Esta ACTIVIDAD DE CLASE deberá realizarse descargando los documentos *NB* incompletos correspondiente a estos ejercicios de clase. Deberás seleccionar en el siguiente panel el enlace correspondiente al número que se te ha asignado en la cuenta del material personalizado de la actividad ***m1-a1a***.

16-CP-C2-Mathematica-C

001 EJERCICIO 1 CURSO 2004-5

EXERCISE 14.1

[A:25] Suppose that the structural material is isotropic, with elastic modulus E and Poisson's ratio ν. The in-plane stress-strain relations for plane stress ($\sigma_{zz} = \sigma_{xz} = \sigma_{yz} = 0$) and plane strain ($e_{zz} = e_{xz} = e_{yz} = 0$) as given in any textbook on elasticity, are

$$\text{plane stress:} \quad \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix},$$

$$\text{plane strain:} \quad \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix}. \qquad \text{(E14.1)}$$

Show that the constitutive matrix of plane strain can be formally obtained by replacing E by a fictitious modulus E^* and ν by a fictitious Poisson's ratio ν^* in the plane stress constitutive matrix and suppressing the stars. Find the expression of E^* and ν^* in terms of E and ν. This device permits "reusing" a plane stress FEM program to do plane strain, as long as the material is isotropic.

001 EJERCICIO 2 CURSO 2004-5

EXERCISE 14.2

[A:25] In the finite element formulation of near incompressible isotropic materials (as well as plasticity and viscoelasticity) it is convenient to use the so-called *Lamé constants* λ and μ instead of E and ν in the constitutive equations. Both λ and μ have the physical dimension of stress and are related to E and ν by

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad \mu = G = \frac{E}{2(1+\nu)}. \qquad \text{(E14.2)}$$

Conversely

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}, \quad \nu = \frac{\lambda}{2(\lambda + \mu)}. \qquad \text{(E14.3)}$$

Substitute (E14.3) into (E14.1) to express the two stress-strain matrices in terms of λ and μ. Then split the stress-strain matrix $\mathbf{E}$ of plane strain as

$$\mathbf{E} = \mathbf{E}_\mu + \mathbf{E}_\lambda \qquad \text{(E14.4)}$$

in which $\mathbf{E}_\mu$ and $\mathbf{E}_\lambda$ contain only μ and λ, respectively, with $\mathbf{E}_\mu$ diagonal and $E_{\lambda 33} = 0$. This is the Lamé or $\{\lambda, \mu\}$ splitting of the plane strain constitutive equations, which leads to the so-called **B**-bar formulation of near-incompressible finite elements.[3] Express $\mathbf{E}_\mu$ and $\mathbf{E}_\lambda$ also in terms of E and ν.

For the plane stress case perform a similar splitting in which where $\mathbf{E}_\lambda$ contains only $\bar{\lambda} = 2\lambda\mu/(\lambda + 2\mu)$ with $E_{\lambda 33} = 0$, and $\mathbf{E}_\mu$ is a diagonal matrix function of μ and $\bar{\lambda}$.[4] Express $\mathbf{E}_\mu$ and $\mathbf{E}_\lambda$ also in terms of E and ν.

001 EJERCICIO 3 CURSO 2004-5

EXERCISE 14.5

[A:25=5+5+15] A plate is in linearly elastic plane stress. It is shown in courses in elasticity that the internal strain energy density stored per unit volume is

$$\mathcal{U} = \tfrac{1}{2}(\sigma_{xx}e_{xx} + \sigma_{yy}e_{yy} + \sigma_{xy}e_{xy} + \sigma_{yx}e_{yx}) = \tfrac{1}{2}(\sigma_{xx}e_{xx} + \sigma_{yy}e_{yy} + 2\sigma_{xy}e_{xy}). \quad \text{(E14.5)}$$

(a) Show that (E14.5) can be written in terms of strains only as

$$\mathcal{U} = \tfrac{1}{2}\mathbf{e}^T\mathbf{E}\,\mathbf{e}, \quad \text{(E14.6)}$$

and hence justify (14.13).

(b) Show that (E14.5) can be written in terms of stresses only as

$$\mathcal{U} = \tfrac{1}{2}\sigma^T\mathbf{C}\,\sigma, \quad \text{(E14.7)}$$

where $\mathbf{C} = \mathbf{E}^{-1}$ is the elastic compliance (strain-stress) matrix.

(c) Suppose you want to write (E14.5) in terms of the extensional strains $\{e_{xx}, e_{yy}\}$ and of the shear stress $\sigma_{xy} = \sigma_{yx}$. This is known as a mixed representation. Show that

$$\mathcal{U} = \tfrac{1}{2}\begin{bmatrix} e_{xx} \\ e_{yy} \\ \sigma_{xy} \end{bmatrix}^T \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ \sigma_{xy} \end{bmatrix}, \quad \text{(E14.8)}$$

and explain how the entries A_{ij} can be calculated[5] in terms of the elastic moduli E_{ij}.

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#21	#22	#23	#24	#25	#26	#27	#28	#29	#30
#31	#32	#33	#34	#35	#36	#37	#38	#39	#40
#41	#42	#43	#44	#45	#46	#47	#48	#49	#50
#51	#52	#53	#54	#55	#56	#57	#58	#59	#60
#61	#62	#63	#64	#65	#66	#67	#68	#69	#70
#71	#72	#73	#74	#75	#76	#77	#78	#79	#80
#81	#82	#83	#84	#85	#86	#87	#88	#89	#90
Each student must download the one corresponding to the number assigned to them									

Una vez completados, deberán subirse adecuadamente denominado a la cuenta de entrega personal, seleccionando del siguiente panel el enlace correspondiente al numero que se te ha asignado en la cuenta del material personalizado de la actividad ***m1-a1a***.

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#81	#82	#83	#84	#85	#86	#87	#88	#89	#90
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