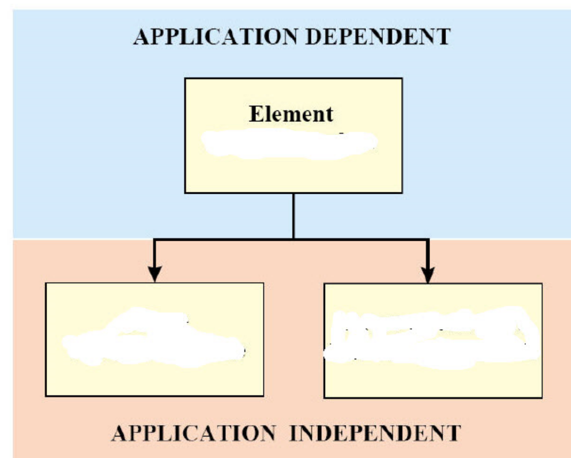
IMPLEMENTACION COMPUTACIONAL ELEMENTOS CUADRILATERO LINEAL 4 NODOS

001 **ELEMENTO CUADRILATERO BILINEAL D 4 NODOS ISOPARAMETRICO** **CURSO 2005-06**



001 **ORGANIZACIÓN DEL CÁLCULO DE LA MATRIZ DE RIGIDEZ** **CURSO 2005-06**



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001 MODULO INFORMACION PUNTOS DE GAUSS CURSO 2005-06

Recall from §17.3 that Gauss quadrature rules for isoparametric quadrilateral elements have the canonical form

$$\int_{-1}^{1}\int_{-1}^{1} \mathbf{F}(\xi,\eta)\, d\xi\, d\eta = \int_{-1}^{1} d\eta \int_{-1}^{1} \mathbf{F}(\xi,\eta)\, d\xi \doteq \sum_{i=1}^{p_1}\sum_{j=1}^{p_2} \qquad . \qquad (23.1)$$

Here $\mathbf{F} = h\mathbf{B}^T\mathbf{E}\mathbf{B}\,J$ is the matrix to be integrated, and p_1 and p_2 are the number of Gauss points in the ξ and η directions, respectively. Often, but not always, the same number $p = p_1 = p_2$ is chosen in both directions. A formula with $p_1 = p_2$ is called an *isotropic integration rule* because directions ξ and η are treated alike.

QuadGaussRuleInfo is an application independent module QuadGaussRuleInfo that implements the two-dimensional product Gauss rules with 1 through 4 points in each direction. The number of points in each direction may be the same or different. Use of this module was described in detail in §17.3.4. For the readers convenience it is listed, along with its subordinate module LineGaussRuleInfo, in Figure 23.3.

```
              [{rule_,numer_},point_]:= Module[
 {xi,eta,p1,p2,i1,i2,w1,w2,k,info=Null},
  If [Length[rule]==2,  {p1,p2}=rule, p1=p2=rule];
  If [Length[point]==2, {i1,i2}=point,
      k=point; i2=Floor[(k-1)/p1]+1; i1=k-p1*(i2-1) ];
  {xi, w1}=                [{p1,numer},i1];
  {eta,w2}=                [{p2,numer},i2];
  info={{xi,eta},w1*w2};
  If [numer, Return[N[info]], Return[Simplify[info]]];
];
              [{rule_,numer_},point_]:= Module[
  {g2={-1,1}/Sqrt[3],w3={5/9,8/9,5/9},
   g3={-Sqrt[3/5],0,Sqrt[3/5]},
   w4={(1/2)-Sqrt[5/6]/6, (1/2)+Sqrt[5/6]/6,
       (1/2)+Sqrt[5/6]/6, (1/2)-Sqrt[5/6]/6},
   g4={-Sqrt[(3+2*Sqrt[6/5])/7],-Sqrt[(3-2*Sqrt[6/5])/7],
        Sqrt[(3-2*Sqrt[6/5])/7], Sqrt[(3+2*Sqrt[6/5])/7]},
   i,info=Null},  i=point;
  If [rule==1, info={0,2}];
  If [rule==2, info={g2[[i]],1}];
  If [rule==3, info={g3[[i]],w3[[i]]}];
  If [rule==4, info={g4[[i]],w4[[i]]}];
  If [numer, Return[N[info]], Return[Simplify[info]]];
];
```


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001 MODULO FUNCIONES DE FORMA CURSO 2005-06

Quad4IsoPShapeFunDer is an application independent module that computes the shape functions $N_i^{(e)}$, $i = 1, 2, 3, 4$ and its x-y partial derivatives at the sample integration points. The logic, listed in Figure 23.4, is straightforward and follows closely the description of Chapter 17.

The arguments of the module are the $\{x, y\}$ quadrilateral corner coordinates, which are passed in ncoor, and the two quadrilateral coordinates $\{\xi, \eta\}$, which are passed in qcoor. The former have the same configuration as described for the element stiffness module below.

The quadrilateral coordinates define the element location at which the shape functions and their derivatives are to be evaluated. For the stiffness formation these are Gauss points, but for strain and stress computations these may be other points, such as corner nodes.

Quad4IsoPShapeFunDer returns the two-level list { }, in which the first three are 4-entry lists. List Nf collects the shape function values, Nx the shape function x-derivatives, Ny the shape function y-derivatives, and Jdet is the Jacobian determinant called J in Chapters 17.

```
                    [ncoor_,qcoor_]:= Module[
 {Nf,dNx,dNy,dNξ,dNη,i,J11,J12,J21,J22,Jdet,ξ,η,x,y},
 {ξ,η}=qcoor;
  Nf={(1-ξ)*(1-η),(1+ξ)*(1-η),(1+ξ)*(1+η),(1-ξ)*(1+η)}/4;
  dNξ ={-(1-η), (1-η),(1+η),-(1+η)}/4;
  dNη= {-(1-ξ),-(1+ξ),(1+ξ), (1-ξ)}/4;
  x=Table[ncoor[[i,1]],{i,4}]; y=Table[ncoor[[i,2]],{i,4}];
  J11=dNξ.x; J21=dNξ.y; J12=dNη.x; J22=dNη.y;
  Jdet=Simplify[J11*J22-J12*J21];
  dNx= ( J22*dNξ-J21*dNη)/Jdet;  dNx=Simplify[dNx];
  dNy= (-J12*dNξ+J11*dNη)/Jdet;  dNy=Simplify[dNy];
  Return[{Nf,dNx,dNy,Jdet}]
];
```

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001 MODULO MATRIZ DE RIGIDEZ CURSO 2005-06

Module `Quad4IsoPMembraneStiffness` computes the stiffness matrix of a four-noded isoparametric quadrilateral element in plane stress. The module configuration is typical of isoparametric elements in any number of dimensions. It follows closely the procedure outlined in Chapter 17. The module logic is listed in Figure 23.5. The statements at the bottom of the module box (not shown in Figure 23.5) test it for a specific configuration.

The arguments of the module are:

`ncoor` Quadrilateral node coordinates arranged in two-dimensional list form: { }}.

`mprop` Material properties supplied as the list `{Emat,rho,alpha}`. `Emat` is a two-dimensional list storing the 3×3 plane stress matrix of elastic moduli:

$$\mathbf{E} = \left[\quad\right] \qquad (23.2)$$

If the material is isotropic with elastic modulus E and Poisson's ratio ν, this matrix becomes

$$\mathbf{E} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \qquad (23.3)$$

The other two items in `mprop` are not used in this module so zeros may be inserted as placeholders.

`fprop` Fabrication properties. The plate thickness specified as a four-entry list: `{h1,h2,h3,h4}`, a one-entry list: `{h}`, or an empty list: `{ }`.

The first form is used to specify an element of variable thickness, in which case the entries are the four corner thicknesses and h is interpolated bilinearly. The second form specifies uniform thickness h. If an empty list appears the module assumes a uniform unit thickness.

`options` Processing options. This list may contain two items: `{numer,p}` or one: `{numer}`.

`numer` is a logical flag with value `True` or `False`. If , the computations are forced to proceed in floating point arithmetic. For symbolic or exact arithmetic work set `numer` to .

p specifies the Gauss product rule to have p points in each direction. p may be 1 through 4. For rank sufficiency, p must be 2 or higher. If p is 1 the element will be rank deficient by two. If omitted p = 2 is assumed.

```
Quad4IsoPMembraneStiffness[ncoor_,mprop_,fprop_,options_]:=
  Module[{i,k,p=2,numer=False,Emat,th=1,h,qcoor,c,w,Nf,
   dNx,dNy,Jdet,B,Ke=Table[0,{8},{8}]},  Emat=mprop[[1]];
  If [Length[options]==2, {numer,p}=options, {numer}=options];
  If [Length[fprop]>0, th=fprop[[1]]];
  If [p<1||p>4, Print["p out of range"];Return[Null]];
  For [k=1, k<=p*p, k++,
       {qcoor,w}= QuadGaussRuleInfo[{p,numer},k];
       {Nf,dNx,dNy,Jdet}=Quad4IsoPShapeFunDer[ncoor,qcoor];
        If [Length[th]==0, h=th, h=th.Nf]; c=w*Jdet*h;
        B={ Flatten[Table[{dNx[[i]],        0},{i,4}]],
            Flatten[Table[{0,        dNy[[i]]},{i,4}]],
            Flatten[Table[{dNy[[i]],dNx[[i]]},{i,4}]]};
        Ke+=Simplify[c*Transpose[B].(Emat.B)];
      ]; Return[Simplify[Ke]]
   ];
```

The module returns Ke as an 8×8 symmetric matrix pertaining to the following arrangement of nodal displacements:

$$\mathbf{u}^{(e)} = [\quad]^T. \qquad (23.4)$$

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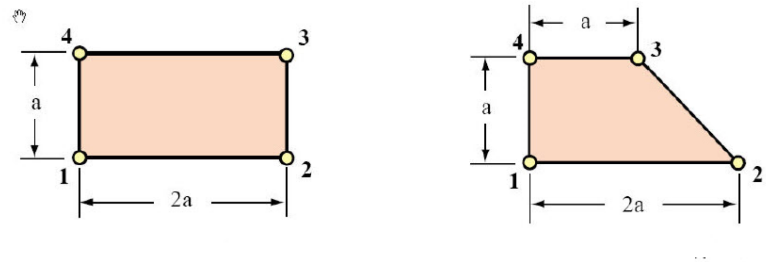
The stiffness module is tested on the two quadrilateral geometries shown in Figure 23.6. Both elements have unit thickness and isotropic material. The left one is a rectangle of base $2a$ and height a. The right one is a right trapezoid with base $2a$, top width a and height a. Both geometries will be used to illustrate the effect of the numerical integration rule.



Main purpose is to illustrate effect of changing Gauss integration rule

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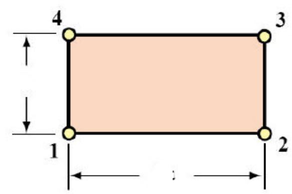


```
ClearAll[Em,nu,a,b,e,h,p,num];  h=1;
Em=96; nu=1/3;    (* isotropic material *)
Emat=Em/(1-nu^2)*{{1,nu,0},{nu,1,0},{0,0,(1-nu)/2}};
Print["Emat=",Emat//MatrixForm];
ncoor={{0,0},{2*a,0},{2*a,a},{0,a}}; (* 2:1 rectangular geometry *)

p=2; (* 2 x 2 Gauss rule *) num=False; (* exact symbolic arithmetic *)
Ke=Quad4IsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{num,p}];
Ke=Simplify[Chop[Ke]];  Print["Ke=",Ke//MatrixForm];
Print["Eigenvalues of Ke=",Chop[Eigenvalues[N[Ke]],.0000001]];
```

Uniform thickness $h =$., isotropic material with $E =$ and $\nu =$.
Rectangle dimension a cancels out in forming stiffness.

$$\mathbf{E} = \left[\quad\right]$$

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Note that the rectangle dimension a does not appear in (23.6). This is a general property: *the stiffness matrix of plane stress elements is independent of inplane dimension scalings*. This follows from the fact that entries of the strain-displacement matrix $\mathbf{B}$ have dimensions $1/L$, where L denotes a characteristic inplane length. Consequently entries of $\mathbf{B}^T\mathbf{B}$ have dimension $1/L^2$. Integration over the element area cancels out L^2.

Using a higher order Gauss integration rule, such as 3×3 and 4×4, reproduces exactly (23.6). This is a property characteristic of the rectangular geometry, since in that case the entries of $\mathbf{B}$ vary linearly in ξ and η, and J is constant. Therefore the integrand $h\,\mathbf{B}^T\,\mathbf{E}\,\mathbf{B}J$ is at most quadratic in ξ and η, and 2 Gauss points in each direction suffice to compute the integral exactly. Using a 1×1 rule yields a rank-deficiency matrix, a result illustrated in detail in §23.2.2.

Stiffness matrix computed by p x p rule, p =2,3,4,...

$$\mathbf{K}^{(e)} = \begin{bmatrix} 42 & 18 & -6 & 0 & -21 & -18 & -15 & 0 \\ 18 & 78 & 0 & 30 & -18 & -39 & 0 & -69 \\ -6 & 0 & 42 & -18 & -15 & 0 & -21 & 18 \\ 0 & 30 & -18 & 78 & 0 & -69 & 18 & -39 \\ -21 & -18 & -15 & 0 & 42 & 18 & -6 & 0 \\ -18 & -39 & 0 & -69 & 18 & 78 & 0 & 30 \\ -15 & 0 & -21 & 18 & -6 & 0 & 42 & -18 \\ 0 & -69 & 18 & -39 & 0 & 30 & -18 & 78 \end{bmatrix}$$

Eigenvalues of stiffness matrix:

$$[\,223.64 \quad 90 \quad 78 \quad 46.3603 \quad 42 \qquad\qquad]$$

which verifies that $\mathbf{K}^{(e)}$ has the correct rank of five ( ).

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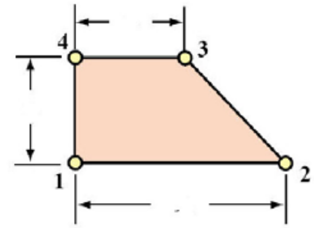


```
ClearAll[Em,nu,h,a,p];  h=1;
Em=48*63*13*107; nu=1/3;
Emat=Em/(1-nu^2)*{{1,nu,0},{nu,1,0},{0,0,(1-nu)/2}};
ncoor={{0,0},{2*a,0},{a,a},{0,a}};
For [p=1,p<=4,p++,
   Ke=Quad4IsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
   Ke=Rationalize[Ke,0.0000001]; Print["Ke=",Ke//MatrixForm];
   Print["Eigenvalues of Ke=",Chop[Eigenvalues[N[Ke]],.0000001]]
   ];
```

Strange value of E = = is to get exact entries in the stiffness matrix computed by Gauss rules p = 1,2,3,4.

The trapezoidal element geometry of Figure 23.6(b) is used to illustrate the effect of changing the $p \times p$ Gauss integration rule. Unlike the rectangular case, the element stiffness keeps changing as p is varied from 1 to 4. The element is rank sufficient, however, for $p \geq 2$ in agreement with the analysis of Chapter 19.

The computations are driven with the script shown in Figure 23.8. The value of p is changed in a loop. The flag `numer` is set to `True` to use floating-point computation for speed (see Remark 23.1). The computed entries of $\mathbf{K}^{(e)}$ are transformed to the nearest rational number (exact integers in this case) using the built-in function `Rationalize`. The strange value of $E = 48 \times 63 \times 13 \times 107 = 4206384$, in conjunction with $\nu = 1/3$, makes all entries of $\mathbf{K}^{(e)}$ exact integers when computed with the first 4 Gauss rules. This device facilitates visual comparison between the computed stiffness matrices: