

APELLIDOS, NOMBRE: _____ E-MAIL (UPV): _____

Estas ACTIVIDADES DE CLASE deberá realizarse descargando los documentos NB disponibles en las páginas web, completandolos adecuadamente, denominandolos de la forma especificada y subiendolos a tu cuenta de entrega personal. En este documento PDF habrá que contestar a las PREGUNTAS que planteo a lo largo de la grabación en video correspondiente a la clase.

Para familiarizarnos con la **formulacion del Elemento Triangular Lineal** para el problema de la Tensión Plana, con el concepto de **Representación Isoparamétrica** y con la **Formulacion del Elemento Cuadrilátero**, su definición, su terminología y su planteamiento; durante las explicaciones en clase habrá que completar este documento PDF.

Estas son imágenes de algunos de los ejercicios considerados en las ACTIVIDADES de esta CLASE:

17-CP-C3-Mathematica-C

EXERCISE 15.4
 [A/C:20] Derive the formula for the consistent force vector f^{int} of a linear triangle of constant thickness h , if side 1-2 ($\xi_1 = 0, \xi_2 = 1 - \xi_1$), is subject to a linearly varying boundary force $q = h\xi$ such that

$$q_1 = q_1 \xi_1 + q_2 \xi_2 = q_1(1 - \xi_2) + q_2 \xi_2, \quad q_2 = q_1 \xi_1 + q_2 \xi_2 = q_1(1 - \xi_2) + q_2 \xi_2. \quad (E15.3)$$

This "line force" q has dimension of force per unit of side length.

Proceedure. Use the last term of the line integral (14.21), in which Γ is replaced by q/h , and show that since the contribution of sides 2-3 and 3-1 to the line integral vanish,

$$f^{int} = \int_{\Gamma} h \mathbf{u}^T \mathbf{b} d\Gamma^{int} + \int_{\Gamma} h \mathbf{u}^T \mathbf{t} d\Gamma^{int} \quad (14.21)$$

$$f^{int} = (\mathbf{u}^{int})^T \mathbf{f}^{int} = \int_{\Gamma} \mathbf{u}^T \mathbf{q} d\Gamma^{int} = \int_{\Gamma} \mathbf{u}^T \mathbf{q} L_{21} d\xi_1 \quad (E15.4)$$

where L_{21} is the length of side 1-2. Replace $\mathbf{u}(\xi_1) = \mathbf{u}_1(1 - \xi_1) + \mathbf{u}_2 \xi_1$ likewise for u_1, u_2 , and q_1, q_2 , integrate and identify with the inner product shown as the second term in (E15.4). Partial result: $f_{11} = L_{21} C_1^T (q_1 + q_2)/6$, $f_{12} = f_{21} = 0$. Note: The following Mathematica script solves this Exercise. If you decide to use it, explain the logic.

```
ClearAll[ux1, uy1, ux2, uy2, ux3, uy3, z2, L12];
ux=ux1*(1-z2)+ux2*z2; uy=uy1*(1-z2)+uy2*z2;
qx=qx1*(1-z2)+qx2*z2; qy=qy1*(1-z2)+qy2*z2;
Do[Print["L12=Integrate[qx*ux+qy*uy, {z2,0,1}]]];
f=Table[Coefficient[Do[ux1,uy1,ux2,uy2,ux3,uy3][L12],{xi,1,6}];
f=Simplify[f]; Print["f==",f];
```

17-CP-C4-Mathematica-C

EXERCICIO 0 - EXPLICACION ETIQUETAS
 Exerices

Most Chapters are followed by a list of homework exercises that pose problems of varying difficulty. Each exercise is labeled by a tag of the form

[Type:rating]

The type is indicated by letters A, C, D or N for exercises to be answered primarily by analytical work, computer programming, descriptive narration, and numerical calculations, respectively. Some exercises involve a combination of these traits, in which case a combination of letters separated by + is used; for example A/N indicates analytical derivation followed by numerical work. For some problems heavy analytical work may be helped by the use of a computer-algebra system, in which case the type is identified as A/C.

The rating is a number between 5 and 50 that estimates the degree of difficulty of an Exercise, in the following "logarithmic" scale:

- 5 A simple question that can be answered in seconds, or is already answered in the text if the student has read and understood the material.
- 10 A straightforward question that can be answered in minutes.
- 15 A relatively simple question that requires some thinking, and may take on the order of half to one hour to answer.
- 20 Either a problem of moderate difficulty, or a straightforward one requiring lengthy computations or some programming, normally taking one to six hours of work.
- 25 A scaled up version of the above, estimated to require six hours to one day of work.
- 30 A problem of moderate difficulty that normally requires on the order of one or two days of work. Answering at the answer may involve a combination of techniques, some background or reference material, or lengthy but straightforward programming.
- 40 A difficult problem that may be solvable only by gifted and well prepared individual students, or a team. Difficulties may be due to the need of correct formulations, advanced mathematics, or high level programming. With the proper preparation, background and tools these problems may be solved in days or weeks, while remaining inaccessible to unprepared or average students.
- 50 A research problem, worthy of publication if solved.

Most Exercises have a rating of 15 or 20. Assigning three or four per week puts a load of roughly 5-10 hours of solution work, plus the time needed to prepare the answer material. Assignments of difficulty 25 or 30 are better handled by groups, or given in take-home exams. Assignments of difficulty beyond 30 are never assigned in the course, but listed as a challenge for an elite group.

Occasionally an Exercise has two or more distinct but related parts identified as items. In that case a rating may be given for each item. For example: [A/C:15-20]. This does not mean that the exercise as a whole has a difficulty of 35, because the scale is roughly logarithmic; the numbers simply rate the expected effort per item.

17-CP-C5-Mathematica-C

SIGNIFICADO DE LOS ARGUMENTOS
 CURSO 2004-5

noor Quadrilateral node coordinates arranged in two-dimensional list form: $\{(x1,y1),(x2,y2),(x3,y3),(x4,y4)\}$.

nprop Material properties supplied as the list {Emat, rho, alpha}. Emat is a two-dimensional list storing the 3×3 plane stress matrix of elastic moduli:

$$\mathbf{E} = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \quad (E17.1)$$

If the material is isotropic with elastic modulus E and Poisson's ratio ν , this matrix becomes

$$\mathbf{E} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1(1-\nu^2) \end{bmatrix} \quad (E17.2)$$

The other two items in nprop are not used in this module so zeros may be inserted as placeholders.

fprop Fabrication properties. The plate thickness specified as a four-entry list: {h1,h2,h3,h4} a one-entry list: {h}, or an empty list: {}.

The first form is used to specify an element of variable thickness, in which case the entries are the four corner thicknesses and h is interpolated bilinearly. The second form specifies uniform thickness h . If an empty list appears the module assumes a uniform unit thickness.

options Processing options. This list may contain two items: {number,p} or one: {number}. number is a logical flag with value True or False. If True, the computations are forced to proceed in floating point arithmetic. For symbolic or exact arithmetic work set number to False.

p specifies the Gauss product rule to have p points in each direction. p may be 1 through 4. For rank sufficiency, p must be 2 or higher. If p is 1 the element will be rank deficient by two. If omitted p=2 is assumed.

The module returns fe as an 8×8 symmetric matrix pertaining to the following arrangement of nodal displacements:

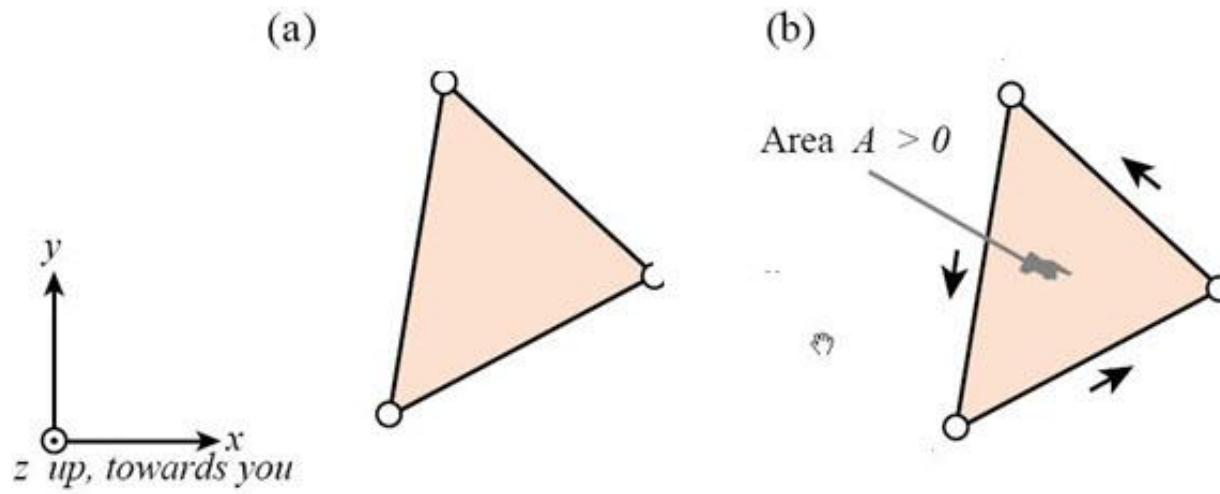
$$\mathbf{u}^{int} = [u_{11} \ u_{12} \ u_{21} \ u_{22} \ u_{31} \ u_{32} \ u_{41} \ u_{42}]^T \quad (E17.3)$$

PREGUNTAS Y TUS CONTESTACIONES:

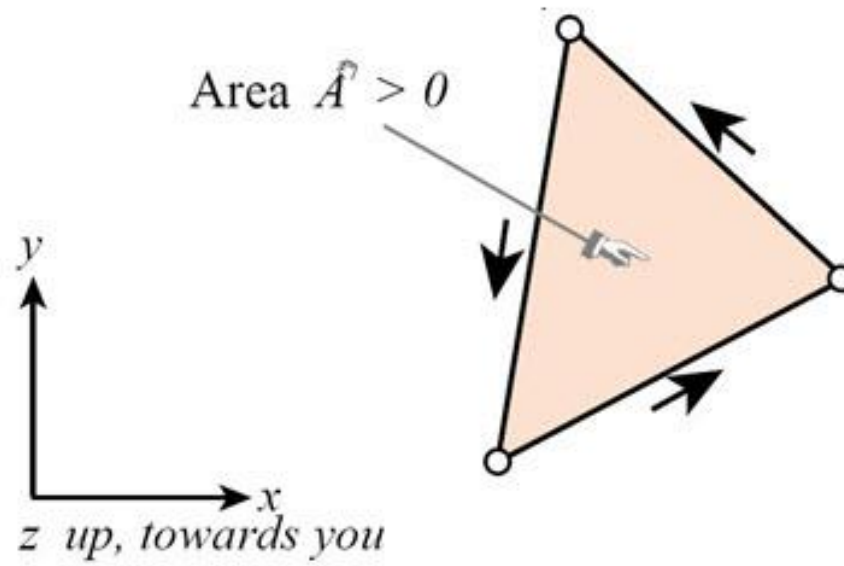
DOCUMENTO PDF A COMPLETAR:

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DEFINICION DEL ELEMENTO TRIANGULAR



AREA ES POSITIVA SI ...

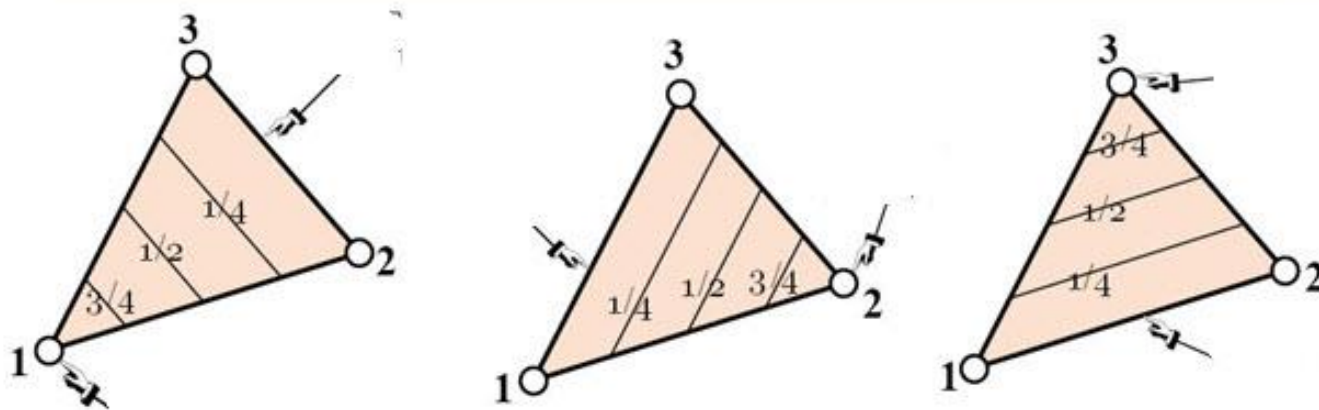


The area of the triangle is denoted by A and is given by

$$2A = \det \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$$

COORDENADAS PARAMETRICAS - COORDENADAS TRIANGULARES (CT)

ζ_1 ζ_2 ζ_3

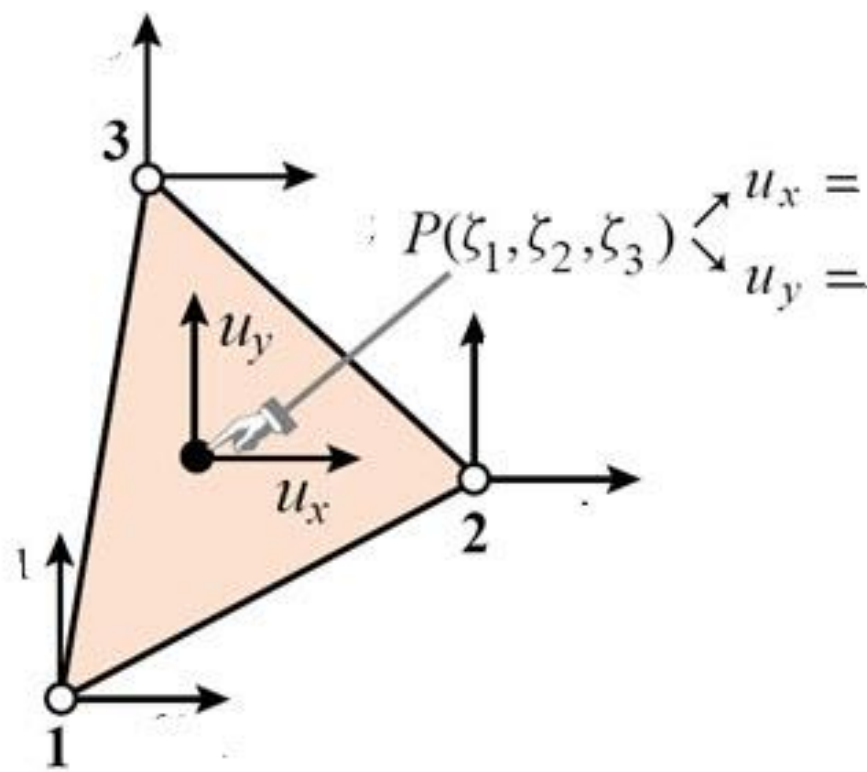


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TRANSFORMACION DE COORDENADAS TRIANGULARES - CARTESIANAS

FUNCIONES DE FORMA DEL ETL = COORDENADAS TRIANGULARES

FORMUALCION ETL - INTERPOLACION DESPLAZAMIENTOS



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FORMULACION ETL - OBTENCION DE LAS DEFORMACIONES

FORMULACION ETL - OBTENCION DE LAS TENSIONES

FORMULACION ETL - MATRIZ DE RIGIDEZ DEL ELEMENTO

ESPESOR h CONSTANTE

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REPRESENTACION ISO-P DE ELEMENTOS 2D CON N NODOS PROBLEMA TENSION PLANA

Iso-P Representation of 2D Plane Stress Elements with n Nodes

Element Geometry:

$$1 = \sum_{i=1}^n \quad , \quad x = \sum_{i=1}^n \quad , \quad y = \sum_{i=1}^n$$

Displacement Interpolation

$$u_x = \sum_{i=1}^n \quad , \quad u_y = \sum_{i=1}^n u_{yi} N_i^{(e)}$$

Matrix Form of Above

$$\begin{bmatrix} 1 \\ x \\ y \\ u_x \\ u_y \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ y_1 & y_2 & \dots & y_n \\ u_{y1} & u_{y2} & \dots & u_{yn} \end{bmatrix} \begin{bmatrix} N_1^{(e)} \\ \vdots \\ N_n^{(e)} \end{bmatrix}$$

ES POSIBLE INTERPOLAR MAGNITUDES ADICIONALES

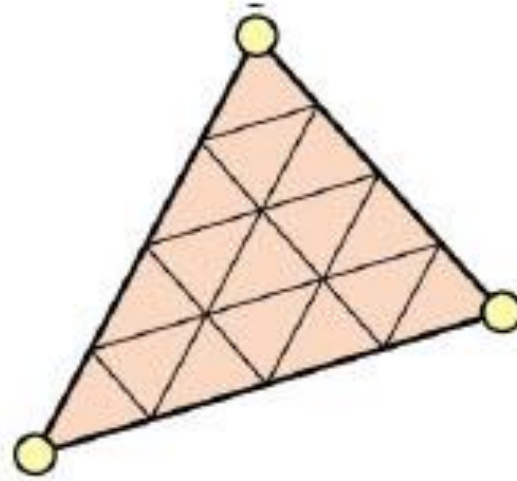
thickness h
temperature T

$$\begin{bmatrix} 1 \\ x \\ y \\ u_x \\ u_y \\ T \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \\ u_{x1} & u_{x2} & \dots & u_{xn} \\ u_{y1} & u_{y2} & \dots & u_{yn} \\ T_1 & T_2 & \dots & T_n \end{bmatrix} \begin{bmatrix} N_1^{(e)} \\ \vdots \\ N_n^{(e)} \end{bmatrix}$$

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REPRESENTACION ISOPARAMETRICA DEL ELEMENTO TRIANGULAR LINEAL

The Linear Triangle

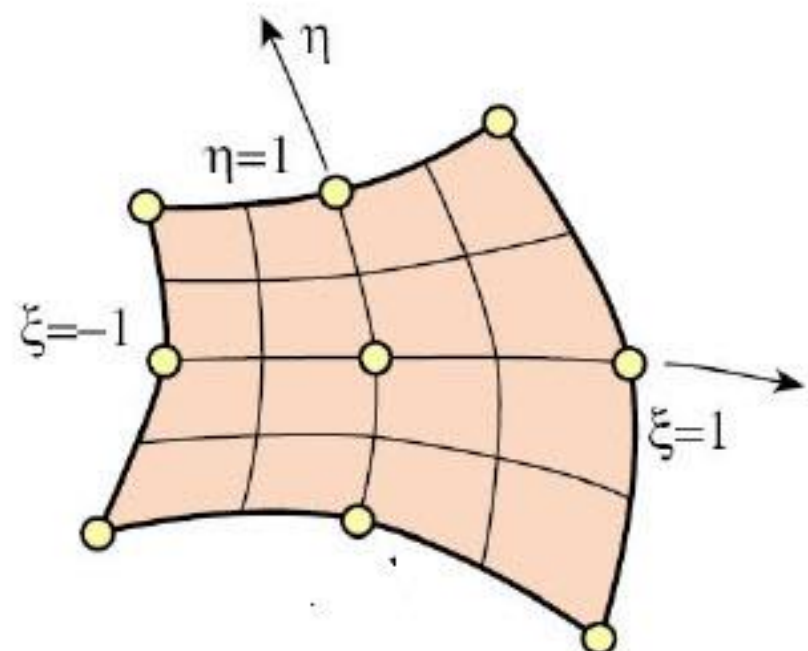
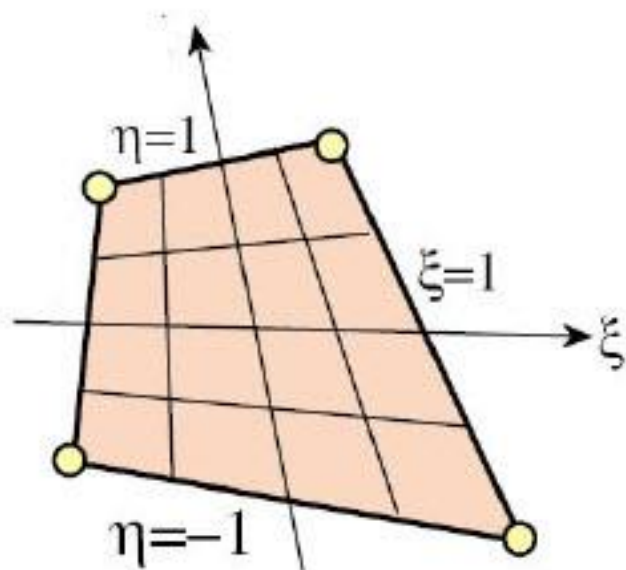


$$\begin{bmatrix} 1 \\ y \\ u_y \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ y_1 & y_2 & y_3 \\ u_{y1} & u_{y2} & u_{y3} \end{bmatrix} \begin{bmatrix} N_1^{(e)} \\ N_2^{(e)} \\ N_3^{(e)} \end{bmatrix}$$

$$N_1^{(e)} = \zeta_1, \quad N_2^{(e)} = \dots, \quad N_3^{(e)} = \dots$$

COORDENADAS DEL CUADRILATERO

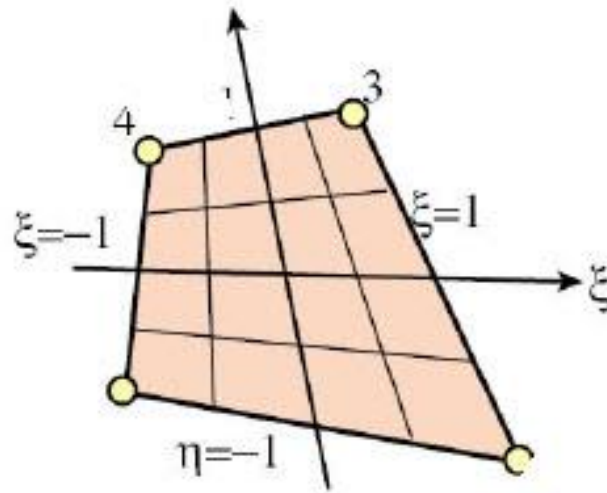
Quadrilateral Coordinates ξ, η



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REPRESENTACION ISOPARAMETRICA DEL CUADRILATERO BILINEAL DE 4 NODOS

4-Node Bilinear Quadrilateral



$$\begin{bmatrix} 1 \\ x \\ y \\ u_y \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ u_{y1} & u_{y2} & u_{y3} & u_{y4} \end{bmatrix} \begin{bmatrix} N_1^{(e)} \\ N_2^{(e)} \\ N_3^{(e)} \\ N_4^{(e)} \end{bmatrix} \quad \begin{aligned} N_1^{(e)} &= \frac{1}{4}(1 - \xi)(1 - \eta) \\ N_2^{(e)} &= \frac{1}{4}(1 + \xi)(1 - \eta) \\ N_3^{(e)} &= \frac{1}{4}(1 + \xi)(1 + \eta) \\ N_4^{(e)} &= \frac{1}{4}(1 - \xi)(1 + \eta) \end{aligned}$$

NECESITAMOS CALCULAR LAS MATRICES JACOBIANA Y SU INVERSA

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{bmatrix} d\xi \\ d\eta \end{bmatrix} = \mathbf{J}^T \begin{bmatrix} d\xi \\ d\eta \end{bmatrix}$$

$$\begin{bmatrix} d\xi \\ d\eta \end{bmatrix} = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \xi}{\partial y} \\ \frac{\partial \eta}{\partial x} & \frac{\partial \eta}{\partial y} \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} = \mathbf{J}^{-T} \begin{bmatrix} dx \\ dy \end{bmatrix}$$

in which

$$\mathbf{J} = \frac{\partial(x, y)}{\partial(\xi, \eta)} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}, \quad \mathbf{J}^{-1} = \frac{\partial(\xi, \eta)}{\partial(x, y)} = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} \\ \frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y} \end{bmatrix}$$

$$J = |\mathbf{J}| = \det \mathbf{J}$$

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Y LAS DERIVADAS PARCIALES DE LAS FUNCIONES DE FORMA

$$\frac{\partial N_i^{(e)}}{\partial x} = \frac{\partial N_i^{(e)}}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial N_i^{(e)}}{\partial \eta} \frac{\partial \eta}{\partial x}$$

$$\frac{\partial N_i^{(e)}}{\partial y} = \frac{\partial N_i^{(e)}}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial N_i^{(e)}}{\partial \eta} \frac{\partial \eta}{\partial y}$$

Main problem is to get $\frac{\partial \xi}{\partial x}$ $\frac{\partial \eta}{\partial x}$ $\frac{\partial \xi}{\partial y}$ $\frac{\partial \eta}{\partial y}$

LOS ELEMENTOS DE LA INVERSA DE LA MATRIZ JACOBIANA SERAN LOS QUE NECESITAMOS

Compute the 2 x 2 Jacobian matrix

$$\mathbf{J} = \frac{\partial(x, y)}{\partial(\xi, \eta)} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

Then invert to get

$$\mathbf{J}^{-1} = \frac{\partial(\xi, \eta)}{\partial(x, y)} = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} \\ \frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y} \end{bmatrix}$$

These are the quantities we need for the S.F. partials

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INTEGRACION NUMERICA MEDIANTE REGLAS DE GAUSS

REGLAS UNIDIMENSIONALES

$$\int_{-1}^1 F(\xi) d\xi = \sum_{i=1}^p w_i F(\xi_i).$$

One point: $\int_{-1}^1 F(\xi) d\xi \doteq 2F(0),$

Two points: $\int_{-1}^1 F(\xi) d\xi \doteq F(-1/\sqrt{3}) + F(1/\sqrt{3}),$

Three points: $\int_{-1}^1 F(\xi) d\xi \doteq \frac{5}{9}F(-\sqrt{3/5}) + \frac{8}{9}F(0) + \frac{5}{9}F(\sqrt{3/5})$

REPRESENTACION GRAFICA DE LAS REGLAS UNIDIMENSIONALES

