UNIVERSIDAD POLITECNICA DE VALENCIA

DEPARTAMENTO DE INGENIERIA MECANICA Y DE MATERIALES

ELEMENTOS FINITOS

(E.T.S.I.I.V)

IMPLEMENTACION COMPUTACIONAL DEL METODO EF

LECCION 13.- CALCULO TENSIONES

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001 ¿POR QUE CALCULAR LAS TENSIONES? CURSO 2004-5

Stress calculations are of interest because in structural analysis and design the stresses are often more important to the engineer than displacements.

001 ¿QUÉ NOMBRE RECIBEN LAS TECNICAS QUE SE UTILIZAN? CURSO 2004-5

These procedures receive the generic name *stress recovery techniques* in the finite element literature. In the following sections we cover the simplest stress recovery

001 ¿COMO CALCULAR LAS TENSIONES A PARTIR DE LOS DESPLAZAMIENTOS? CURSO 2004-5

SE HA SOLUCIONADO EL PROBLEMA DE LOS DESPLAZAMIENTOS EN LOS NODOS

$$\mathbf{K}\mathbf{u} = \mathbf{f}$$

SE CALCULAN LAS DEFORMACIONES EN CUALQUIER PUNTO DE CUALQUIER ELEMENTO

$$\mathbf{e} = \mathbf{B}\mathbf{u}^{(e)}$$

where $\mathbf{B}$ is the strain-displacement matrix (14.18) assembled with the x and y derivatives of the element shape functions evaluated at the point where we are calculating strains.

Y SE CALCULAN LAS TENSIONES EN CUALQUIER PUNTO DE CUALQUIER ELMENTO COMO

$$\boldsymbol{\sigma} = \mathbf{E}\mathbf{e} = \mathbf{E}\mathbf{B}\mathbf{u}$$

001 ¿EN QUE PUNTOS SE DEBEN CALCULAR LAS TENSIONES Y QUE SUCEDE CON ELLAS? CURSO 2004-5

NORMALMENTE EN LOS NODOS DE LOS ELEMENTOS

In the applications it is of interest to evaluate and report these stresses at the *element nodal points* located on the corners and possibly midpoints of the element. These are called *element nodal point stresses*.

It is important to realize that the stresses computed at the same nodal point from adjacent elements *will not generally be the same*, since stresses are not required to be continuous in displacement-assumed finite elements. This suggests some form of stress averaging can be used to improve the stress accuracy, and indeed this is part of the stress recovery technique further discussed in §29.5. The results from this averaging procedure are called *nodal point stresses*.

001 ¿COMO CALCULAR LAS TENSIONES EN LOS NODOS? CURSO 2004-5

1. Evaluate directly $\boldsymbol{\sigma}$ at the element node locations by substituting the natural coordinates of the nodal points as arguments to the shape function modules. These modules return $\mathbf{q}_x$ and $\mathbf{q}_y$ and direct application of (29.2)-(29.4) yields the strains and stresses at the nodes.

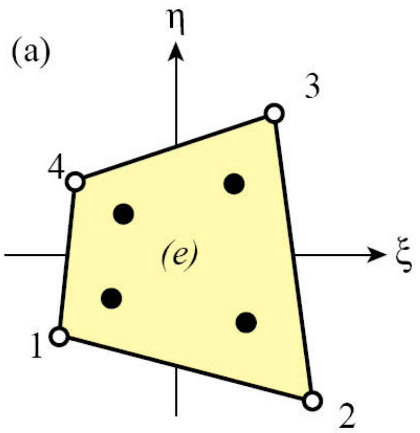
2. Evaluate $\boldsymbol{\sigma}$ at the Gauss integration points used in the element stiffness integration rule and then extrapolate to the element node points.

001 ¿CUAL DE LAS DOS TECNICAS ES MAS ES MAS CONVENIENTE Y EN QUE TIPOS DE ELEMENTOS? **CURSO 2004-5**

Empirical evidence indicates that the second approach generally delivers better stress values for *quadrilateral* elements whose geometry departs substantially from the rectangular shape. This is backed up by "superconvergence" results in finite element approximation theory. For rectangular elements there is no difference.

For isoparametric *triangles* both techniques deliver similar results (identical if the elements are straight sided with midside nodes at midpoints) and so the advantages of the second one are marginal. Both approaches are covered in the sequel.

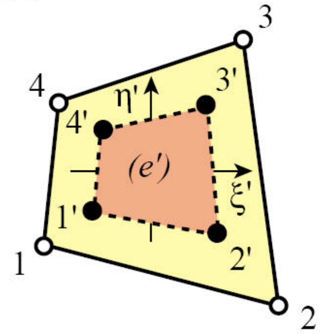
001 TECNICA DE EXTRAPOLACION DESDE LOS PUNTOS DE GAUSS ELEMENTO CUADRILATERO 4 NODOS **CURSO 2004-5**



Corner node	ξ	η	ξ'	η'	Gauss node	ξ	η	ξ'	η'
1	-1	-1	$-\sqrt{3}$	$-\sqrt{3}$	1'	$-1/\sqrt{3}$	$-1/\sqrt{3}$	-1	-1
2	$+1$	-1	$+\sqrt{3}$	$-\sqrt{3}$	2'	$+1/\sqrt{3}$	$-1/\sqrt{3}$	$+1$	-1
3	$+1$	$+1$	$+\sqrt{3}$	$+\sqrt{3}$	3'	$+1/\sqrt{3}$	$+1/\sqrt{3}$	$+1$	$+1$
4	-1	$+1$	$-\sqrt{3}$	$+\sqrt{3}$	4'	$-1/\sqrt{3}$	$+1/\sqrt{3}$	-1	$+1$

The stresses are calculated at the Gauss points, which are identified as $1'$, $2'$, $3'$ and $4'$ in Figure 29.1. Point i' is closest to node i so it is seen that Gauss point numbering essentially follows element node numbering in the counterclockwise sense. The natural coordinates of these points are listed in Table 29.1. The stresses are evaluated at these Gauss points by passing these natural coordinates to the shape function subroutine. Then each stress component is "carried" to the corner nodes 1 through 4 through a bilinear extrapolation based on the computed values at $1'$ through $4'$.

(b)



ELEMENTO DE GAUSS

To understand the extrapolation procedure more clearly it is convenient to consider the region bounded by the Gauss points as an "internal element" or "Gauss element". This interpretation is depicted in Figure 29.1(b). The Gauss element, denoted by (e'), is also a four-node quadrilateral. Its quadrilateral (natural) coordinates are denoted by ξ' and η'. These are linked to ξ and η by the simple relations

$$\xi = \xi'/\sqrt{3}, \quad \eta = \eta'/\sqrt{3}, \qquad \xi' = \xi\sqrt{3}, \quad \eta' = \eta\sqrt{3}.$$

Any scalar quantity w whose values w'_i at the Gauss element corners are known can be interpolated through the usual bilinear shape functions now expressed in terms of ξ' and η':

$$w(\xi', \eta') = [\, w'_1 \quad w'_2 \quad w'_3 \quad w'_4 \,] \begin{bmatrix} N_1^{(e')} \\ N_2^{(e')} \\ N_3^{(e')} \\ N_4^{(e')} \end{bmatrix}$$

$$\begin{aligned} N_1^{(e')} &= \tfrac{1}{4}(1-\xi')(1-\eta'), \\ N_2^{(e')} &= \tfrac{1}{4}(1+\xi')(1-\eta'), \\ N_3^{(e')} &= \tfrac{1}{4}(1+\xi')(1+\eta'), \\ N_4^{(e')} &= \tfrac{1}{4}(1-\xi')(1+\eta'). \end{aligned}$$

To extrapolate w to corner 1, say, we replace its ξ' and η' coordinates, namely $\xi' = \eta' = -\sqrt{3}$, into the above formula. Doing that for the four corners we obtain

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 1+\frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1-\frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ -\frac{1}{2} & 1+\frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1-\frac{1}{2}\sqrt{3} \\ 1-\frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1+\frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ -\frac{1}{2} & 1-\frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1+\frac{1}{2}\sqrt{3} \end{bmatrix} \begin{bmatrix} w'_1 \\ w'_2 \\ w'_3 \\ w'_4 \end{bmatrix}$$

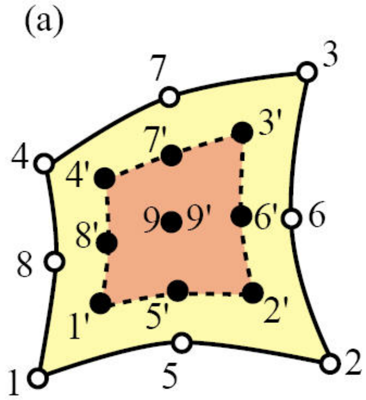
Note that the sum of the coefficients in each row is one, as it should be. For stresses we apply this formula taking w to be each of the three stress components, σ_{xx}, σ_{yy} and τ_{xy}, in turn.

001 TECNICA DE EXTRAPOLACION DESDE LOS PUNTOS DE GAUSS CURSO 2004-5

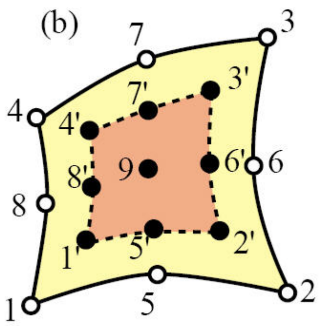
ELEMENTOS DE ORDEN SUPERIOR

For eight-node and nine-node isoparametric quadrilaterals the usual Gauss integration rule is 3×3, and the Gauss elements are nine-noded quadrilaterals that look as in Figure 29.2(a) and (b) above. For six-node triangles the usual quadrature is the 3-point rule with internal sampling points, and the Gauss element is a three-node triangle as shown in Figure 29.2(c).

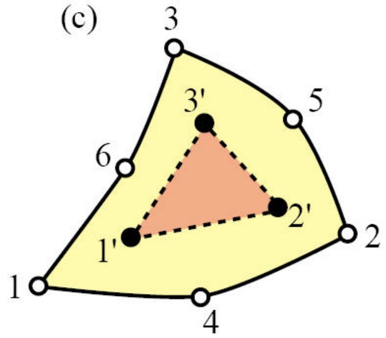
ELEMENTO CUADRILATERO DE 9 NODOS



ELEMENTO CUADRILATERO DE 8 NODOS



ELEMENTO TRIANGULO DE 6 NODOS



001 PROMEDIADO ENTRE ELEMENTOS CURSO 2004-5

The stresses computed in element-by-element fashion as discussed above, whether by direct evaluation at the nodes or by extrapolation, will generally exhibit jumps between elements. For printing and plotting purposes it is usually convenient to "smooth out" those jumps by computing *averaged nodal stresses*. This averaging may be done in two ways:

(I) Unweighted averaging: assign same weight to all elements that meet at a node;

(II) Weighted averaging: the weight assigned to element contributions depends on the stress component and the element geometry and possibly the element type.

Several weighted average schemes have been proposed in the finite element literature, but they do require additional programming.