Esta ACTIVIDAD DE CLASE deberá realizarse descargando los documentos *NB* incompletos correspondientes a estos ejercicios de clase. Deberás seleccionar en el siguiente panel el enlace correspondiente al número que se te ha asignado en la cuenta del material personalizado de la actividad ***m1-a1a***.

17-CP-C5-Mathematica-C

001 SIGNIFICADO DE LOS ARGUMENTOS CURSO 2004-5

`ncoor` Quadrilateral node coordinates arranged in two-dimensional list form: `{{x1,y1},{x2,y2},{x3,y3},{x4,y4}}`.

`mprop` Material properties supplied as the list `{Emat,rho,alpha}`. `Emat` is a two-dimensional list storing the 3 × 3 plane stress matrix of elastic moduli:

$$\mathbf{E} = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{12} & E_{22} & E_{23} \\ E_{13} & E_{23} & E_{33} \end{bmatrix} \qquad \text{(E17.1)}$$

If the material is isotropic with elastic modulus E and Poisson's ratio ν, this matrix becomes

$$\mathbf{E} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \qquad \text{(E17.2)}$$

The other two items in `mprop` are not used in this module so zeros may be inserted as placeholders.

`fprop` Fabrication properties. The plate thickness specified as a four-entry list: `{h1,h2,h3,h4}`, a one-entry list: `{h}`, or an empty list: `{ }`.

The first form is used to specify an element of variable thickness, in which case the entries are the four corner thicknesses and h is interpolated bilinearly. The second form specifies uniform thickness `h`. If an empty list appears the module assumes a uniform unit thickness.

`options` Processing options. This list may contain two items: `{numer,p}` or one: `{numer}`.

`numer` is a logical flag with value `True` or `False`. If `True`, the computations are forced to proceed in floating point arithmetic. For symbolic or exact arithmetic work set `numer` to `False`.[6]

p specifies the Gauss product rule to have p points in each direction. p may be 1 through 4. For rank sufficiency, p must be 2 or higher. If p is 1 the element will be rank deficient by two.[7] If omitted p = 2 is assumed.

The module returns `Ke` as an 8 × 8 symmetric matrix pertaining to the following arrangement of nodal displacements:

$$\mathbf{u}^{(e)} = [\, u_{x1} \quad u_{y1} \quad u_{x2} \quad u_{y2} \quad u_{x3} \quad u_{y3} \quad u_{x4} \quad u_{y4} \,]^T . \qquad \text{(E17.3)}$$

001 EJERCICIO 1 CURSO 2004-5

[C:15] Exercise the *Mathematica* module of Figure E17.1 with the following script:

```
ClearAll[Em,nu,a,b,h];  Em=48; h=1; a=4; b=2; nu=0;
ncoor={{0,0},{a,0},{a,b},{0,b}};
Emat=Em/(1-nu^2)*{{1,nu,0},{nu,1,0},{0,0,(1-nu)/2}};
For [p=1, p<=4, p++,
     Ke= Quad4IsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
     Print["Gauss integration rule: ",p," x ",p];
     Print["Ke=",Chop[Ke]//MatrixForm];
     Print["Eigenvalues of Ke=",Chop[Eigenvalues[N[Ke]]]]
];
```

Verify that for integration rules `p= 2,3,4` the stiffness matrix does not change and has three zero eigenvalues, which correspond to the three two-dimensional rigid body modes. On the other hand, for p = 1 the stiffness matrix is different and displays five zero eigenvalues, which is physically incorrect. (This phenomenon is discussed in detail in Chapter 19.) Question: why does the stiffness matrix stays exactly the same for $p \geq 2$? Hint: take a look at the entries of the integrand $h\mathbf{B}^T\mathbf{E}\mathbf{B}\,J$ — for a *rectangular geometry* are those polynomials in ξ and η, or rational functions?

001 **EJERCICIO 2** **CURSO 2004-5**

[C:20] Check the rectangular element stiffness closed form given in (E17.4). This may be done by hand (takes a few days) or running the following script that calls the *Mathematica* module of Figure E17.1:

```
ClearAll[Em,ν,a,b,h];   b=γ*a;
ncoor={{0,0},{a,0},{a,b},{0,b}};
Emat=Em/(1-ν^2)*{{1,ν,0},{ν,1,0},{0,0,(1-ν)/2}};
Ke= Quad4IsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{False,2}];
scaledKe=Simplify[Ke*(24*γ*(1-ν^2)/(Em*h))];
Print["Ke=",Em*h/(24*γ*(1-ν^2)),"*\n",scaledKe//MatrixForm];
```

Figure E17.3. Script suggested for Exercise E17.2.

The scaling introduced in the last two lines is for matrix visualization convenience. Verify (E17.4) by printout inspection and report any typos to instructor.

ELEMENTO CUADRILATERO 4 NODOS - TENSION PLANA

INICIO

```
Off[General::"spell1"]
Off[General::"spell"]
```

MODULOS FORMULACION ELEMENTO CUADRILATERO 4 NODOS

```
Quad4IsoPMembraneStiffness[ncoor_, mprop_, fprop_, options_] := Module[{i, k, p = 2, numer = False, Emat,
    th = 1, h, qcoor, c, w, Nf, dNx, dNy, Jdet, B, Ke = Table[0, {8}, {8}]}, Emat = mprop[[1]];
   If[Length[options] == 2, {numer, p} = options, {numer} = options];
   If[Length[fprop] > 0, th = fprop[[1]]];
   If[p < 1 || p > 4, Print["p out of range"]; Return[Null]];
   For[k = 1, k ≤ p * p, k++, {qcoor, w} = QuadGaussRuleInfo[{p, numer}, k];
    {Nf, dNx, dNy, Jdet} = Quad4IsoPShapeFunDer[ncoor, qcoor];
    If[Length[th] == 0, h = th, h = th.Nf]; c = w * Jdet * h;
    B = {Flatten[Table[{dNx[[i]], 0}, {i, 4}]],
      Flatten[Table[{0, dNy[[i]]}, {i, 4}]], Flatten[Table[{dNy[[i]], dNx[[i]]}, {i, 4}]]};
    Ke += Simplify[c * Transpose[B].(Emat.B)];]; Return[Simplify[Ke]]];
```

```
Quad4IsoPShapeFunDer[ncoor_, qcoor_] :=
  Module[{Nf, dNx, dNy, dNξ, dNη, i, J11, J12, J21, J22, Jdet, ξ, η, x, y}, {ξ, η} = qcoor;
   Nf = {(1 - ξ) * (1 - η), (1 + ξ) * (1 - η), (1 + ξ) * (1 + η), (1 - ξ) * (1 + η)} / 4;
   dNξ = {- (1 - η), (1 - η), (1 + η), - (1 + η)} / 4;
   dNη = {- (1 - ξ), - (1 + ξ), (1 + ξ), (1 - ξ)} / 4;
   x = Table[ncoor[[i, 1]], {i, 4}]; y = Table[ncoor[[i, 2]], {i, 4}];
   J11 = dNξ.x; J21 = dNξ.y; J12 = dNη.x; J22 = dNη.y;
   Jdet = Simplify[J11 * J22 - J12 * J21];
   dNx = (J22 * dNξ - J21 * dNη) / Jdet; dNx = Simplify[dNx];
   dNy = (-J12 * dNξ + J11 * dNη) / Jdet; dNy = Simplify[dNy];
   Return[{Nf, dNx, dNy, Jdet}]];
```

```
QuadGaussRuleInfo[{rule_, numer_}, point_] := Module[
   {xi, eta, p1, p2, i1, i2, w1, w2, k, info = Null}, If[Length[rule] == 2, {p1, p2} = rule, p1 = p2 = rule];
   If[Length[point] == 2, {i1, i2} = point, k = point; i2 = Floor[(k - 1) / p1] + 1; i1 = k - p1 * (i2 - 1)];
   {xi, w1} = LineGaussRuleInfo[{p1, numer}, i1];
   {eta, w2} = LineGaussRuleInfo[{p2, numer}, i2];
   info = {{xi, eta}, w1 * w2};
   If[numer, Return[N[info]], Return[Simplify[info]]];];
```

```
LineGaussRuleInfo[{rule_, numer_}, point_] :=
  Module[{g2 = {-1, 1} / Sqrt[3], w3 = {5 / 9, 8 / 9, 5 / 9}, g3 = {-Sqrt[3 / 5], 0, Sqrt[3 / 5]},
    w4 = {(1 / 2) - Sqrt[5 / 6] / 6, (1 / 2) + Sqrt[5 / 6] / 6, (1 / 2) + Sqrt[5 / 6] / 6, (1 / 2) - Sqrt[5 / 6] / 6},
    g4 = {-Sqrt[(3 + 2 * Sqrt[6 / 5]) / 7], -Sqrt[(3 - 2 * Sqrt[6 / 5]) / 7],
      Sqrt[(3 - 2 * Sqrt[6 / 5]) / 7], Sqrt[(3 + 2 * Sqrt[6 / 5]) / 7]}, i, info = Null}, i = point;
   If[rule == 1, info = {0, 2}];
   If[rule == 2, info = {g2[[i]], 1}];
   If[rule == 3, info = {g3[[i]], w3[[i]]}];
   If[rule == 4, info = {g4[[i]], w4[[i]]}];
   If[numer, Return[N[info]], Return[Simplify[info]]];];
```

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#11	#12	#13	#14	#15	#16	#17	#18	#19	#20
#21	#22	#23	#24	#25	#26	#27	#28	#29	#30
#31	#32	#33	#34	#35	#36	#37	#38	#39	#40
#41	#42	#43	#44	#45	#46	#47	#48	#49	#50
#51	#52	#53	#54	#55	#56	#57	#58	#59	#60
#61	#62	#63	#64	#65	#66	#67	#68	#69	#70
#71	#72	#73	#74	#75	#76	#77	#78	#79	#80
#81	#82	#83	#84	#85	#86	#87	#88	#89	#90
Each student must download the one corresponding to the number assigned to them									

Una vez completado, deberá subirse adecuadamente denominado a la cuenta de entrega personal, seleccionando del siguiente panel el enlace correspondiente al numero que se te ha asignado en la cuenta del material personalizado de la actividad ***m1-a1a***.

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#51	#52	#53	#54	#55	#56	#57	#58	#59	#60
#61	#62	#63	#64	#65	#66	#67	#68	#69	#70
#71	#72	#73	#74	#75	#76	#77	#78	#79	#80
#81	#82	#83	#84	#85	#86	#87	#88	#89	#90
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