Esta ACTIVIDAD DE CLASE deberá realizarse descargando los documentos *NB* incompletos correspondientes a estos ejercicios de clase. Deberás seleccionar en el siguiente panel el enlace correspondiente al número que se te ha asignado en la cuenta del material personalizado de la actividad ***m1-a1a***.

17-CP-C3-Mathematica-C

001 **EJERCICIO 3** **CURSO 2004-5**

EXERCISE 15.3

[A/C:20] Compute the consistent node force vector $\mathbf{f}^{(e)}$ for body loads over a linear triangle, if the element thickness varies as per (E15.1), $b_x = 0$, and $b_y = b_{y1}\zeta_1 + b_{y2}\zeta_2 + b_{y3}\zeta_3$. Check that for $h_1 = h_2 = h_3 = h$ and $b_{y1} = b_{y2} = b_{y3} = b_y$ you recover (15.26). For the integrals over the triangle area use the formula (E15.2).

Partial result: $f_{y1} = (A/60)[b_{y1}(6h_1 + 2h_2 + 2h_3) + b_{y2}(2h_1 + 2h_2 + h_3) + b_{y3}(2h_1 + h_2 + 2h_3)]$.

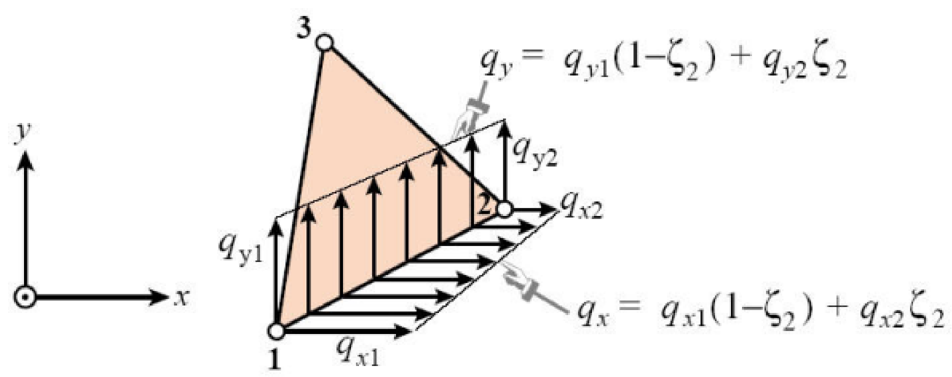
001 **EJERCICIO 4** **CURSO 2004-5**

EXERCISE 15.4

[A/C:20] Derive the formula for the consistent force vector $\mathbf{f}^{(e)}$ of a linear triangle of constant thickness h, if side 1–2 ($\zeta_3 = 0$, $\zeta_2 = 1 - \zeta_1$), is subject to a linearly varying boundary force $\mathbf{q} = h\hat{\mathbf{t}}$ such that

$$q_x = q_{x1}\zeta_1 + q_{x2}\zeta_2 = q_{x1}(1 - \zeta_2) + q_{x2}\zeta_2, \quad q_y = q_{y1}\zeta_1 + q_{y2}\zeta_2 = q_{y1}(1 - \zeta_2) + q_{y2}\zeta_2. \tag{E15.3}$$

This "line force" $\mathbf{q}$ has dimension of force per unit of side length.



Procedure. Use the last term of the line integral (14.21), in which $\hat{\mathbf{t}}$ is replaced by $\mathbf{q}/h$, and show that since the contribution of sides 2-3 and 3-1 to the line integral vanish,

$$W^{(e)} = \int_{\Omega^{(e)}} h\,\mathbf{u}^T\mathbf{b}\,d\Omega^{(e)} + \int_{\Gamma^{(e)}} h\,\mathbf{u}^T\,\hat{\mathbf{t}}\,d\Gamma^{(e)} \tag{14.21}$$

$$W^{(e)} = \left(\mathbf{u}^{(e)}\right)^T \mathbf{f}^{(e)} = \int_{\Gamma^{(e)}} \mathbf{u}^T\mathbf{q}\,d\Gamma^{(e)} = \int_0^1 \mathbf{u}^T\mathbf{q}\,L_{21}\,d\zeta_2, \tag{E15.4}$$

where L_{21} is the length of side 1-2. Replace $u_x(\zeta_2) = u_{x1}(1 - \zeta_2) + u_{x2}\zeta_2$; likewise for u_y, q_x and q_y, integrate and identify with the inner product shown as the second term in (E15.4). Partial result: $f_{x1} = L_{21}(2q_{x1}+q_{x2})/6$, $f_{x3} = f_{y3} = 0$. *Note.* The following *Mathematica* script solves this Exercise. If you decide to use it, explain the logic.

```
ClearAll[ux1,uy1,ux2,uy2,ux3,uy3,z2,L12];
ux=ux1*(1-z2)+ux2*z2; uy=uy1*(1-z2)+uy2*z2;
qx=qx1*(1-z2)+qx2*z2; qy=qy1*(1-z2)+qy2*z2;
We=Simplify[L12*Integrate[qx*ux+qy*uy,{z2,0,1}]];
fe=Table[Coefficient[We,{ux1,uy1,ux2,uy2,ux3,uy3}[[i]]],{i,1,6}];
fe=Simplify[fe]; Print["fe=",fe];
```

001 **EJERCICIO 5** **CURSO 2004-5**

[C+N:15] Compute the entries of $\mathbf{K}^{(e)}$ for the following plane stress triangle:

$$x_1 = 0,\ y_1 = 0,\ x_2 = 3,\ y_2 = 1,\ x_3 = 2,\ y_3 = 2,$$

$$\mathbf{E} = \begin{bmatrix} 100 & 25 & 0 \\ 25 & 100 & 0 \\ 0 & 0 & 50 \end{bmatrix}, \qquad h = 1. \tag{E15.5}$$

This may be done by hand (it is a good exercise in matrix multiplication) or (more quickly) using the following *Mathematica* script:

```
Stiffness3NodePlaneStressTriangle[{{x1_,y1_},{x2_,y2_},{x3_,y3_}},
 Emat_,{h_}]:=Module[{x21,x13,x32,y12,y31,y23,A,Be,Ke},
   A=Simplify[(x2*y3-x3*y2+(x3*y1-x1*y3)+(x1*y2-x2*y1))/2];
   {x21,x13,x32}={x2-x1,x1-x3,x3-x2};
   {y12,y31,y23}={y1-y2,y3-y1,y2-y3};
   Be={{y23,0,y31,0,y12,0},{0,x32,0,x13,0,x21},
       {x32,y23,x13,y31,x21,y12}}/(2*A);
   Ke=A*h*Transpose[Be].Emat.Be;Return[Ke]];

Ke=Stiffness3NodePlaneStressTriangle[{{0,0},{3,1},{2,2}},
     {{100,25,0},{25,100,0},{0,0,50}},{1}];
Print["Ke=",Ke//MatrixForm];
Print["eigs of Ke=",Chop[Eigenvalues[N[Ke]]]];
Show[Graphics[Line[{{0,0},{3,1},{2,2},{0,0}}]],Axes->True];
```

Check it out: $K_{11} = 18.75$, $K_{66} = 118.75$. The last statement draws the triangle.

001 **EJERCICIO 7** **CURSO 2004-5**

EXERCISE 15.7

[A/C:30] Let $p(\zeta_1, \zeta_2, \zeta_3)$ represent a *polynomial* expression in the natural coordinates. The integral

$$\int_{\Omega^{(e)}} p(\zeta_1, \zeta_2, \zeta_3)\, d\Omega \tag{E15.6}$$

over a straight-sided triangle can be computed symbolically by the following *Mathematica* module:

```
IntegrateOverTriangle[expr_,tcoord_,A_,max_]:=Module [{p,i,j,k,z1,z2,z3,c,s=0},
   p=Expand[expr];  {z1,z2,z3}=tcoord;
   For [i=0,i<=max,i++, For [j=0,j<=max,j++, For [k=0,k<=max,k++,
      c=Coefficient[Coefficient[Coefficient[p,z1,i],z2,j],z3,k];
      s+=2*c*(i!*j!*k!)/((i+j+k+2)!);
      ]]];
   Return[Simplify[A*s]] ];
```

This is referenced as `int=IntegrateOverTriangle[p,{z1,z2,z3},A,max]`. Here `p` is the polynomial to be integrated, `z1`, `z2` and `z3` denote the symbols used for the triangular coordinates, `A` is the triangle area and `max` the highest exponent appearing in a triangular coordinate. The module name returns the integral. For example, if `p=16+5*b*z2^2+z1^3+z2*z3*(z2+z3)` the call `int=IntegrateOverTriangle[p,{z1,z2,z3},A,3]` returns `int=A*(97+5*b)/6`. Explain how the module works.

001 **EJERCICIO 8** CURSO 2004-5

EXERCISE 15.8

[C+D:25] Access the file `Trig3PlaneStress.nb` from the course Web site by clicking on the appropriate link in Chapter 15 Index. This is a *Mathematica* 4.1 Notebook that does plane stress FEM analysis using the 3-node linear triangle.

Download the Notebook into your directory. Load into *Mathematica*. Execute the top 7 input cells (which are actually initialization cells) so the necessary modules are compiled. Each cell is preceded by a short comment cell which outlines the purpose of the modules it holds. Notes: (1) the plot-module cell may take a while to run through its tests; be patient; (2) to get rid of unsightly messages and silly beeps about similar names, initialize each cell twice.

After you are satisfied everything works fine, run the cantilever beam problem, which is defined in the last input cell.

After you get a feel of how this code operate, study the source. Prepare a hierarchical diagram of the modules,[4] beginning with the main program of the last cell. Note which calls what, and briefly explain the purpose of each module. Return this diagram as answer to the homework. You do not need to talk about the actual run and results; those will be discussed in Part III.

[4] A hierarchical diagram is a list of modules and their purposes, with indentation to show dependence, similar to the table of contents of a book. For example, if module AAAA calls BBBB and CCCC, and BBBB calld DDDD, the hierarchical diagram may look like:

```
AAAA -  purpose of AAAA
  BBBB  - purpose of BBBB
    DDDD - purpose of DDDD
  CCCC  - purpose of CCCC
```

Hint: a hierarchical diagram for `Trig3PlaneStress.nb` begins like

```
Main program in Cell 8 - drives the FEM analysis
   GenerateNodes - generates node coordinates of regular mesh
   GenerateTriangles - generate element node lists of regular mesh
   ........
```

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Custom Material ready to use in the *Mathematica (v8) environment*									
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#11	#12	#13	#14	#15	#16	#17	#18	#19	#20
#21	#22	#23	#24	#25	#26	#27	#28	#29	#30
#31	#32	#33	#34	#35	#36	#37	#38	#39	#40
#41	#42	#43	#44	#45	#46	#47	#48	#49	#50
#51	#52	#53	#54	#55	#56	#57	#58	#59	#60
#61	#62	#63	#64	#65	#66	#67	#68	#69	#70
#71	#72	#73	#74	#75	#76	#77	#78	#79	#80
#81	#82	#83	#84	#85	#86	#87	#88	#89	#90
Each student must download the one corresponding to the number assigned to them									

Una vez completado, deberá subirse adecuadamente denominado a la cuenta de entrega personal, seleccionando del siguiente panel el enlace correspondiente al numero que se te ha asignado en la cuenta del material personalizado de la actividad ***m1-a1a***.

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#21	#22	#23	#24	#25	#26	#27	#28	#29	#30
#31	#32	#33	#34	#35	#36	#37	#38	#39	#40
#41	#42	#43	#44	#45	#46	#47	#48	#49	#50
#51	#52	#53	#54	#55	#56	#57	#58	#59	#60
#61	#62	#63	#64	#65	#66	#67	#68	#69	#70
#71	#72	#73	#74	#75	#76	#77	#78	#79	#80
#81	#82	#83	#84	#85	#86	#87	#88	#89	#90
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