

UNIVERSIDAD POLITECNICA DE VALENCIA
DEPARTAMENTO DE INGENIERIA MECANICA Y DE MATERIALES

ELEMENTOS FINITOS
(E.T.S.I.I.V)

IMPLEMENTACION COMPUTACIONAL DEL METODO EF

LECCION 12.- PROGRAMA COMPLETO DE EF
PROBLEMA TENSION PLANA

J. L. OLIVER
Dr. Ingeniero Industrial

Valencia, 2005

Preprocessing: problem definition

Processing: problem solution

Postprocessing: showing results

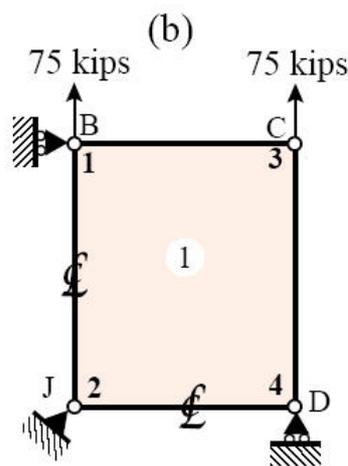
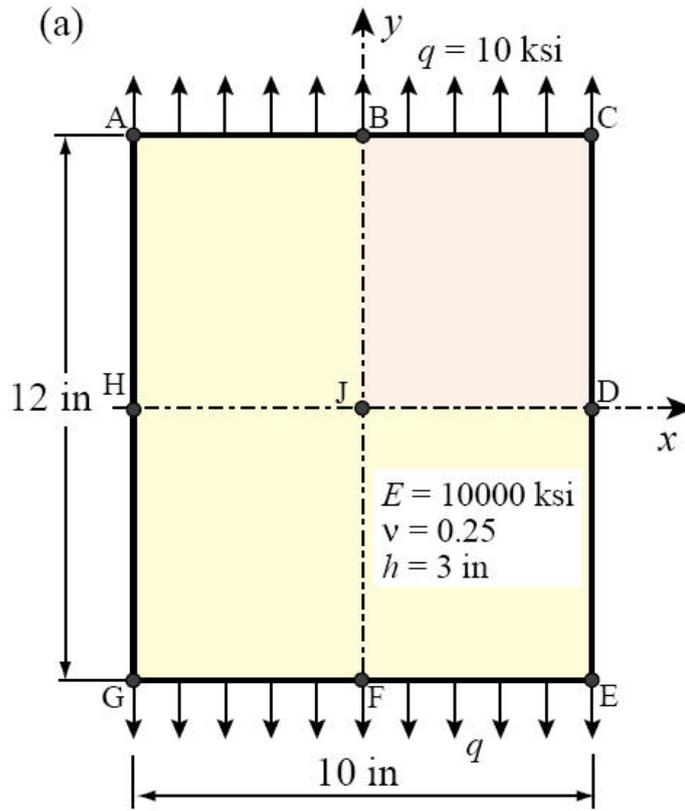
Geometry Data Set:
NodeCoordinates

Element Data Set:
ElemTypes, ElemNodeLists,
ElemMaterial, ElemFabrication

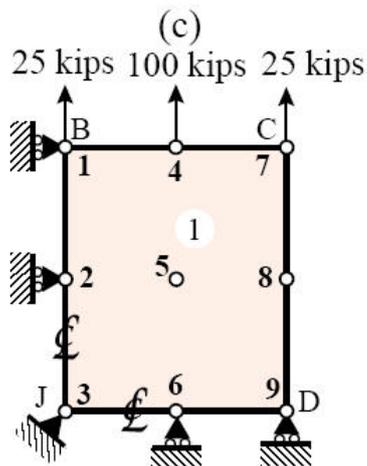
Degree of Freedom Data Set:
FreedomTags, FreedomValues

Miscellaneous Processing Data Set:
ProcessOptions

Figure 27.1 is a rectangular plate in plane stress under uniform uniaxial loading. Its analytical solution is $\sigma_{yy} = q$, $\sigma_{xx} = \sigma_{xy} = 0$, $u_y = qy/E$, $u_x = -vqx/E$. This problem should be solved exactly by *any* finite element mesh. In particular, the two one-element models shown on the right of that figure.

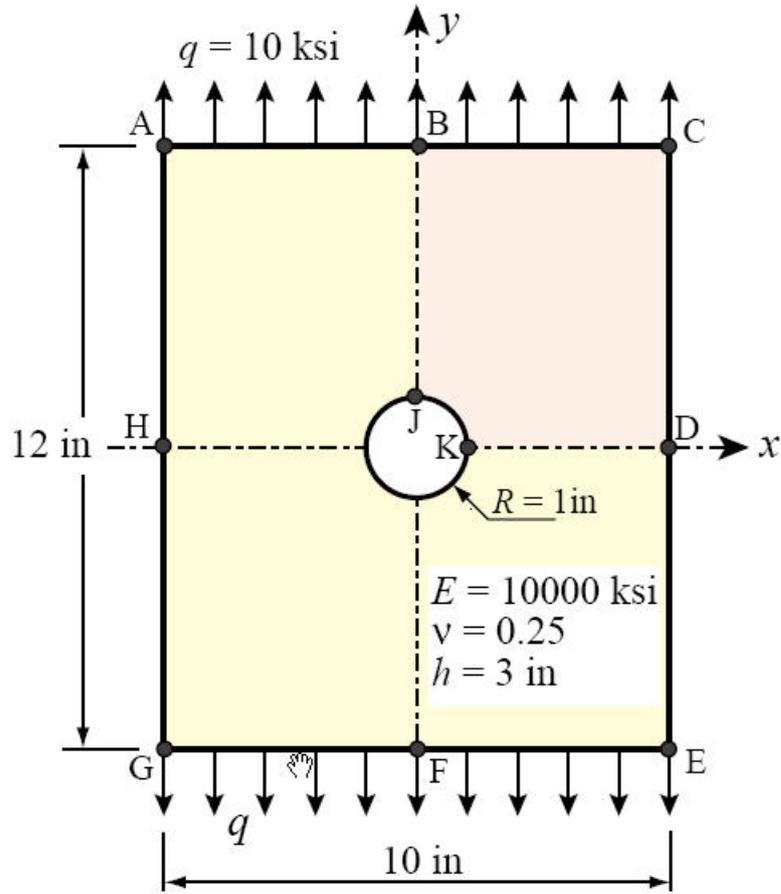


Model (I):
 4 nodes, 8 DOFs,
 1 bilinear quad

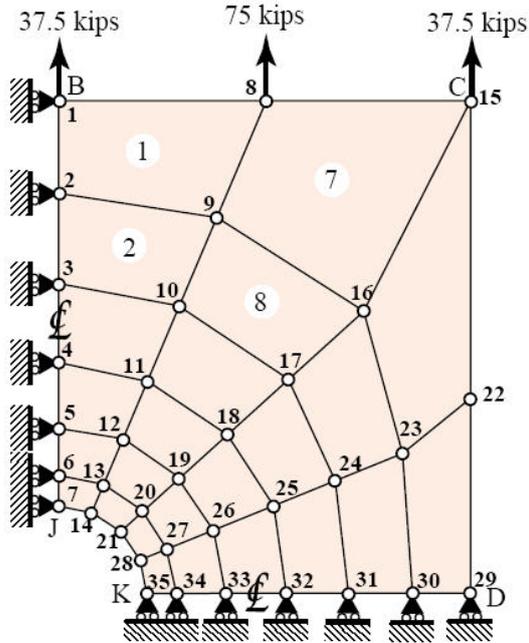


Model (II):
 9 nodes, 18 DOFs,
 1 biquadratic quad

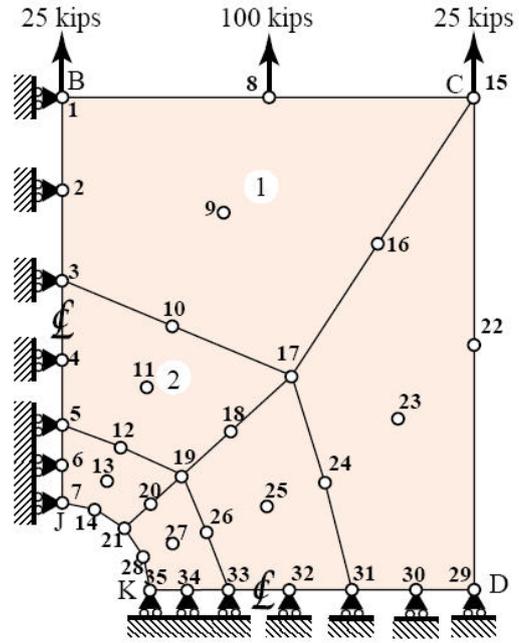
Global node numbers shown



Node 8 is exactly midway between 1 and 15



Model (I): 35 nodes, 70 DOFs,
 24 bilinear quads



Model (II): 35 nodes, 70 DOFs,
 6 biquadratic quads

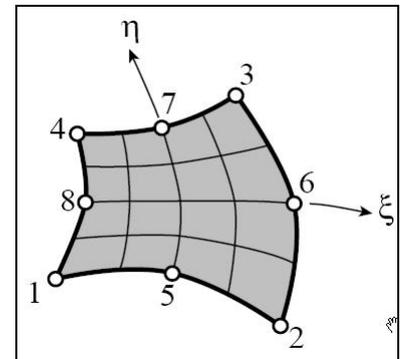
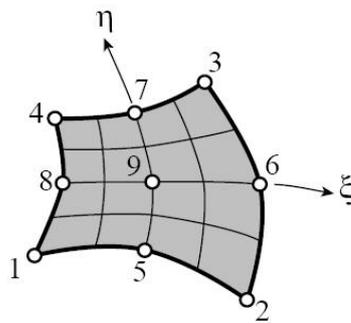
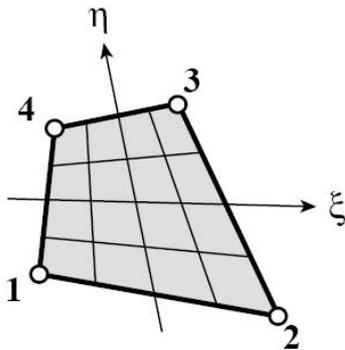
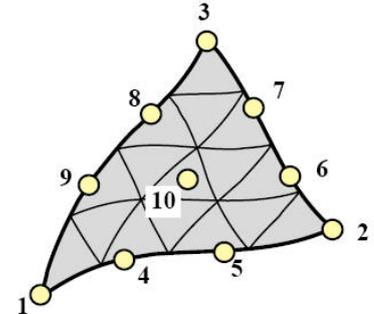
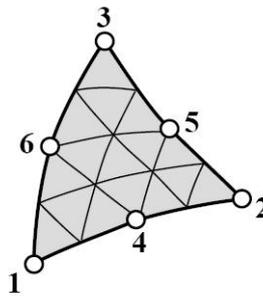
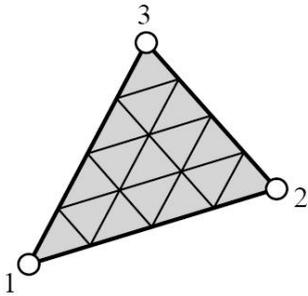
COORDENADAS NODALES

$$\text{NodeCoordinates} = \{ \{x_1, y_1\}, \{x_2, y_2\}, \dots, \{x_N, y_N\} \}$$

TIPO DE ELEMENTO

$$\text{ElemTypes} = \{ \{ \text{etyp}^{(1)} \}, \{ \text{etyp}^{(2)} \}, \dots, \{ \text{etyp}^{(N_e)} \} \}$$

| Identifier | Plane Stress Model |
|------------|---------------------------|
| "Trig3" | 3-node linear triangle |
| "Trig6" | 6-node quadratic triangle |
| "Trig10" | 10-node cubic triangle |
| "Quad4" | 4-node bilinear quad |
| "Quad9" | 9-node biquadratic quad |



CONECTIVIDAD ELEMENTOS

$$\text{ElemNodeLists} = \{ \{ \text{eNL}^{(1)} \}, \{ \text{eNL}^{(2)} \}, \dots, \{ \text{eNL}^{(N_e)} \} \}$$

Element boundaries must be traversed counterclockwise but you can start at any corner. Numbering elements with midnodes requires more care: first list corners counterclockwise, followed by midpoints (first midpoint is the one that follows first corner when going counterclockwise). When elements have an interior node, as in the 9-node biquadratic quadrilateral, that node goes last.

PROPIEDADES DEL MATERIAL

$$\text{ElemMaterial} = \{ \{ E^{(1)}, \nu^{(1)} \}, \{ E^{(2)}, \nu^{(2)} \}, \dots \{ E^{(N_e)}, \nu^{(N_e)} \} \}$$

In the common case in which all elements have the same E and ν , this list can be easily generated by a Table instruction.

$$\text{ElemMaterial} = \text{Table} [\{ \text{Em}, \text{nu} \}, \{ \text{numele} \}]$$

$$\text{numele} = \text{Length} [\text{ElemNodeLists}]$$

PROPIEDADES GEOMETRICAS

$$\text{ElemFabrication} = \{ \{ h^{(1)} \}, \{ h^{(2)} \}, \dots \{ h^{(N_e)} \} \}$$

INDICADORES DE LIBERTAD - DISPLAZAMIENTO/FUERZA

FreedomTags labels each node degree of freedom as to whether the load or the displacement is specified. The configuration of this list is similar to that of NodeCoordinates:

$$\text{FreedomTags} = \{ \{ \text{tag}_{x1}, \text{tag}_{y1} \}, \{ \text{tag}_{x2}, \text{tag}_{y2} \}, \dots \{ \text{tag}_{xN}, \text{tag}_{yN} \} \}$$

The tag value is 0 if the force is specified and 1 if the displacement is specified. When there are a lot of nodes, the quickest way to specify this list is to start from all zeros, and then insert the boundary conditions appropriately.

VALORES DE LIBERTAD - DISPLAZAMIENTO/FUERZA

FreedomValues has the same node by node configuration as FreedomTags. It lists the specified values of the applied node force component if the corresponding tag is zero, and of the prescribed displacement component if the tag is one. Typically most of the list entries are zero.

OPCIONES DE PROCESADO

$$\text{ProcessingOptions} = \{ \text{True} \};$$

This specifies floating point numerical computations.

VISUALIZACION DE LA DISCRETIZACION (MALLA)

$$\text{aspect} = 6/5;$$

$$\text{Plot2DElementsAndNodes} [\text{NodeCoordinates}, \text{ElemNodeLists}, \text{aspect}, \\ \text{"Plate with circular hole - 4-node quad model"}, \text{True}, \text{True}];$$

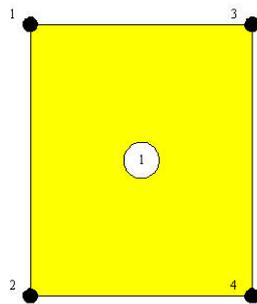
Here aspect is the plot frame aspect ratio (y dimension over x dimension), and the last two True argument values specify that node labels and element labels, respectively, be shown.

001

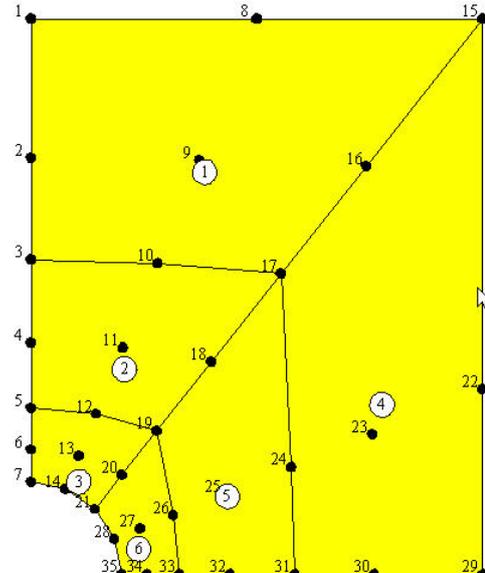
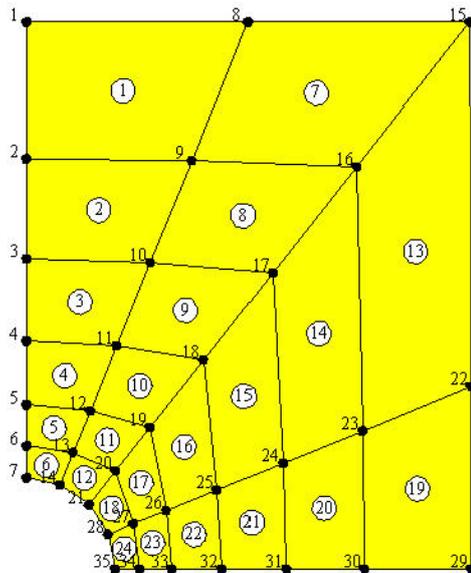
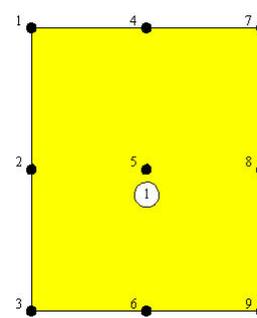
VISUALIZACION MALLAS EJEMPLOS

CURSO 2004-5

MALLA UN ELEMENTO - CUADRILATERO 4 NODOS



UN SOLO ELEMENTO - CUADRILATERO 9 NODOS



001

PROCESADO DEL MODELO

CURSO 2004-5

```
{u,f,sig}=MembraneSolution[NodeCoordinates,ElemTypes,
  ElemNodeLists,ElemMaterial,ElemFabrication,
  ProcessingOptions,FreedomTags,FreedomValues];
```

```
MembraneSolution[nodcoor_,eletyp_,elenod_,elemat_,
  elefab_,eleopt_,dof>tag_,dofval_] := Module[{K,Kmod,u,f,sig,j,n,ns,
  supdof,supval,numnod=Length[nodcoor],numele=Length[elenod]},
  u=f=sig={};
  K=MembraneMasterStiffness[nodcoor,
    eletyp_,elenod_,elemat_,elefab_,eleopt_]; K=N[K];
  ns=0; Do [Do [If[dof>tag>[n,j]]>0,ns++],{j,1,2}],{n,1,numnod}];
  supdof=supval=Table[0,{ns}];
  k=0; Do [Do [If[dof>tag>[n,j]]>0,k++;supdof[[k]]=2*(n-1)+j;
    supval[[k]]=dofval[[n,j]]],{j,1,2}],{n,1,numnod}];
  f=ModifiedNodeForces[supdof,supval,K,Flatten[dofval]];
  Kmod=ModifiedMasterStiffness[supdof,K];
  u=Simplify[Inverse[Kmod].f]; u=Chop[u];
  f=Simplify[K.u]; f=Chop[f];
  sig=MembraneNodalStresses[nodcoor,
    eletyp_,elenod_,elemat_,elefab_,eleopt_,u];
  sig=Chop[sig];
  Return[{u,f,sig}];
];
```

1 - ENSAMBLADO DE LA MATRIZ RIGIDEZ GLOBAL

The function begins by assembling the free-free master stiffness matrix K by calling the module MembraneMasterStiffness. this module is listed in Figure 27.5. As a study of its code reveals, it can handle the five element types described in the previous section. The modules that compute the element stiffness matrices have been studied in previous Chapters and need not be listed here.

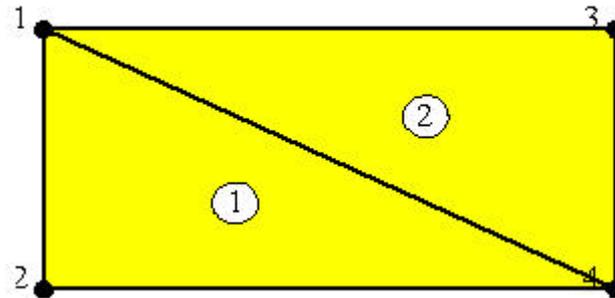
```

MembraneMasterStiffness[nodcoor_,eletyp_,elenod_,
  elemat_,elefab_,eleopt_]:=
Module[{numele=Length[elenod],numnod=Length[nodcoor],numer,
  ne,eNL,eftab,neldof,i,n,Em,v,Emat,th,ncoor,Ke,K},
K=Table[0,{2*numnod},{2*numnod}]; numer=eleopt[[1]];
For [ne=1,ne<=numele,ne++,
  {type}=eletyp[[ne]];
  If [type!="Trig3"&&type!="Trig6"&&type!="Trig10"&&
    type!="Quad4"&&type!="Quad9",
    Print["Illegal element type,ne=",ne]; Return[Null]];
  eNL=elenod[[ne]]; n=Length[eNL];
  eftab=Flatten[Table[{2*eNL[[i]]-1,2*eNL[[i]]},{i,n}]];
  ncoor=Table[nodcoor[[eNL[[i]]]},{i,n}];
  {Em,v}=elemat[[ne]];
  Emat=Em/(1-v^2)*{{1,v,0},{v,1,0},{0,0,(1-v)/2}};
  {th}=elefab[[ne]];
  If [type=="Trig3", Ke=Trig3IsoPMembraneStiffness[ncoor,
    {Emat,0,0},{th},{numer}] ];
  If [type=="Quad4", Ke=Quad4IsoPMembraneStiffness[ncoor,
    {Emat,0,0},{th},{numer,2}] ];
  If [type=="Trig6", Ke=Trig6IsoPMembraneStiffness[ncoor,
    {Emat,0,0},{th},{numer,3}] ];
  If [type=="Quad9", Ke=Quad9IsoPMembraneStiffness[ncoor,
    {Emat,0,0},{th},{numer,3}] ];
  If [type=="Trig10",Ke=Trig10IsoPMembraneStiffness[ncoor,
    {Emat,0,0},{th},{numer,3}] ];
  neldof=Length[Ke];
  For [i=1,i<=neldof,i++,ii=eftab[[i]];
    For [j=i,j<=neldof,j++,jj=eftab[[j]];
      K[[jj,ii]]=K[[ii,jj]]+=Ke[[i,j]]
    ];
  ];
  ];
Return[K];
];

```

EJEMPLO ENSAMBLADO DE LAS MATRICES DE DOS TRIANGULOS

MALLA DOS ELEMENTOS - TRIANGULOS 3 NODOS



Out[37]//MatrixForm=

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 7 \\ 8 \end{array} \begin{pmatrix} \frac{24}{5} & 0 & -\frac{24}{5} & -\frac{12}{5} & 0 & \frac{12}{5} \\ 0 & \frac{64}{5} & -\frac{8}{5} & -\frac{64}{5} & \frac{8}{5} & 0 \\ -\frac{24}{5} & -\frac{8}{5} & 8 & 4 & -\frac{16}{5} & -\frac{12}{5} \\ -\frac{12}{5} & -\frac{64}{5} & 4 & 14 & -\frac{8}{5} & -\frac{6}{5} \\ 0 & \frac{8}{5} & -\frac{16}{5} & -\frac{8}{5} & \frac{16}{5} & 0 \\ \frac{12}{5} & 0 & -\frac{12}{5} & -\frac{6}{5} & 0 & \frac{6}{5} \end{pmatrix}$$

Out[36]//MatrixForm=

$$\begin{array}{c} 1 \\ 2 \\ 7 \\ 8 \\ 5 \\ 6 \end{array} \begin{pmatrix} 2 & 0 & 0 & 2 & -2 & -2 \\ 0 & \frac{1}{2} & 1 & 0 & -1 & -\frac{1}{2} \\ 0 & 1 & 2 & 0 & -2 & -1 \\ 2 & 0 & 0 & 8 & -2 & -8 \\ -2 & -1 & -2 & -2 & 4 & 3 \\ -2 & -\frac{1}{2} & -1 & -8 & 3 & \frac{17}{2} \end{pmatrix}$$

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{pmatrix} \frac{34}{5} & 0 & -\frac{24}{5} & -\frac{12}{5} & -2 & -2 & 0 & \frac{22}{5} \\ 0 & \frac{133}{10} & -\frac{8}{5} & -\frac{64}{5} & -1 & -\frac{1}{2} & \frac{13}{5} & 0 \\ -\frac{24}{5} & -\frac{8}{5} & 8 & 4 & 0 & 0 & -\frac{16}{5} & -\frac{12}{5} \\ -\frac{12}{5} & -\frac{64}{5} & 4 & 14 & 0 & 0 & -\frac{8}{5} & -\frac{6}{5} \\ -2 & -1 & 0 & 0 & 4 & 3 & -2 & -2 \\ -2 & -\frac{1}{2} & 0 & 0 & 3 & \frac{17}{2} & -1 & -8 \\ 0 & \frac{13}{5} & -\frac{16}{5} & -\frac{8}{5} & -2 & -1 & \frac{26}{5} & 0 \\ \frac{22}{5} & 0 & -\frac{12}{5} & -\frac{6}{5} & -2 & -8 & 0 & \frac{46}{5} \end{pmatrix}$$

MATRIZ RIGIDEZ GLOBAL

2 - APLICACIÓN CONDICIONES DE CONTORNO Y CARGAS

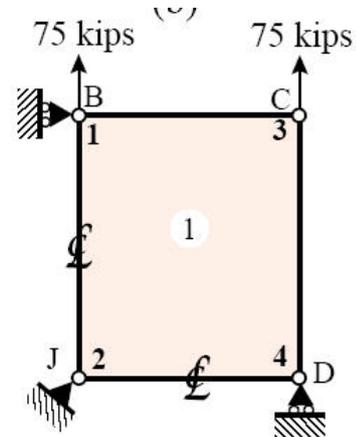
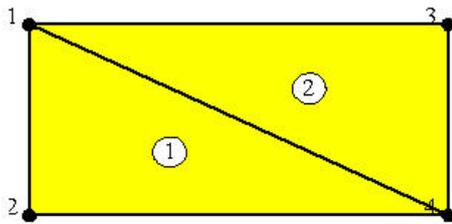
The displacement boundary conditions are applied by modules ModifiedMasterStiffness and ModifiedNodeForces, which return the modified stiffness matrix $\hat{\mathbf{K}}$ and the modified force vector $\hat{\mathbf{f}}$ in Khat and fnat, respectively.

```
ModifiedMasterStiffness[pdof_, Km_] := Module[{i, j, k, K}, K = Km;
  For[k = 1, k ≤ Length[pdof], k++, i = pdof[[k]];
  For[j = 1, j ≤ Length[K], j++, K[[i, j]] = K[[j, i]] = 0]; K[[i, i]] = 1;]; Return[K];;
```

```
ModifiedNodeForces[pdof_, pdofv_, Km_, nfv_] :=
  Module[{i, j, k, l, d, kk = Length[pdof], n = Length[Km], fixed, rhs}, rhs = nfv;
  d = pdofv; fixed = Table[False, {n}];
  Do[i = pdof[[k]]; fixed[[i]] = True, {k, 1, kk}];
  For[k = 1, k ≤ kk, k++, i = pdof[[k]];
  For[j = 1, j ≤ n, j++, If[fixed[[j]], Continue[]];
  rhs[[j]] = rhs[[j]] - Km[[i, j]] * d[[k]];]; rhs[[i]] = d[[k]];];
  Return[rhs];;
```

EJEMPLO DOS TRIANGULOS

MALLA DOS ELEMENTOS - TRIANGULOS 3 NODOS



$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{133}{10} & 0 & 0 & -1 & -\frac{1}{2} & \frac{13}{5} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 4 & 3 & -2 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 3 & \frac{17}{2} & -1 & 0 \\ 0 & \frac{13}{5} & 0 & 0 & -2 & -1 & \frac{26}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

MATRIZ RIGIDEZ GLOBAL MODIFICADA CON CONDICIONES CONTORNO

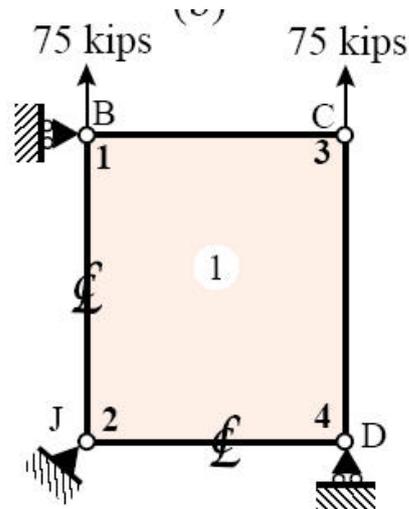
3 - OBTENCION DESPLAZAMIENTOS NODALES Y FUERZAS DE REACCION

The unknown node displacements u are then obtained through the built in `LinearSolve` function, as $u = \text{LinearSolve}[K_{\text{hat}}, f_{\text{hat}}]$. This solution strategy is of course restricted to very small systems, but it has the advantages of simplicity.

The function returns arrays u , f and p , which are lists of length 12, 12 and 13, respectively. Array u contains the computed node displacements ordered $u_{x1} < u_{y1}, u_{x2}, \dots, u_{y8}$. Array f contains the node forces recovered from $f = Ku$; this includes the reactions f_{x1}, f_{y1} and f_{y8} .

```
MembraneSolution[nodcoor_, eletyp_, elenod_, elemat_, elefab_, eleopt_,
  doftag_, dofval_] :=
Module[{K, Kmod, u, f, sig, j, n, ns, supdof, supval, numnod = Length[nodcoor],
  numele = Length[elenod]}, u = f = sig = {};
K = MembraneMasterStiffness[nodcoor, eletyp, elenod, elemat, elefab, eleopt];
Print["Master Stiff K"]; Print[MatrixForm[K]]; K = N[K];
Print["eigs of K=", Chop[Eigenvalues[K]]]; ns = 0;
Do[Do[If[doftag[[n, j]] > 0, ns++], {j, 1, 2}], {n, 1, numnod}];
Print["doftag=", doftag]; Print["ns=", ns]; supdof = supval = Table[0, {ns}];
k = 0; Do[Do[If[doftag[[n, j]] > 0, k++]; supdof[[k]] = 2 * (n - 1) + j;
  supval[[k]] = dofval[[n, j]], {j, 1, 2}], {n, 1, numnod}]; Print["supdof=", supdof];
f = ModifiedNodeForces[supdof, supval, K, Flatten[dofval]];
Print["Vector Fuerzas f"]; Print[MatrixForm[f]];
Kmod = ModifiedMasterStiffness[supdof, K]; Print["Matriz Rigidez Modificada Kmod"];
Print[MatrixForm[Kmod]];
Print["eigs of Kmod=", Eigenvalues[Kmod]]; u = Simplify[Inverse[Kmod].f];
u = Chop[u]; Print["Desplazamientos Nodales u=", MatrixForm[u]];
f = Simplify[K.u]; f = Chop[f]; Print["Fuerzas de Reaccion f=", MatrixForm[f]];
sig = MembraneNodalStresses[nodcoor, eletyp, elenod, elemat, elefab, eleopt, u];
sig = Chop[sig];
Return[{u, f, sig}];
];
```

EJEMPLO PLACA CON UN ELEMENTO CUADRILATERO 4 NODOS



```
{u, f, sig} = MembraneSolution[NodeCoordinates, ElemTypes, ElemNodeLists,
  ElemMaterial, ElemFabrication, ElemOptions, FreedomTags, FreedomValues];
```

Master Stiff K

$$\begin{pmatrix} 16133.3 & -5000. & 3066.67 & -1000. & -11133.3 & 1000. & -8066.67 & 5000. \\ -5000. & 13688.9 & 1000. & -6488.89 & -1000. & -355.556 & 5000. & -6844.44 \\ 3066.67 & 1000. & 16133.3 & 5000. & -8066.67 & -5000. & -11133.3 & -1000. \\ -1000. & -6488.89 & 5000. & 13688.9 & -5000. & -6844.44 & 1000. & -355.556 \\ -11133.3 & -1000. & -8066.67 & -5000. & 16133.3 & 5000. & 3066.67 & 1000. \\ 1000. & -355.556 & -5000. & -6844.44 & 5000. & 13688.9 & -1000. & -6488.89 \\ -8066.67 & 5000. & -11133.3 & 1000. & 3066.67 & -1000. & 16133.3 & -5000. \\ 5000. & -6844.44 & -1000. & -355.556 & 1000. & -6488.89 & -5000. & 13688.9 \end{pmatrix}$$

eigs of K={42453.9, 24400., 22612.8, 16133.3, 13688.9, 0, 0, 0}

MATRIZ RIGIDEZ GLOBAL Y VALORES PROPIOS

doftag={{1, 0}, {1, 1}, {0, 0}, {0, 1}}

ns=4

supdof={1, 3, 4, 8}

GRADOS DE LIBERTAD RESTRINGIDOS - CONDICIONES CONTORNO

Vector Fuerzas f

$$\begin{pmatrix} 0 \\ 75 \\ 0 \\ 0 \\ 0 \\ 75 \\ 0 \\ 0 \end{pmatrix}$$

VECTOR FUERZAS NODALES APLICADAS

Matriz Rigidez Modificada Kmod

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 13688.9 & 0 & 0 & -1000. & -355.556 & 5000. & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1000. & 0 & 0 & 16133.3 & 5000. & 3066.67 & 0 \\ 0 & -355.556 & 0 & 0 & 5000. & 13688.9 & -1000. & 0 \\ 0 & 5000. & 0 & 0 & 3066.67 & -1000. & 16133.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

eigs of Kmod={21227., 19575.4, 11306.4, 7535.67, 1., 1., 1., 1.}

MATRIZ RIGIDEZ GLOBAL MODIFICADAS CON CONDICIONES CONTORNO

$$\mathbf{Ku} = \mathbf{f}$$

PROBLEMA A RESOLVER

$$\text{Desplazamientos Nodales } u = \begin{pmatrix} 0 \\ 0.006 \\ 0 \\ 0 \\ -0.00125 \\ 0.006 \\ -0.00125 \\ 0 \end{pmatrix}$$

SOLUCION DESPLAZAMIENTOS NODALES

$$u_y = qy/E, u_x = -\nu qx/E$$

SOLUCION EXACTA

$$\text{Fuerzas de Reaccion } f = \begin{pmatrix} 0 \\ 75. \\ 0 \\ -75. \\ 0 \\ 75. \\ 0 \\ -75. \end{pmatrix}$$

VECTOR FUERZAS APLICADAS Y DE REACCION

1 - OBTENCION TENSIONES NODALES

Finally, array sig contains the nodal stresses σ_{xx} , σ_{yy} , σ_{xy} at each node, recovered from the displacement solution. This computation is driven by module MembraneNodeStresses, which is

```

MembraneMasterStiffness[nodcoor_, elotyp_, elenod_,
  elemat_, elefab_, eleopt_] :=
Module[{numele=Length[elenod], numnod=Length[nodcoor], numer,
  ne, eNL, eftab, neldof, i, n, Em, v, Emat, th, ncoor, Ke, K},
K=Table[0, {2*numnod}, {2*numnod}]; numer=eleopt[[1]];
For [ne=1, ne<=numele, ne++,
  {type}=elotyp[[ne]];
  If [type!="Trig3"&&type!="Trig6"&&type!="Trig10"&&
    type!="Quad4"&&type!="Quad9",
    Print["Illegal element type, ne=", ne]; Return[Null]];
  eNL=elenod[[ne]]; n=Length[eNL];
  eftab=Flatten[Table[{2*eNL[[i]]-1, 2*eNL[[i]]}, {i, n}]];
  ncoor=Table[nodcoor[[eNL[[i]]]], {i, n}];
  {Em, v}=elemat[[ne]];
  Emat=Em/(1-v^2)*{{1, v, 0}, {v, 1, 0}, {0, 0, (1-v)/2}};
  {th}=elefab[[ne]];
  If [type=="Trig3", Ke=Trig3IsoPMembraneStiffness[ncoor,
    {Emat, 0, 0}, {th}, {numer}]]];
  If [type=="Quad4", Ke=Quad4IsoPMembraneStiffness[ncoor,
    {Emat, 0, 0}, {th}, {numer, 2}]]];
  If [type=="Trig6", Ke=Trig6IsoPMembraneStiffness[ncoor,
    {Emat, 0, 0}, {th}, {numer, 3}]]];
  If [type=="Quad9", Ke=Quad9IsoPMembraneStiffness[ncoor,
    {Emat, 0, 0}, {th}, {numer, 3}]]];
  If [type=="Trig10", Ke=Trig10IsoPMembraneStiffness[ncoor,
    {Emat, 0, 0}, {th}, {numer, 3}]]];
  neldof=Length[Ke];
  For [i=1, i<=neldof, i++, ii=eftab[[i]];
    For [j=i, j<=neldof, j++, jj=eftab[[j]];
      K[[jj, ii]]=K[[ii, jj]]+=Ke[[i, j]]
    ];
  ];
];
Return[K];
];

```

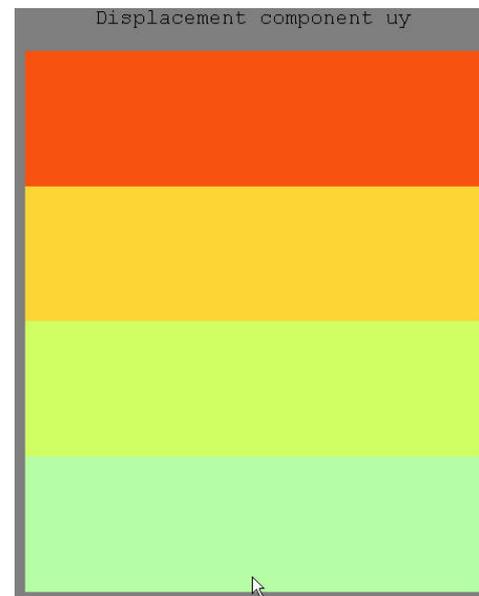
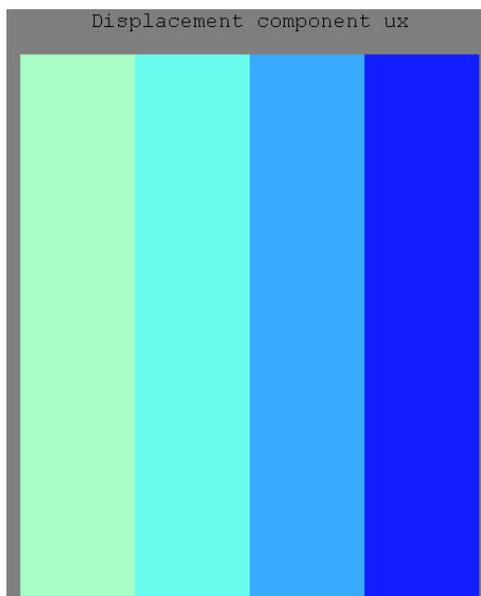
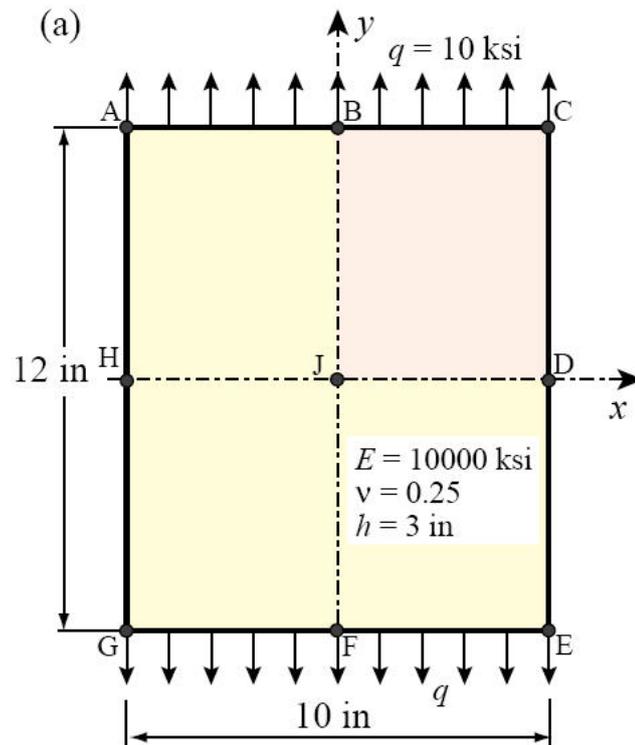
2 VISUALIZACION DESPLAZAMIENTOS

```

ux = uy = Table[0, {numnod}];
Do[ux[[n]] = u[[2*n - 1]]; uy[[n]] = u[[2*n]], {n, 1, numnod}];
uxmax = uymax = 0;
Do[uxmax = Max[Abs[ux[[n]]], uxmax]; uymax = Max[Abs[uy[[n]]], uymax], {n, 1, numnod}];
ContourPlotNodeFuncOver2DMesh[NodeCoordinates, ElemNodeLists, ux, uxmax,
  Nsub, aspect, "Displacement component ux"];
ContourPlotNodeFuncOver2DMesh[NodeCoordinates, ElemNodeLists, uy, uymax,
  Nsub, aspect, "Displacement component uy"];

```

EJEMPLO PLACA CON UN ELEMENTO CUADRILATERO 4 NODOS



2 VISUALIZACION TENSIONES

```

sxx = syy = sxy = Table[0, {numnod}];
Do[{sxx[[n]], syy[[n]], sxy[[n]]} = sig[[n]], {n, 1, numnod}];
sxxmax = syymin = sxymax = 0;
Do[sxxmax = Max[Abs[sxx[[n]]], sxxmax];
  syymin = Max[Abs[syy[[n]]], syymin];
  sxymax = Max[Abs[sxy[[n]]], sxymax], {n, 1, numnod}];
ContourPlotNodeFuncOver2DMesh[NodeCoordinates, ElemNodeLists, sxx, sxxmax,
  Nsub, aspect, "Nodal stress sig-xx"];
ContourPlotNodeFuncOver2DMesh[NodeCoordinates, ElemNodeLists, syy, syymin,
  Nsub, aspect, "Nodal stress sig-yy"];
ContourPlotNodeFuncOver2DMesh[NodeCoordinates, ElemNodeLists, sxy, sxymax,
  Nsub, aspect, "Nodal stress sig-xy"];

```

EJEMPLO PLACA CON UN ELEMENTO CUADRILATERO 4 NODOS

```
sig // MatrixForm
```

$$\begin{pmatrix} 0 & 10. & 0 \\ 0 & 10. & 0 \\ 0 & 10. & 0 \\ 0 & 10. & 0 \end{pmatrix}$$

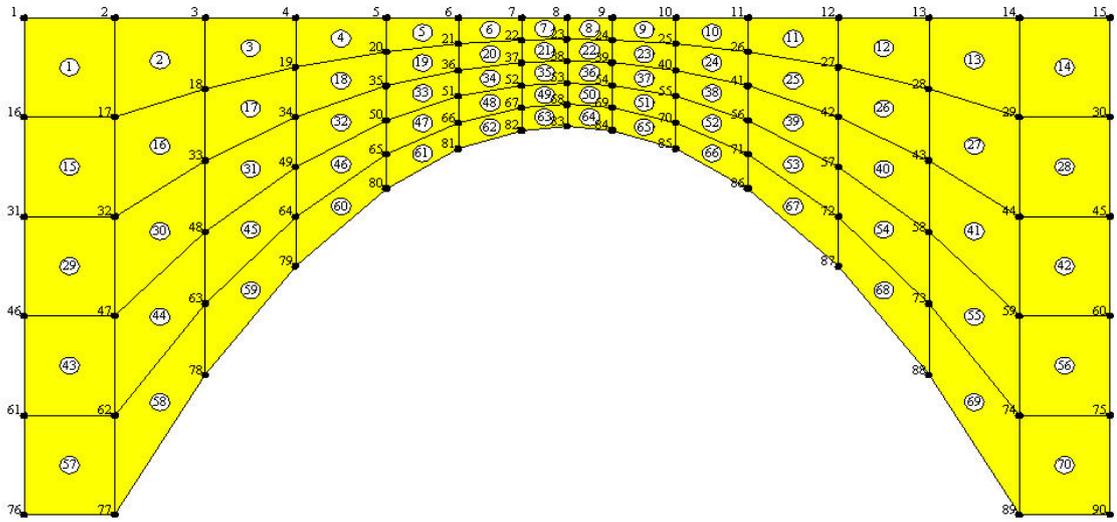
SOLUCION TENSIONES NODALES PROMEDIADAS

$$\sigma_{yy} = q, \sigma_{xx} = \sigma_{xy} = 0$$

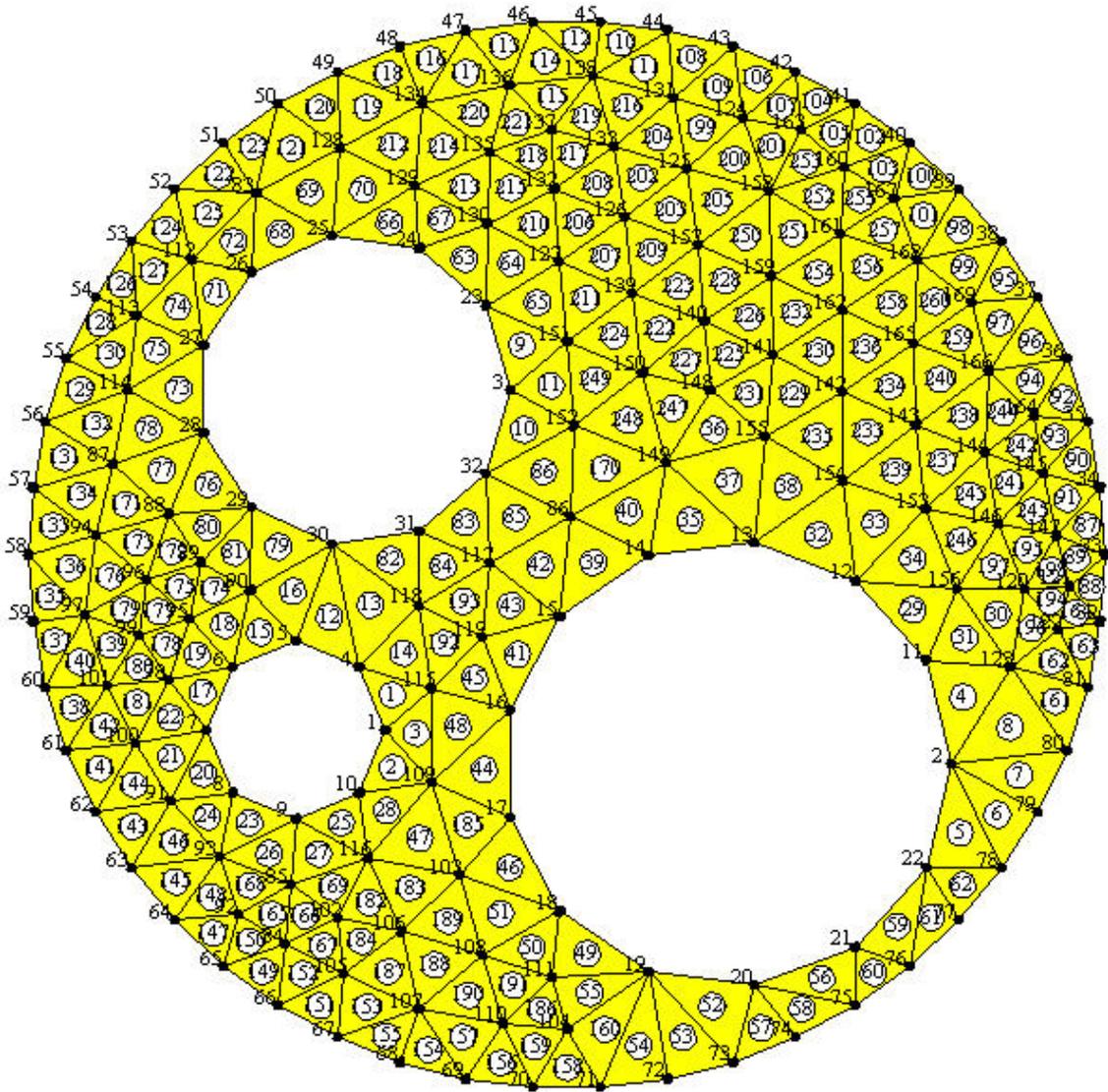
SOLUCION EXACTA



Explicar cual es el problema que se plantea y resuelve en el ejemplo "animacion.nb". Explicar los resultados. Modelarlo con ANSYS, y comparar los resultados.



Explicar cual es el problema que se plantea y resuelve en el ejemplo "importacion.nb". Explicar los resultados. Modelarlo con ANSYS, y comparar los resultados.



Utilizando el fichero "importa.nb", importar varias discretizaciones del ejercicio de la biela bidimensional realizado en la prácticas con ANSYS. Utilizar todos los elementos que sea posible, dentro de los disponibles.

