ELEMENTO TRIANGULAR REGULAR - 10 NODOS

Implementación en Mathematica

v.2018

1. DATOS INICIALES

▪ INICIO

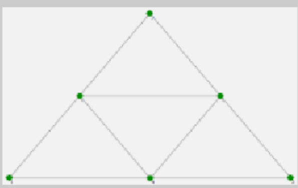
```
Off[General::"spell1"]
Off[General::"spell"]
```

```
SetDirectory[NotebookDirectory[]]
```

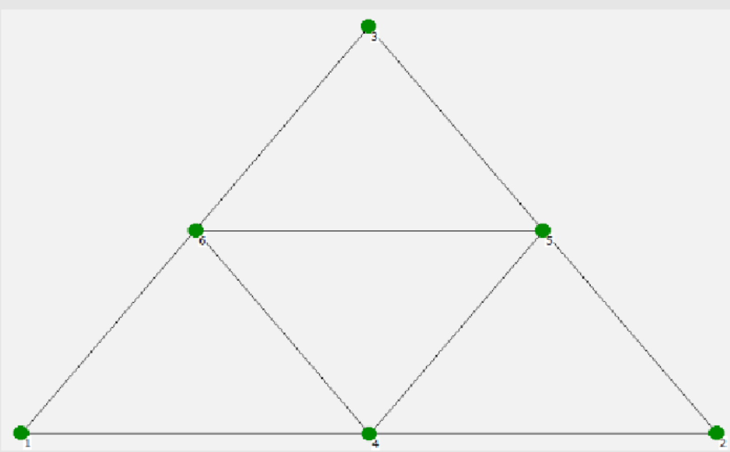
```
C:\#0-Modulos-M30x_MeF-10\#M309-m6-a6a-sws\12-I-triangulo-i
```

▪ DEFINICION ELEMENTO TRIANGULAR REGULAR DE 10 NODOS - COORDENADAS TRIANGULARES

▫ DEFINICION GRAFICA

TrianR6 =  ;

```
TrianR6r = Show[TrianR6, ImageSize → 350]
```



▫ COORDENADAS TRIANGULARES NODOS

$$Cnt = \left\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}, \left\{\frac{1}{2}, \frac{1}{2}, 0\right\}, \left\{0, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0, \frac{1}{2}\right\}\right\};$$

```
NNodos = Dimensions[Cnt][[1]]
```

```
6
```

DEFINICION ELEMENTO BASE REAL - COORDENADAS CARTESIANAS

COORDENADAS CARTESIANAS NODOS ELEMENTO BASE REAL

```
NNodosB = 3;
```

```
Cne = Table[{0, 0}, {i, NNodosB}];
```

```
Cne[[1]] = {-10, 0}; Cne[[2]] = {10, 0}; Cne[[3]] = {0, 10*Sqrt[3]};
```

FUNCION TRANSFORMACION DE COORDENADAS DE TRIANGULARES A CARTESIANAS

$$TtC[\xi1\_, \xi2\_, \xi3\_] = \begin{pmatrix} 1 & 1 & 1 \\ Cne[[1]][[1]] & Cne[[2]][[1]] & Cne[[3]][[1]] \\ Cne[[1]][[2]] & Cne[[2]][[2]] & Cne[[3]][[2]] \end{pmatrix} . \begin{pmatrix} \xi1 \\ \xi2 \\ \xi3 \end{pmatrix};$$

COORDENADAS CARTESIANAS DE LOS NODOS

```
Cnc = Table[{0, 0}, {i, NNodos}];
```

```
Do[
  Cnc[[i]] = {TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[2, 1]],
    TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[3, 1]]},
  {i, NNodos}
 ];
```

IMAGEN DEL ELEMENTO

FUNCION REPRESENTACION GRAFICA ELEMENTO Y NODOS

```
ElementPlot[b_List, options___] := Module[{asa, color, nr, circles, lines},
   asa = Select[{options}, ((! SameQ[#[[1]], NodeColor]) && (! SameQ[#[[1]], NodeSize])) &];
   {color, nr} = {NodeColor, NodeSize} /. {options} /. {NodeColor → GrayLevel[0], NodeSize → PointSize[0.06]};
   circles = Map[Point[#] &, Partition[Flatten[b], 2]];
   lines = Line[Append[b[[1]], First[b[[1]]]]];
   Show[Graphics[{nr, color, circles}], Graphics[lines], Evaluate[Sequence[##] & @@ asa]]];
```

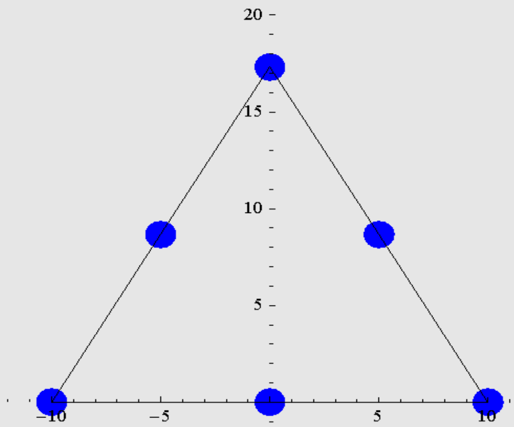
DEFINICION VECTOR DE NODOS-

```
ptsexteriores = {Cnc[[1]], Cnc[[4]], Cnc[[2]], Cnc[[5]], Cnc[[3]], Cnc[[6]]};
```

```
ptsinteriores = {};
```

▫ IMAGEN DE COMPROBACION

```
Imagen = Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
   Axes → True, PlotRange → {{-12, 12}, {-2, 20}}, ImageSize → 250, NodeColor → RGBColor[0, 0, 1]]
```



3. FUNCIONES DE FORMA - METODO PRODUCTO DE CURVAS -

▪ CURVAS A CONSIDERAR

```
Cu = Table[0, {i, 9}];
```

▫ LADOS

```
Cu[[1]] = ξ3; Cu[[2]] = ξ1; Cu[[3]] = ξ2;
```

▫ MEDIANAS

```
Cu[[4]] = (ξ1 - 1 / 2);
```

```
Cu[[5]] = (ξ2 - 1 / 2);
```

```
Cu[[6]] = (ξ3 - 1 / 2);
```

▪ DEFINICION PRODUCTOS DE CURVAS EN CADA NODO

```
Nc = Table[0, {i, NNodos}];
```

▫ Tipo 1 - ESQUINA

```
Nc[[1]] = Cu[[2]] * Cu[[4]];
```

```
Nc[[2]] = Cu[[3]] * Cu[[5]];
```

```
Nc[[3]] = Cu[[1]] * Cu[[6]];
```

▫ Tipo 2 - LADOS

```
Nc[[4]] = Cu[[2]] * Cu[[3]];
```

```
Nc[[5]] = Cu[[3]] * Cu[[1]];
```

```
Nc[[6]] = Cu[[1]] * Cu[[2]];
```

OBTENCION FUNCIONES DE FORMA

```
Clear[Nf]
```

```
Nfp = Table[0, {i, NNodos}];
```

```
Nf = Table[0, {i, NNodos}];
```

```
Do[
  Nfp[[i]] = a * Nc[[i]];
  eq = 1 == Nfp[[i]] /. {ξ1 -> Cnt[[i, 1]], ξ2 -> Cnt[[i, 2]], ξ3 -> Cnt[[i, 3]]};
  as = a /. Solve[eq, a][[1]]; Print["Nodo ", i];
  Nf[[i]] = Simplify[Nfp[[i]] /. {a → as}],
  {i, NNodos}
 ];
```

Nodo 1

Nodo 2

Nodo 3

Nodo 4

Nodo 5

Nodo 6

```
MatrixForm[Nf]
```

$$\begin{pmatrix} \xi1\,(-1+2\,\xi1) \\ \xi2\,(-1+2\,\xi2) \\ \xi3\,(-1+2\,\xi3) \\ 4\,\xi1\,\xi2 \\ 4\,\xi2\,\xi3 \\ 4\,\xi1\,\xi3 \end{pmatrix}$$

COMPROBACION SUMA UNIDAD

$$\text{Suma} = \sum_{i=1}^{\text{NNodos}} \text{Nf}[[i]]$$

$$\xi1\,(-1+2\,\xi1) + 4\,\xi1\,\xi2 + \xi2\,(-1+2\,\xi2) + 4\,\xi1\,\xi3 + 4\,\xi2\,\xi3 + \xi3\,(-1+2\,\xi3)$$

```
Simplify[Suma /. {ξ1 → 1 - ξ2 - ξ3}]
```

1

OK.

Representación Gráfica.

Función Representación Gráfica Funciones de Forma

```
PlotTriangleShapeFunction[xytrig_, f_, Nsub_, aspect_] :=
  Module[{Ni, line3D = {}, poly3D = {}, zc1, zc2, zc3, xyf1, xyf2, xyf3, xc, yc, x1, x2, x3, y1, y2,
    y3, z1, z2, z3, iz1, iz2, iz3, d}, {{x1, y1, z1}, {x2, y2, z2}, {x3, y3, z3}} = Take[xytrig, 3];
   xc = {x1, x2, x3}; yc = {y1, y2, y3}; Ni = Nsub * 3; Do[Do[iz3 = Ni - iz1 - iz2; If[iz3 ≤ 0, Continue[]]; d = 0;
     If[Mod[iz1 + 2, 3] == 0 && Mod[iz2 - 1, 3] == 0, d = 1]; If[Mod[iz1 - 2, 3] == 0 && Mod[iz2 + 1, 3] == 0, d = -1];
     If[d == 0, Continue[]]; zc1 = N[{iz1 + d + d, iz2 - d, iz3 - d} / Ni]; zc2 = N[{iz1 - d, iz2 + d + d, iz3 - d} / Ni];
     zc3 = N[{iz1 - d, iz2 - d, iz3 + d + d} / Ni]; xyf1 = {xc.zc1, yc.zc1, f[zc1[[1]], zc1[[2]], zc1[[3]]]};
     xyf2 = {xc.zc2, yc.zc2, f[zc2[[1]], zc2[[2]], zc2[[3]]]};
     xyf3 = {xc.zc3, yc.zc3, f[zc3[[1]], zc3[[2]], zc3[[3]]]}; AppendTo[poly3D, Polygon[{xyf1, xyf2, xyf3}]];
     AppendTo[line3D, Line[{xyf1, xyf2, xyf3, xyf1}]], {iz2, 1, Ni - iz1}], {iz1, 1, Ni}];
   Show[Graphics3D[RGBColor[1, 0, 0]], Graphics3D[poly3D], Graphics3D[Thickness[.002]],
    Graphics3D[line3D], Graphics3D[RGBColor[0, 0, 0]], Graphics3D[Thickness[.005]],
    Graphics3D[Line[xytrig]], PlotRange → All, BoxRatios → {1, 1, aspect}, Boxed → False]];
```

Representación Gráfica Funciones Forma Elemento.

```
Ng = Table[0, {i, NNodos}];
```

```
xyc1 = {0, 0, 0}; xyc2 = {3, 0, 0}; xyc3 = {Sqrt[3], 3 / 2, 0}; xytrig = N[{xyc1, xyc2, xyc3, xyc1}];
```

Control de Cuadricula

```
Nsub = 15;
```

```
Do[
  fi[ξ1_, ξ2_, ξ3_] = Nf[[i]];
  Ng[[i]] = PlotTriangleShapeFunction[xytrig, fi, Nsub, 1 / 2],
  {i, NNodos}
 ];
```

4. RESULTADOS INTERACTIVOS -

```
Manipulate[{Imagen, Ng[[n]], Nf[[n]]}, {n, 1, Dimensions[Nf][[1]], 1}, {n, Range[Dimensions[Nf][[1]]]},
 FrameLabel → {"FUNCION DE FORMA EN NODO n – TRIANGULO REGULAR 10 NODOS"}, SaveDefinitions → True]
```

n

n 1

FUNCION DE FORMA EN NODO n – TRIANGULO REGULAR 10 NODOS

5. DERIVADAS FUNCIONES DE FORMA Y JACOBIANO -

- **Inicializaciones Necesarias - #**

```
ClearAll[x1, x2, x3, x4, x5, x6, y1, y2, y3, y4, y5, y6, ξ1, ξ2, ξ3];
```

- **1 - Definción Isoparamétrica del Elemento - #**

IsoP =

$$\begin{bmatrix} 1 \\ x \\ y \\ u_x \\ u_y \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ u_{x1} & u_{x2} & \cdots & u_{xn} \\ u_{y1} & u_{y2} & \cdots & u_{yn} \end{bmatrix} \begin{bmatrix} N_1^e \\ N_2^e \\ \vdots \\ N_n^e \end{bmatrix}. \quad (16.6)$$

;

```
IsoPr = Show[IsoP, ImageSize → 650]
```

$$\begin{bmatrix} 1 \\ x \\ y \\ u_x \\ u_y \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \\ u_{x1} & u_{x2} & \dots & u_{xn} \\ u_{y1} & u_{y2} & \dots & u_{yn} \end{bmatrix} \begin{bmatrix} N_1^{(e)} \\ N_2^{(e)} \\ \vdots \\ N_n^{(e)} \end{bmatrix}. \qquad (16.6)$$

```
x = {x1, x2, x3, x4, x5, x6};
y = {y1, y2, y3, y4, y5, y6};
```

▪ 2 - Funciones de Forma <<< -----

```
Nf
```

```
{ξ1 (-1 + 2 ξ1), ξ2 (-1 + 2 ξ2), ξ3 (-1 + 2 ξ3), 4 ξ1 ξ2, 4 ξ2 ξ3, 4 ξ1 ξ3}
```

▪ 3 - Derivadas Funciones de Forma respecto Coordenadas Naturales.

```
Nf1 = D[Nf, ξ1];
Nf2 = D[Nf, ξ2];
Nf3 = D[Nf, ξ3];
{Nf1, Nf2, Nf3} = Simplify[{Nf1, Nf2, Nf3}];
```

```
Nf1 // MatrixForm
```

$$\begin{pmatrix} -1 + 4\,\xi 1 \\ 0 \\ 0 \\ 4\,\xi 2 \\ 0 \\ 4\,\xi 3 \end{pmatrix}$$

```
Nf2 // MatrixForm
```

$$\begin{pmatrix} 0 \\ -1 + 4\,\xi 2 \\ 0 \\ 4\,\xi 1 \\ 4\,\xi 3 \\ 0 \end{pmatrix}$$

```
Nf3 // MatrixForm
```

$$\begin{pmatrix} 0 \\ 0 \\ -1 + 4\,\xi 3 \\ 0 \\ 4\,\xi 2 \\ 4\,\xi 1 \end{pmatrix}$$

■ 4 - Derivadas Coordenadas Triangulares respecto a las Cartesianas - Desarrollo

□ Calculo Elementos Matriz Jacobiana - Elemento Considerado

```
SistemA = [image];
```

```
SistemAr = Show[SistemA, ImageSize → 650]
```

$$\begin{bmatrix} 1 & 1 & 1 \\ \sum x_i \frac{\partial N_i}{\partial \zeta_1} & \sum x_i \frac{\partial N_i}{\partial \zeta_2} & \sum x_i \frac{\partial N_i}{\partial \zeta_3} \\ \sum y_i \frac{\partial N_i}{\partial \zeta_1} & \sum y_i \frac{\partial N_i}{\partial \zeta_2} & \sum y_i \frac{\partial N_i}{\partial \zeta_3} \end{bmatrix} \begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (24.18)$$

```
{Nf1x, Nf2x, Nf3x} = Simplify[{x.Nf1, x.Nf2, x.Nf3}];
{Nf1y, Nf2y, Nf3y} = Simplify[{y.Nf1, y.Nf2, y.Nf3}];
```

```
{Nf1x, Nf2x, Nf3x} // MatrixForm
```

$$\begin{pmatrix} -x1 + 4\,x1\,\xi1 + 4\,x4\,\xi2 + 4\,x6\,\xi3 \\ -x2 + 4\,x4\,\xi1 + 4\,x2\,\xi2 + 4\,x5\,\xi3 \\ -x3 + 4\,x6\,\xi1 + 4\,x5\,\xi2 + 4\,x3\,\xi3 \end{pmatrix}$$

```
{Nf1y, Nf2y, Nf3y} // MatrixForm
```

$$\begin{pmatrix} -y1 + 4\,y1\,\xi1 + 4\,y4\,\xi2 + 4\,y6\,\xi3 \\ -y2 + 4\,y4\,\xi1 + 4\,y2\,\xi2 + 4\,y5\,\xi3 \\ -y3 + 4\,y6\,\xi1 + 4\,y5\,\xi2 + 4\,y3\,\xi3 \end{pmatrix}$$

```
JacobianA = [image];
```

```
JacobianAr = Show[JacobianA, ImageSize → 650]
```

rewritten

$$\mathbf{J\,P} = \begin{bmatrix} 1 & 1 & 1 \\ J_{x1} & J_{x2} & J_{x3} \\ J_{y1} & J_{y2} & J_{y3} \end{bmatrix} \begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (24.19)$$

Para mostrar el planteamiento genérico, de momento no aplicamos estas definiciones:

```
Clear[Jx1, Jx2, Jx3, Jy1, Jy2, Jy3]
```

```
(*Jx1=Nf1x; Jx2=Nf2x; Jx3=Nf3x;*)
```

```
(*Jy1=Nf1y; Jy2=Nf2y; Jy3=Nf3y;*)
```

▫ Definición Matriz Jacobiana y Sistema de Ecuaciones a Resolver - Planteamiento Genérico

$$J = \begin{pmatrix} 1 & 1 & 1 \\ Jx1 & Jx2 & Jx3 \\ Jy1 & Jy2 & Jy3 \end{pmatrix};$$

▫ Determinante de la Matriz Jacobiana - J

```
Jdet = Det[J]
```

```
-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3
```

▫ Defincion de Jc

```
Jc = 1 / 2 * Jdet
```

$$\frac{1}{2}\,(-Jx2\,Jy1 + Jx3\,Jy1 + Jx1\,Jy2 - Jx3\,Jy2 - Jx1\,Jy3 + Jx2\,Jy3)$$

▫ Matriz Derivadas Coordenadas Triangulares respecto a las Cartesianas - Matriz Incognitas

$$P = \begin{pmatrix} f1x & f1y \\ f2x & f2y \\ f3x & f3y \end{pmatrix};$$

▫ Matriz de Terminos Independientes

$$Ti = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix};$$

▫ Solución del Sistema de Ecuaciones

```
P = Inverse[J].Ti
```

```
{{(Jy2 - Jy3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (-Jx2 + Jx3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)},
 {(-Jy1 + Jy3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (Jx1 - Jx3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)},
 {(Jy1 - Jy2) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (-Jx1 + Jx2) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)}}
```

▫ Defínción Matriz Jacobiana Inversa Modificada - Jim

JacobianaModificadA = $\begin{bmatrix} \frac{\partial\zeta_1}{\partial x} & \frac{\partial\zeta_1}{\partial y} \\ \frac{\partial\zeta_2}{\partial x} & \frac{\partial\zeta_2}{\partial y} \\ \frac{\partial\zeta_3}{\partial x} & \frac{\partial\zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J}\begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \quad (24.20)$;

```
JacobianaModificadAr = Show[JacobianaModificadA, ImageSize → 650]
```

$$\begin{bmatrix} \frac{\partial\zeta_1}{\partial x} & \frac{\partial\zeta_1}{\partial y} \\ \frac{\partial\zeta_2}{\partial x} & \frac{\partial\zeta_2}{\partial y} \\ \frac{\partial\zeta_3}{\partial x} & \frac{\partial\zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J}\begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \quad (24.20)$$

```
Jc
```

$$\frac{1}{2}\,(-Jx2\,Jy1 + Jx3\,Jy1 + Jx1\,Jy2 - Jx3\,Jy2 - Jx1\,Jy3 + Jx2\,Jy3)$$

```
P
```

```
{{(Jy2 - Jy3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (-Jx2 + Jx3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)},
 {(-Jy1 + Jy3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (Jx1 - Jx3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)},
 {(Jy1 - Jy2) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (-Jx1 + Jx2) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)}}
```

```
Jim = 2 * Jc * P
```

```
{{Jy2 - Jy3, -Jx2 + Jx3}, {-Jy1 + Jy3, Jx1 - Jx3}, {Jy1 - Jy2, -Jx1 + Jx2}}
```

```
Jim // MatrixForm
```

$$\begin{pmatrix} Jy2 - Jy3 & -Jx2 + Jx3 \\ -Jy1 + Jy3 & Jx1 - Jx3 \\ Jy1 - Jy2 & -Jx1 + Jx2 \end{pmatrix}$$

$$\begin{pmatrix} Jy23 & Jx32 \\ Jy31 & Jx13 \\ Jy12 & Jx21 \end{pmatrix} = Jim;$$

```
Jx13
```

```
Jx1 - Jx3
```

Necesario para mantener el planteamiento genérico:

```
Clear[Jdet, Jc, Jy23, Jx32, Jy31, Jx13, Jy12, Jx21]
```

```
Jc = 1 / 2 * Jdet;
```

```
JacobianaModificadAr
```

$$\begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J} \begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \qquad (24.20)$$

$$\begin{pmatrix} f1x & f1y \\ f2x & f2y \\ f3x & f3y \end{pmatrix} = \frac{1}{2 * Jc} * \begin{pmatrix} Jy23 & Jx32 \\ Jy31 & Jx13 \\ Jy12 & Jx21 \end{pmatrix};$$

```
f1x
```

$$\frac{Jy23}{Jdet}$$

▪ 5 - Derivadas de las Funciones de Forma respecto a las Coordenadas Cartesianas - Desarrollo

▫ Definición de las Derivadas de las Funciones de Forma respecto a las Coordenadas Cartesianas - Planteamiento Genérico

```
DerivadasFuncionesFormaCartesianaS =
```

$$\frac{\partial N_i}{\partial x} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{y23} + \frac{\partial N_i}{\partial \zeta_2} J_{y31} + \frac{\partial N_i}{\partial \zeta_3} J_{y12}\right),$$
$$\frac{\partial N_i}{\partial y} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{x32} + \frac{\partial N_i}{\partial \zeta_2} J_{x13} + \frac{\partial N_i}{\partial \zeta_3} J_{x21}\right). \quad (24.22)$$

;

```
DerivadasFuncionesFormaCartesianaSr = Show[DerivadasFuncionesFormaCartesianaS, ImageSize → 650]
```

$$\frac{\partial N_i}{\partial x} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{y23} + \frac{\partial N_i}{\partial \zeta_2} J_{y31} + \frac{\partial N_i}{\partial \zeta_3} J_{y12}\right),$$
$$\frac{\partial N_i}{\partial y} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{x32} + \frac{\partial N_i}{\partial \zeta_2} J_{x13} + \frac{\partial N_i}{\partial \zeta_3} J_{x21}\right). \quad (24.22)$$

```
DerivadasFuncionesFormaCartesianasM =
```

In matrix form:

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = \mathbf{P}^T \begin{bmatrix} \frac{\partial N_i}{\partial \zeta_1} & \frac{\partial N_i}{\partial \zeta_2} & \frac{\partial N_i}{\partial \zeta_3} \end{bmatrix}^T. \quad (24.23)$$

;

```
DerivadasFuncionesFormaCartesianasMr = Show[DerivadasFuncionesFormaCartesianasM, ImageSize → 650]
```

In matrix form:

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = \mathbf{P}^T \begin{bmatrix} \frac{\partial N_i}{\partial \zeta_1} & \frac{\partial N_i}{\partial \zeta_2} & \frac{\partial N_i}{\partial \zeta_3} \end{bmatrix}^T. \quad (24.23)$$

$$\begin{pmatrix} dNx \\ dNy \end{pmatrix} = \text{Transpose}\left[\begin{pmatrix} f1x & f1y \\ f2x & f2y \\ f3x & f3y \end{pmatrix}\right].\begin{pmatrix} Nf1 \\ Nf2 \\ Nf3 \end{pmatrix};$$

▫ dNx & dNy <<< -----

```
dNx
```

$$\left\{\frac{Jy23\,(-1+4\,\xi 1)}{Jdet}, \frac{Jy31\,(-1+4\,\xi 2)}{Jdet}, \frac{Jy12\,(-1+4\,\xi 3)}{Jdet}, \frac{4\,Jy31\,\xi 1}{Jdet} + \frac{4\,Jy23\,\xi 2}{Jdet}, \frac{4\,Jy12\,\xi 2}{Jdet} + \frac{4\,Jy31\,\xi 3}{Jdet}, \frac{4\,Jy12\,\xi 1}{Jdet} + \frac{4\,Jy23\,\xi 3}{Jdet}\right\}$$

```
dNy
```

$$\left\{\frac{Jx32\,(-1+4\,\xi 1)}{Jdet}, \frac{Jx13\,(-1+4\,\xi 2)}{Jdet}, \frac{Jx21\,(-1+4\,\xi 3)}{Jdet}, \frac{4\,Jx13\,\xi 1}{Jdet} + \frac{4\,Jx32\,\xi 2}{Jdet}, \frac{4\,Jx21\,\xi 2}{Jdet} + \frac{4\,Jx13\,\xi 3}{Jdet}, \frac{4\,Jx21\,\xi 1}{Jdet} + \frac{4\,Jx32\,\xi 3}{Jdet}\right\}$$

Se observa quedan en función de los elementos de la Matriz Jacobiana Inversa Modificada - Jim

Matriz Jacobiana Inversa Modificada - Jim - Planteamiento Genérico

```
JacobianaModificadAr
```

$$\begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J} \begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \qquad (24.20)$$

$$\begin{pmatrix} Jy23 & Jx32 \\ Jy31 & Jx13 \\ Jy12 & Jx21 \end{pmatrix} = Jim;$$

```
Jx13
```

```
Jx1 - Jx3
```

Elemetos Matriz Jacobiana - Elemento Considerado

```
JacobianAr
```

rewritten

$$\mathbf{J\,P} = \begin{bmatrix} 1 & 1 & 1 \\ J_{x1} & J_{x2} & J_{x3} \\ J_{y1} & J_{y2} & J_{y3} \end{bmatrix} \begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (24.19)$$

Aplicamos las deiniciones correspondientes al alemento considerado:

```
Jx1 = Nf1x; Jx2 = Nf2x; Jx3 = Nf3x;
```

```
Jy1 = Nf1y; Jy2 = Nf2y; Jy3 = Nf3y;
```

Matriz Jacobiana Inversa Modificada - Jim - Elemento Considerado

$$\begin{pmatrix} Jy23 & Jx32 \\ Jy31 & Jx13 \\ Jy12 & Jx21 \end{pmatrix};$$

```
Jy23
```

```
-y2 + y3 + 4 y4 ξ1 - 4 y6 ξ1 + 4 y2 ξ2 - 4 y5 ξ2 - 4 y3 ξ3 + 4 y5 ξ3
```

```
Jx32
```

```
x2 - x3 - 4 x4 ξ1 + 4 x6 ξ1 - 4 x2 ξ2 + 4 x5 ξ2 + 4 x3 ξ3 - 4 x5 ξ3
```

```
Jy31
```

```
y1 - y3 - 4 y1 ξ1 + 4 y6 ξ1 - 4 y4 ξ2 + 4 y5 ξ2 + 4 y3 ξ3 - 4 y6 ξ3
```

```
Jx13
```

```
-x1 + x3 + 4 x1 ξ1 - 4 x6 ξ1 + 4 x4 ξ2 - 4 x5 ξ2 - 4 x3 ξ3 + 4 x6 ξ3
```

```
Jy12
```

```
-y1 + y2 + 4 y1 ξ1 - 4 y4 ξ1 - 4 y2 ξ2 + 4 y4 ξ2 - 4 y5 ξ3 + 4 y6 ξ3
```

```
Jx21
```

```
x1 - x2 - 4 x1 ξ1 + 4 x4 ξ1 + 4 x2 ξ2 - 4 x4 ξ2 + 4 x5 ξ3 - 4 x6 ξ3
```

MODULO CALCULO FUNCIONES DE FORMA Y JACOBIANO

MODULO GENERICO A COMPLETAR -

MODULO GENERICO A COMPLETAR: XX = No. Nodos, {xI,yI}= nodo i-esimo

```
(*TrigXXIsoPShapeFunDer[ncoor_,tcoor_]:=Module[{ξ1,ξ2,ξ3,x1,x2,x3,x4,x5,x6,x7,x8,x9,xI,y1,y2,
    y3,y4,y5,y6,y7,y8,y9,yI,Jx21,Jx32,Jx13,Jy12,Jy23,Jy31,Nf,dNx,dNy,Jdet},{ξ1,ξ2,ξ3}=tcoor;
   {{x1,y1},{x2,y2},{x3,y3},{x4,y4},{x5,y5},{x6,y6},{x7,y7},{x8,y8},{x9,y9},{xI,yI}}=ncoor;
   Nf=(*Nf*);
   Jx21=(*Jx21*);
   Jy12=(*Jy12*);
   Jx13=(*Jx13*);
   Jy31=(*Jy31*);
   Jx32=(*Jx32*);
   Jy23=(*Jy23*);
   Jdet=Jx21*Jy31-Jy12*Jx13;
   dNx=(*dNx*);
   dNy=(*dNy*);
   Return[Simplify[{Nf,dNx,dNy,Jdet}]]];*)
```

MODULO COMPLETADO -

```
Trig6IsoPShapeFunDer[ncoor_, tcoor_] :=
  Module[{ξ1, ξ2, ξ3, x1, x2, x3, x4, x5, x6, y1, y2, y3, y4, y5, y6, dx4, dx5, dx6, dy4,
    dy5, dy6, Jx21, Jx32, Jx13, Jy12, Jy23, Jy31, Nf, dNx, dNy, Jdet}, {ξ1, ξ2, ξ3} = tcoor;
   {{x1, y1}, {x2, y2}, {x3, y3}, {x4, y4}, {x5, y5}, {x6, y6}} = ncoor;
   Nf = {ξ1 * (2 * ξ1 - 1), ξ2 * (2 * ξ2 - 1), ξ3 * (2 * ξ3 - 1), 4 * ξ1 * ξ2, 4 * ξ2 * ξ3, 4 * ξ3 * ξ1};
   Jx21 = x1 - x2 - 4 x1 ξ1 + 4 x4 ξ1 + 4 x2 ξ2 - 4 x4 ξ2 + 4 x5 ξ3 - 4 x6 ξ3;
   Jy12 = -y1 + y2 + 4 y1 ξ1 - 4 y4 ξ1 - 4 y2 ξ2 + 4 y4 ξ2 - 4 y5 ξ3 + 4 y6 ξ3;
   Jx13 = -x1 + x3 + 4 x1 ξ1 - 4 x6 ξ1 + 4 x4 ξ2 - 4 x5 ξ2 - 4 x3 ξ3 + 4 x6 ξ3;
   Jy31 = y1 - y3 - 4 y1 ξ1 + 4 y6 ξ1 - 4 y4 ξ2 + 4 y5 ξ2 + 4 y3 ξ3 - 4 y6 ξ3;
   Jx32 = x2 - x3 - 4 x4 ξ1 + 4 x6 ξ1 - 4 x2 ξ2 + 4 x5 ξ2 + 4 x3 ξ3 - 4 x5 ξ3;
   Jy23 = -y2 + y3 + 4 y4 ξ1 - 4 y6 ξ1 + 4 y2 ξ2 - 4 y5 ξ2 - 4 y3 ξ3 + 4 y5 ξ3;
   Jdet = Jx21 * Jy31 - Jy12 * Jx13;
   dNx = { (4 * ξ1 - 1) * Jy23, (4 * ξ2 - 1) * Jy31, (4 * ξ3 - 1) * Jy12,
      4 * (ξ2 * Jy23 + ξ1 * Jy31), 4 * (ξ3 * Jy31 + ξ2 * Jy12), 4 * (ξ1 * Jy12 + ξ3 * Jy23)} / Jdet;
   dNy = { (4 * ξ1 - 1) * Jx32, (4 * ξ2 - 1) * Jx13, (4 * ξ3 - 1) * Jx21, 4 * (ξ2 * Jx32 + ξ1 * Jx13),
      4 * (ξ3 * Jx13 + ξ2 * Jx21), 4 * (ξ1 * Jx21 + ξ3 * Jx32)} / Jdet;
   Return[Simplify[{Nf, dNx, dNy, Jdet}]]];
```

6. MATRIZ DE RIGIDEZ - INTEGRACION NUMERICA -

▪ MODULO DE CALCULO DE LA MATRIZ DE RIGIDEZ

▫ MODULO GENERICO A COMPLETAR -

MODULO GENERICO: XX = No. Nodos, YY = Grados Libertad

```
(*TrigXXIsoPMembraneStiffness[ncoor_,mprop_,fprop_,opt_]:=
  Module[{i,k,l,p=3,numer=False,Emat,th={fprop},h,tcoor,w,c,Nf,dNx,dNy,Jdet,B,Ke=Table[0,{YY},{YY}]},
   Emat=mprop[[1]];If[Length[fprop]>0,th=fprop[[1]]];
   If[Length[opt]>0,numer=opt[[1]]];
   If[Length[opt]>1,p=opt[[2]]];
   If[p≠1&&p≠-3&&p≠3&&p≠6&&p≠-6&&p≠7&&p≠12,Print["Illegal p"];Return[Null]];
   For[k=1,k≤Abs[p],k++,{tcoor,w}=TrigGaussRuleInfo[{p,numer},k];
    {Nf,dNx,dNy,Jdet}=TrigXXIsoPShapeFunDer[ncoor,tcoor];
    If[Length[th]==0,h=th,h=th.Nf];c=w*Jdet*h/2;
    B={Flatten[Table[{dNx[[i]],0},{i,XX}]],
      Flatten[Table[{0,dNy[[i]]},{i,XX}]],Flatten[Table[{dNy[[i]],dNx[[i]]},{i,XX}]]};
    Ke+=c*Transpose[B].(Emat.B);];If[!numer,Ke=Simplify[Ke]];Return[Ke]];*)
```

▫ MODULO COMPLETADO -

```
Trig6IsoPMembraneStiffness[ncoor_, mprop_, fprop_, opt_] :=
  Module[{i, k, l, p = 3, numer = False, Emat, th = {fprop}, h, tcoor, w, c, Nf, dNx, dNy, Jdet,
    B, Ke = Table[0, {12}, {12}]}, Emat = mprop[[1]]; If[Length[fprop] > 0, th = fprop[[1]]];
   If[Length[opt] > 0, numer = opt[[1]]];
   If[Length[opt] > 1, p = opt[[2]]];
   If[p ≠ 1 && p ≠ -3 && p ≠ 3 && p ≠ 6 && p ≠ -6 && p ≠ 7 && p ≠ 12, Print["Illegal p"]; Return[Null]];
   For[k = 1, k ≤ Abs[p], k++, {tcoor, w} = TrigGaussRuleInfo[{p, numer}, k];
    {Nf, dNx, dNy, Jdet} = Trig6IsoPShapeFunDer[ncoor, tcoor];
    If[Length[th] == 0, h = th, h = th.Nf]; c = w * Jdet * h / 2;
    B = {Flatten[Table[{dNx[[i]], 0}, {i, 6}]],
      Flatten[Table[{0, dNy[[i]]}, {i, 6}]], Flatten[Table[{dNy[[i]], dNx[[i]]}, {i, 6}]]};
    Ke += c * Transpose[B].(Emat.B);]; If[! numer, Ke = Simplify[Ke]]; Return[Ke]];
```

- **MODULO REGLAS DE CUADRATURA DE GAUSS**

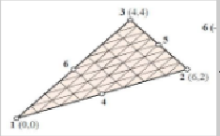
  - **OPCION UNICA: DEFINICION DE CARLOS FELIPPA**

```
TrigGaussRuleInfo[{rule_, numer_}, point_] :=
  Module[{zeta, p = rule, i = point, g1, g2, g3, g4, w1, w2, w3, eps = 10.^(-24),
    jkl = {{1, 2, 3}, {2, 1, 3}, {1, 3, 2}, {3, 1, 2}, {2, 3, 1}, {3, 2, 1}}, info = {{Null, Null, Null}, 0}},
   If[p == 1, info = {{1/3, 1/3, 1/3}, 1}]; If[p == 3, info = {{1, 1, 1}/6, 1/3}; info[[1, i]] = 2/3];
   If[p == -3, info = {{1, 1, 1}/2, 1/3}; info[[1, i]] = 0];
   If[p == 6, g1 = (8 - Sqrt[10] + Sqrt[38 - 44*Sqrt[2/5]])/18; g2 = (8 - Sqrt[10] - Sqrt[38 - 44*Sqrt[2/5]])/18;
    If[i < 4, info = {{g1, g1, g1}, (620 + Sqrt[213125 - 53320*Sqrt[10]])/3720}; info[[1, i]] = 1 - 2*g1];
    If[i > 3, info = {{g2, g2, g2}, (620 - Sqrt[213125 - 53320*Sqrt[10]])/3720}; info[[1, i - 3]] = 1 - 2*g2]];
   If[p == -6, If[i < 4, info = {{1, 1, 1}/6, 3/10}; info[[1, i]] = 2/3];
    If[i > 3, info = {{1, 1, 1}/2, 1/30}; info[[1, i - 3]] = 0]]; If[p == 7, g1 = (6 - Sqrt[15])/21;
    g2 = (6 + Sqrt[15])/21; If[i < 4, info = {{g1, g1, g1}, (155 - Sqrt[15])/1200}; info[[1, i]] = 1 - 2*g1];
    If[i > 3 && i < 7, info = {{g2, g2, g2}, (155 + Sqrt[15])/1200}; info[[1, i - 3]] = 1 - 2*g2];
    If[i == 7, info = {{1/3, 1/3, 1/3}, 9/40}]];
   If[p == 12, g1 = 0.063089014491502228340331602870819157; g2 = 0.249286745170910421291638553107019076;
    g3 = 0.053145049844816947353249671631398147; g4 = 0.310352451033784405416607733956552153;
    If[! numer, {g1, g2, g3, g4} = Rationalize[{g1, g2, g3, g4}, eps]];
    w1 = (30*g2^3*(4*g3^2 + (1 - 2*g4)^2 + 4*g3*(-1 + g4)) + g3^2*(1 - 15*g4) +
         (-1 + g4)*g4 - g3*(-1 + g4)*(-1 + 15*g4) + 2*g2*(1 + 60*g3*g4*(-1 + g3 + g4)) -
         6*g2^2*(3 + 10*(-1 + g4)*g4 + 10*g3^2*(1 + 3*g4) + 10*g3*(-1 + g4)*(1 + 3*g4)))/
       (180*(g1 - g2)*(-(g2*(-1 + 2*g2)*(-1 + g3)*g3) + (-1 + g3)*(g2 - 2*g2^2 - 2*g3 + 3*g2*g3)*g4 -
           (g2*(-1 + 2*g2 - 3*g3) + 2*g3)*g4^2 + 2*g1^2*(g2*(-2 + 3*g2) + g3 - g3^2 + g4 - g3*g4 - g4^2) +
           g1*(-4*g2^2 + (-1 + g3)*g3 + (-1 + g3)*(1 + 3*g3)*g4 +
              (1 + 3*g3)*g4^2 - 2*g2*(-1 + g3^2 + g3*(-1 + g4) + (-1 + g4)*g4))));
    w2 = (-1 + 12*(2 - 3*g1)*g1*w1 + 4*g3^2*(-1 + 3*w1) + 4*g3*(-1 + g4)*(-1 + 3*w1) +
         4*(-1 + g4)*g4*(-1 + 3*w1))/(12*(g2*(-2 + 3*g2) + g3 - g3^2 + g4 - g3*g4 - g4^2));
    w3 = (1 - 3*w1 - 3*w2)/6; If[i < 4, info = {{g1, g1, g1}, w1}; info[[1, i]] = 1 - 2*g1];
    If[i > 3 && i < 7, info = {{g2, g2, g2}, w2}; info[[1, i - 3]] = 1 - 2*g2];
    If[i > 6, {j, k, l} = jkl[[i - 6]]; info = {{0, 0, 0}, w3}; info[[1, j]] = g3; info[[1, k]] = g4;
     info[[1, l]] = 1 - g3 - g4]]; If[numer, Return[N[info]], Return[Simplify[info]]];];
```

7. TEST DEL ELEMENTO DE LADOS RECTOS SUPERPARAMETRICO.

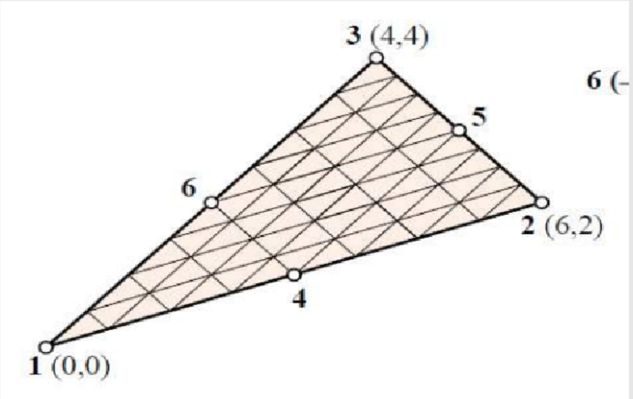
- **DEFINICION DE LA GEOMETRIA**

```
LadosRectosTesT =
```




```
;
```

```
LadosRectosTesTr = Show[LadosRectosTesT, ImageSize → 300]
```



- NODOS ESQUINA DEL ELEMENTO

```
{{x1, y1}, {x2, y2}, {x3, y3}} = {{0, 0}, {6, 2}, {4, 4}};
```

- NODOS MITAD DE LADOS

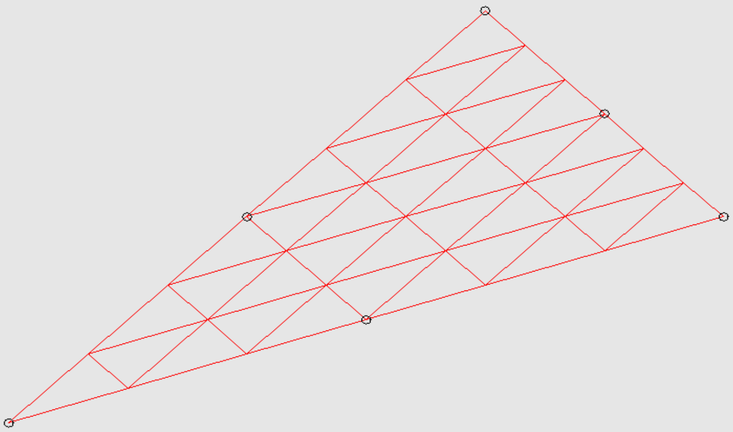
```
x4 = (x1 + x2) / 2; x5 = (x2 + x3) / 2; x6 = (x3 + x1) / 2;
y4 = (y1 + y2) / 2; y5 = (y2 + y3) / 2; y6 = (y3 + y1) / 2;
```

- REPRESENTACION GRAFICA DEL ELEMENTO

```
ncoorr = {{x1, y1}, {x2, y2}, {x3, y3}, {x4, y4}, {x5, y5}, {x6, y6}};
```

```
PlotTrig6Shape[xytrig_, Nsub_, ratio_] :=
  Module[{Ne, Nev, Ni, line2D = {}, nodes = {}, xy1, xy2, xy3, i, j, iz1, iz2, iz3, z1, z2, z3, x1, x2, x3, x4, x5,
     x6, y1, y2, y3, y4, y5, y6, xc, yc}, {{x1, y1}, {x2, y2}, {x3, y3}, {x4, y4}, {x5, y5}, {x6, y6}} = xytrig;
   xc = {x1, x2, x3, x4, x5, x6}; yc = {y1, y2, y3, y4, y5, y6};
   Ne[z1_, z2_, z3_] := N[{z1 (2 z1 - 1), z2 (2 z2 - 1), z3 (2 z3 - 1), 4 z1 z2, 4 z2 z3, 4 z3 z1}];
   Ni = Nsub 3; Do[Do[iz3 = Ni - iz1 - iz2; If[iz3 ≤ 0, Continue[]]; d = 0;
     If[Mod[iz1 + 2, 3] == 0 && Mod[iz2 - 1, 3] == 0, d = 1]; If[Mod[iz1 - 2, 3] == 0 && Mod[iz2 + 1, 3] == 0, d = -1];
     If[d == 0, Continue[]]; {z1, z2, z3} = N[{iz1 + d + d, iz2 - d, iz3 - d}/Ni];
     zc1 = Ne[z1, z2, z3]; {z1, z2, z3} = N[{iz1 - d, iz2 + d + d, iz3 - d}/Ni]; zc2 = Ne[z1, z2, z3];
     {z1, z2, z3} = N[{iz1 - d, iz2 - d, iz3 + d + d}/Ni]; zc3 = Ne[z1, z2, z3]; xy1 = {xc.zc1, yc.zc1};
     xy2 = {xc.zc2, yc.zc2}; xy3 = {xc.zc3, yc.zc3}; AppendTo[line2D, Line[{xy1, xy2, xy3, xy1}]],
     {iz2, 1, Ni - iz1}], {iz1, 1, Ni}]; Do[AppendTo[nodes, Circle[xytrig[[i]], 0.04`]], {i, 1, 6}];
   Show[Graphics[RGBColor[1, 0, 0]], Graphics[Thickness[0.002`]], Graphics[line2D],
    Graphics[RGBColor[0, 0, 0]], Graphics[nodes], PlotRange → All, AspectRatio → ratio]];
```

```
PlotTrig6Shape[ncoorr, 6, 4 / 6]
```



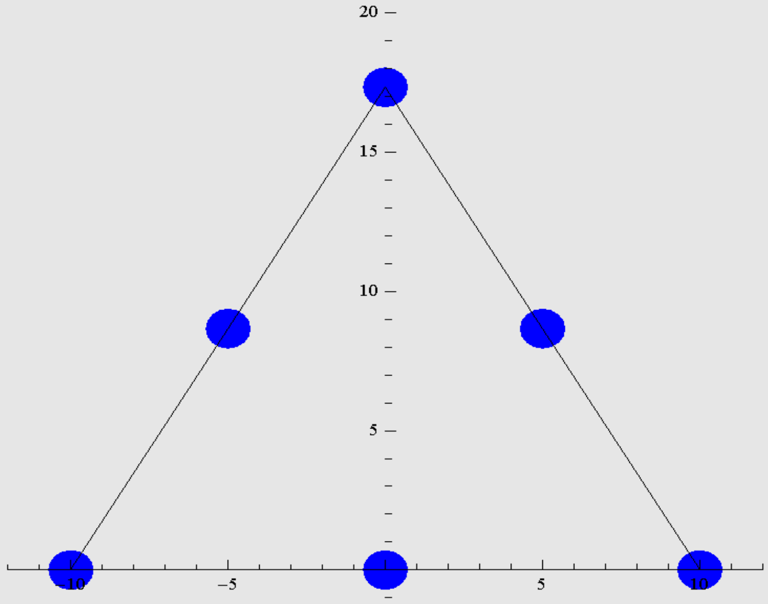
- **DEFINICION DE LOS NODOS ELEMENTO - #**

```
ncoor = Cnc;
```

```
ptsexteriores = {Cnc[[1]], Cnc[[4]], Cnc[[2]], Cnc[[5]], Cnc[[3]], Cnc[[6]]};
```

```
ptsinteriores = {};
```

```
Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
  Axes → True, PlotRange → {{-12, 12}, {-2, 20}}, NodeColor → RGBColor[0, 0, 1]]
```



- **DEFINICION DEL MATERIAL**

```
ClearAll[Em, nu, h];
```

```
h = 1; Em = 288; nu = 1 / 3;
```

```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

```
Print["Emat=", Emat // MatrixForm]
```

Emat= $\begin{pmatrix} 324 & 108 & 0 \\ 108 & 324 & 0 \\ 0 & 0 & 108 \end{pmatrix}$

VERIFICACION DE LA MATRIZ DE RIGIDEZ -

NUMERO DE PUNTOS DE GAUSS MINIMO PARA CONSEGUIR SUFICIENCIA DE RANGO -

```
NF = NNodos * 2.;
```

```
NG = (NF - 3)/3
```

3.

Se necesitan como mínimo 3 Puntos -- Regla 3 minima

BUCLE GENERICO A COMPLETAR -

BUCLE GENERICO: XX = No. Nodos, ZZ = GRADOS DE LIBERTAD - 3

```
(*For [i=1, i≤7,i++,p={1,-3,3,6,-6,7,12}[[i]];
  Ke=TrigXXIsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
  Print["Gauss integration rule: ",p];
  Print["Ke=",Chop[Simplify[Ke]]//MatrixForm];
  Valores=Chop[Eigenvalues[N[Ke]]];
  If[Valores[[ZZ]]!=0,Print["Valores propios matriz Ke=",Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=",p];Break[],
   Print["Valores proprios matriz Ke=",Valores];Print["NO tenemos la suficiencia de rango para p=",p]]
 ];*)
```

DESARROLLO DE LA MATRIZ DE RIGIDEZ -

```
For [i = 1, i ≤ 7, i++, p = {1, -3, 3, 6, -6, 7, 12}[[i]];
  Ke = Trig6IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
  Print["Gauss integration rule: ", p];
  Print["Ke=", Chop[Simplify[Ke]] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]]];
  If[Valores[[9]] != 0, Print["Valores propios matriz Ke=", Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=", p](*;Break[]*),
   Print["Valores proprios matriz Ke=", Valores]; Print["NO tenemos la suficiencia de rango para p=", p]]
 ];
```

Gauss integration rule: 1

$$Ke=\begin{pmatrix} 17.3205 & 6. & -13.8564 & 0 & -3.4641 & -6. & 13.8564 & 24. & -69.282 & -24. & 55.4256 & \\ 6. & 10.3923 & 0 & 0 & -6. & -10.3923 & 24. & 41.5692 & -24. & -41.5692 & 0 & \\ -13.8564 & 0 & 17.3205 & -6. & -3.4641 & 6. & 13.8564 & -24. & 55.4256 & 0 & -69.282 & \\ 0 & 0 & -6. & 10.3923 & 6. & -10.3923 & -24. & 41.5692 & 0 & 0 & 24. & -41 \\ -3.4641 & -6. & -3.4641 & 6. & 6.9282 & 0 & -27.7128 & 0 & 13.8564 & 24. & 13.8564 & - \\ -6. & -10.3923 & 6. & -10.3923 & 0 & 20.7846 & 0 & -83.1384 & 24. & 41.5692 & -24. & 41 \\ 13.8564 & 24. & 13.8564 & -24. & -27.7128 & 0 & 110.851 & 0 & -55.4256 & -96. & -55.4256 & \\ 24. & 41.5692 & -24. & 41.5692 & 0 & -83.1384 & 0 & 332.554 & -96. & -166.277 & 96. & -16 \\ -69.282 & -24. & 55.4256 & 0 & 13.8564 & 24. & -55.4256 & -96. & 277.128 & 96. & -221.703 & \\ -24. & -41.5692 & 0 & 0 & 24. & 41.5692 & -96. & -166.277 & 96. & 166.277 & 0 & \\ 55.4256 & 0 & -69.282 & 24. & 13.8564 & -24. & -55.4256 & 96. & -221.703 & 0 & 277.128 & - \\ 0 & 0 & 24. & -41.5692 & -24. & 41.5692 & 96. & -166.277 & 0 & 0 & -96. & 16 \end{pmatrix}$$

Valores propios matriz Ke={706.677, 353.338, 353.338, 0, 0, 0, 0, 0, 0, 0, 0, 0}

NO tenemos la suficiencia de rango para p=1

Gauss integration rule: -3

$$Ke=\begin{pmatrix} 155.885 & 54. & 41.5692 & 0 & 10.3923 & 18. & -166.277 & 0 & 0 & 0 & -41.5692 & - \\ 54. & 93.5307 & 0 & 0 & 18. & 31.1769 & 0 & 0 & 0 & 0 & -72. & -12 \\ 41.5692 & 0 & 155.885 & -54. & 10.3923 & -18. & -166.277 & 0 & -41.5692 & 72. & 0 & \\ 0 & 0 & -54. & 93.5307 & -18. & 31.1769 & 0 & 0 & 72. & -124.708 & 0 & \\ 10.3923 & 18. & 10.3923 & -18. & 62.3538 & 0 & 0 & 0 & -41.5692 & 72. & -41.5692 & - \\ 18. & 31.1769 & -18. & 31.1769 & 0 & 187.061 & 0 & 0 & 72. & -124.708 & -72. & -12 \\ -166.277 & 0 & -166.277 & 0 & 0 & 0 & 498.831 & 0 & -83.1384 & -144. & -83.1384 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 498.831 & -144. & -249.415 & 144. & -24 \\ 0 & 0 & -41.5692 & 72. & -41.5692 & 72. & -83.1384 & -144. & 498.831 & 0 & -332.554 & \\ 0 & 0 & 72. & -124.708 & 72. & -124.708 & -144. & -249.415 & 0 & 498.831 & 0 & \\ -41.5692 & -72. & 0 & 0 & -41.5692 & -72. & -83.1384 & 144. & -332.554 & 0 & 498.831 & \\ -72. & -124.708 & 0 & 0 & -72. & -124.708 & 144. & -249.415 & 0 & 0 & 0 & 49 \end{pmatrix}$$

Valores propios matriz Ke=
{997.661, 810.227, 810.227, 415.692, 237.634, 237.634, 124.708, 53.7233, 53.7233, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=-3

Gauss integration rule: 3

$$Ke=\begin{pmatrix} 155.885 & 54. & 41.5692 & 0 & 10.3923 & 18. & -166.277 & 0 & 0 & 0 & -41.5692 & - \\ 54. & 93.5307 & 0 & 0 & 18. & 31.1769 & 0 & 0 & 0 & 0 & -72. & -12 \\ 41.5692 & 0 & 155.885 & -54. & 10.3923 & -18. & -166.277 & 0 & -41.5692 & 72. & 0 & \\ 0 & 0 & -54. & 93.5307 & -18. & 31.1769 & 0 & 0 & 72. & -124.708 & 0 & \\ 10.3923 & 18. & 10.3923 & -18. & 62.3538 & 0 & 0 & 0 & -41.5692 & 72. & -41.5692 & - \\ 18. & 31.1769 & -18. & 31.1769 & 0 & 187.061 & 0 & 0 & 72. & -124.708 & -72. & -12 \\ -166.277 & 0 & -166.277 & 0 & 0 & 0 & 498.831 & 0 & -83.1384 & -144. & -83.1384 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 498.831 & -144. & -249.415 & 144. & -24 \\ 0 & 0 & -41.5692 & 72. & -41.5692 & 72. & -83.1384 & -144. & 498.831 & 0 & -332.554 & \\ 0 & 0 & 72. & -124.708 & 72. & -124.708 & -144. & -249.415 & 0 & 498.831 & 0 & \\ -41.5692 & -72. & 0 & 0 & -41.5692 & -72. & -83.1384 & 144. & -332.554 & 0 & 498.831 & \\ -72. & -124.708 & 0 & 0 & -72. & -124.708 & 144. & -249.415 & 0 & 0 & 0 & 49 \end{pmatrix}$$

Valores propios matriz Ke=
{997.661, 810.227, 810.227, 415.692, 237.634, 237.634, 124.708, 53.7233, 53.7233, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=3

Gauss integration rule: 6

$$Ke=\begin{pmatrix} 155.885 & 54. & 41.5692 & 0 & 10.3923 & 18. & -166.277 & 0 & 0 & 0 & -41.5692 & - \\ 54. & 93.5307 & 0 & 0 & 18. & 31.1769 & 0 & 0 & 0 & 0 & -72. & -12 \\ 41.5692 & 0 & 155.885 & -54. & 10.3923 & -18. & -166.277 & 0 & -41.5692 & 72. & 0 & \\ 0 & 0 & -54. & 93.5307 & -18. & 31.1769 & 0 & 0 & 72. & -124.708 & 0 & \\ 10.3923 & 18. & 10.3923 & -18. & 62.3538 & 0 & 0 & 0 & -41.5692 & 72. & -41.5692 & - \\ 18. & 31.1769 & -18. & 31.1769 & 0 & 187.061 & 0 & 0 & 72. & -124.708 & -72. & -12 \\ -166.277 & 0 & -166.277 & 0 & 0 & 0 & 498.831 & 0 & -83.1384 & -144. & -83.1384 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 498.831 & -144. & -249.415 & 144. & -24 \\ 0 & 0 & -41.5692 & 72. & -41.5692 & 72. & -83.1384 & -144. & 498.831 & 0 & -332.554 & \\ 0 & 0 & 72. & -124.708 & 72. & -124.708 & -144. & -249.415 & 0 & 498.831 & 0 & \\ -41.5692 & -72. & 0 & 0 & -41.5692 & -72. & -83.1384 & 144. & -332.554 & 0 & 498.831 & \\ -72. & -124.708 & 0 & 0 & -72. & -124.708 & 144. & -249.415 & 0 & 0 & 0 & 49 \end{pmatrix}$$

Valores propios matriz Ke=
{997.661, 810.227, 810.227, 415.692, 237.634, 237.634, 124.708, 53.7233, 53.7233, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=6

Gauss integration rule: -6

Ke=

155.885	54.	41.5692	0	10.3923	18.	-166.277	0	0	0	-41.5692	-
54.	93.5307	0	0	18.	31.1769	0	0	0	0	-72.	-12
41.5692	0	155.885	-54.	10.3923	-18.	-166.277	0	-41.5692	72.	0	
0	0	-54.	93.5307	-18.	31.1769	0	0	72.	-124.708	0	
10.3923	18.	10.3923	-18.	62.3538	0	0	0	-41.5692	72.	-41.5692	-
18.	31.1769	-18.	31.1769	0	187.061	0	0	72.	-124.708	-72.	-12
-166.277	0	-166.277	0	0	0	498.831	0	-83.1384	-144.	-83.1384	1
0	0	0	0	0	0	0	498.831	-144.	-249.415	144.	-24
0	0	-41.5692	72.	-41.5692	72.	-83.1384	-144.	498.831	0	-332.554	
0	0	72.	-124.708	72.	-124.708	-144.	-249.415	0	498.831	0	
-41.5692	-72.	0	0	-41.5692	-72.	-83.1384	144.	-332.554	0	498.831	
-72.	-124.708	0	0	-72.	-124.708	144.	-249.415	0	0	0	49

Valores propios matriz Ke=
{997.661, 810.227, 810.227, 415.692, 237.634, 237.634, 124.708, 53.7233, 53.7233, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=-6

Gauss integration rule: 7

Ke=

155.885	54.	41.5692	0	10.3923	18.	-166.277	0	0	0	-41.5692	-
54.	93.5307	0	0	18.	31.1769	0	0	0	0	-72.	-12
41.5692	0	155.885	-54.	10.3923	-18.	-166.277	0	-41.5692	72.	0	
0	0	-54.	93.5307	-18.	31.1769	0	0	72.	-124.708	0	
10.3923	18.	10.3923	-18.	62.3538	0	0	0	-41.5692	72.	-41.5692	-
18.	31.1769	-18.	31.1769	0	187.061	0	0	72.	-124.708	-72.	-12
-166.277	0	-166.277	0	0	0	498.831	0	-83.1384	-144.	-83.1384	1
0	0	0	0	0	0	0	498.831	-144.	-249.415	144.	-24
0	0	-41.5692	72.	-41.5692	72.	-83.1384	-144.	498.831	0	-332.554	
0	0	72.	-124.708	72.	-124.708	-144.	-249.415	0	498.831	0	
-41.5692	-72.	0	0	-41.5692	-72.	-83.1384	144.	-332.554	0	498.831	
-72.	-124.708	0	0	-72.	-124.708	144.	-249.415	0	0	0	49

Valores propios matriz Ke=
{997.661, 810.227, 810.227, 415.692, 237.634, 237.634, 124.708, 53.7233, 53.7233, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=7

Gauss integration rule: 12

Ke=

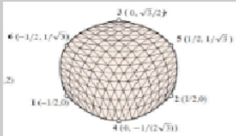
155.885	54.	41.5692	0	10.3923	18.	-166.277	0	0	0	-41.5692	-
54.	93.5307	0	0	18.	31.1769	0	0	0	0	-72.	-12
41.5692	0	155.885	-54.	10.3923	-18.	-166.277	0	-41.5692	72.	0	
0	0	-54.	93.5307	-18.	31.1769	0	0	72.	-124.708	0	
10.3923	18.	10.3923	-18.	62.3538	0	0	0	-41.5692	72.	-41.5692	-
18.	31.1769	-18.	31.1769	0	187.061	0	0	72.	-124.708	-72.	-12
-166.277	0	-166.277	0	0	0	498.831	0	-83.1384	-144.	-83.1384	1
0	0	0	0	0	0	0	498.831	-144.	-249.415	144.	-24
0	0	-41.5692	72.	-41.5692	72.	-83.1384	-144.	498.831	0	-332.554	
0	0	72.	-124.708	72.	-124.708	-144.	-249.415	0	498.831	0	
-41.5692	-72.	0	0	-41.5692	-72.	-83.1384	144.	-332.554	0	498.831	
-72.	-124.708	0	0	-72.	-124.708	144.	-249.415	0	0	0	49

Valores propios matriz Ke=
{997.661, 810.227, 810.227, 415.692, 237.634, 237.634, 124.708, 53.7233, 53.7233, 0, 0, 0}

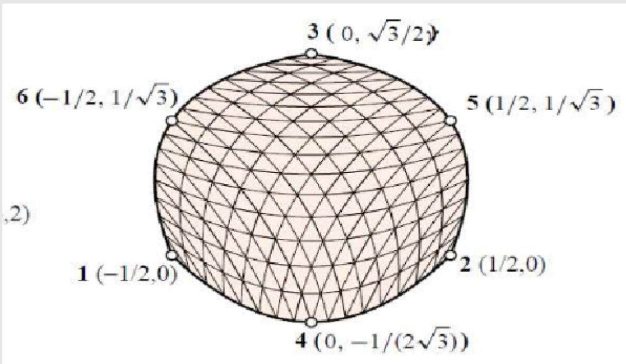
TENEMOS LA SUFICIENCIA DE RANGO PARA p=12

8. TEST DEL ELEMENTO DE LADOS CURVOS.

▪ DEFINICION DE LA GEOMETRIA

LadosRectosTesT =  ;

```
LadosRectosTesTr = Show[LadosRectosTesT, ImageSize → 300]
```



▪ DEFINICION DE LOS NODOS ELEMENTO -

▫ ELEMENTO BASE REAL

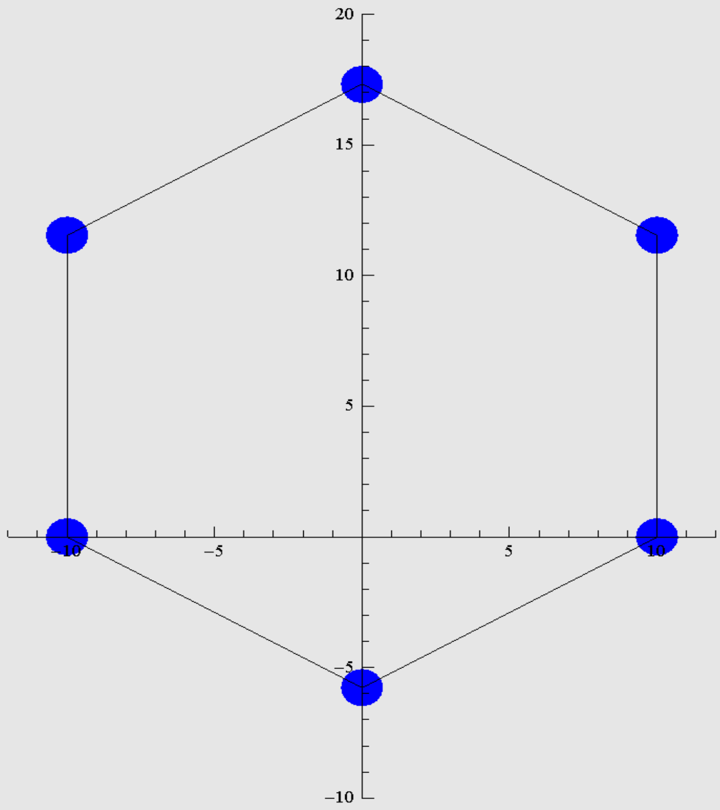
```
Cng = Table[{0, 0}, {i, 6}];
```

```
Cng[[1]] = {-10, 0}; Cng[[2]] = {10, 0}; Cng[[3]] = {0, 10*Sqrt[3]}; Cng[[4]] = {0, -10*Sqrt[3]*1/3};
Cng[[5]] = {10, 10*Sqrt[3]*2/3}; Cng[[6]] = {-10, 10*Sqrt[3]*2/3};
```

```
ptsexteriores = {Cng[[1]], Cng[[4]], Cnc[[2]], Cng[[5]], Cng[[3]], Cng[[6]]};
```

```
ptsinteriores = {};
```

```
Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
  Axes → True, PlotRange → {{-12, 12}, {-10, 20}}, NodeColor → RGBColor[0, 0, 1]]
```



- **TRANSFORMACION DE COORDENADAS TRIANGULARES A CARTESIANAS**

```
Nf6 = {ξ1 * (2 * ξ1 - 1), ξ2 * (2 * ξ2 - 1), ξ3 * (2 * ξ3 - 1), 4 * ξ1 * ξ2, 4 * ξ2 * ξ3, 4 * ξ3 * ξ1};
```

```
TtC[ξ1_, ξ2_, ξ3_] =
  (        1                 1                 1                 1                 1                 1
   Cng[[1]][[1]]  Cng[[2]][[1]]  Cng[[3]][[1]]  Cng[[4]][[1]]  Cng[[5]][[1]]  Cng[[6]][[1]]
   Cng[[1]][[2]]  Cng[[2]][[2]]  Cng[[3]][[2]]  Cng[[4]][[2]]  Cng[[5]][[2]]  Cng[[6]][[2]] ).Nf6;
```

- **COORDENADAS TRIANGULARES DEL ELEMENTO CONSIDERADO**

```
Cnt
```

$$\left\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}, \left\{\frac{1}{2}, \frac{1}{2}, 0\right\}, \left\{0, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{1}{2}, 0, \frac{1}{2}\right\}\right\}$$

- **COORDENADAS CARTESIANAS NODOS ELEMENTO REAL CONSIDERADO Y COMPROBACION GRAFICA - #**

```
Cnc = Table[{0, 0}, {i, NNodos}];
```

```
Do[
  Cnc[[i]] = {TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[2]],
    TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[3]]},
  {i, NNodos}
 ];
```

```
Cnc
```

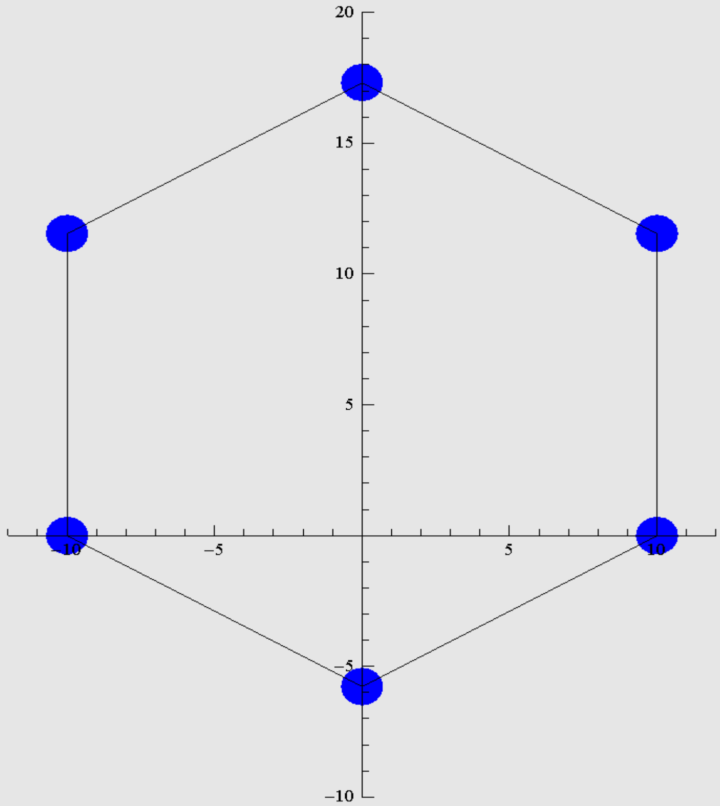
$\{\{-10, 0\}, \{10, 0\}, \{0, 10\sqrt{3}\}, \{0, -\frac{10}{\sqrt{3}}\}, \{10, \frac{20}{\sqrt{3}}\}, \{-10, \frac{20}{\sqrt{3}}\}\}$

```
ncoor = Cnc;
```

```
ptsexteriores = {Cnc[[1]], Cnc[[4]], Cnc[[2]], Cnc[[5]], Cnc[[3]], Cnc[[6]]};
```

```
ptsinteriores = {};
```

```
Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
  Axes → True, PlotRange → {{-12, 12}, {-10, 20}}, NodeColor → RGBColor[0, 0, 1]]
```



- **DEFINICION DEL MATERIAL**

```
ClearAll[Em, nu, h];
```

```
h = 1; Em = 7 * 72; nu = 0; h = 1;
```

```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

- **VERIFICACION DE LA MATRIZ DE RIGIDEZ - #**

- **NUMERO DE PUNTOS DE GAUSS MINIMO PARA CONSEGUIR SUFICIENCIA DE RANGO - #**

```
NF = NNodos * 2.;
```

```
NG = (NF - 3)/3
```

```
3.
```

Se necesitan como mínimo 6 Puntos -- Regla 6 minima

BUCLE GENERICO A COMPLETAR -

BUCLE GENERICO: XX = No. Nodos, ZZ = GRADOS DE LIBERTAD - 3

```
(*For [i=1, i≤7,i++,p={1,-3,3,6,-6,7,12}[[i]];
  Ke=TrigXXIsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
  Print["Gauss integration rule: ",p];
  Print["Ke=",Chop[Simplify[Ke]]//MatrixForm];
  Valores=Chop[Eigenvalues[N[Ke]]];
  If[Valores[[ZZ]]!=0,Print["Valores propios matriz Ke=",Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=",p];Break[],
   Print["Valores proprios matriz Ke=",Valores];Print["NO tenemos la suficiencia de rango para p=",p]]
 ];*)
```

DESARROLLO DE LA MATRIZ DE RIGIDEZ -

```
For [i = 1, i ≤ 7, i++, p = {1, -3, 3, 6, -6, 7, 12}[[i]];
  Ke = Trig6IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
  Print["Gauss integration rule: ", p];
  Print["Ke=", Chop[Simplify[Ke]] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]]];
  If[Valores[[9]] != 0, Print["Valores propios matriz Ke=", Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=", p] (*;Break[]*),
   Print["Valores proprios matriz Ke=", Valores]; Print["NO tenemos la suficiencia de rango para p=", p]]
 ];
```

Gauss integration rule: 1

Ke=

28.2902	7.	-20.2073	-7.	-8.0829	0	32.3316	0	-113.161	-28.	80.829	
7.	20.2073	7.	-4.04145	-14.	-16.1658	56.	64.6632	-28.	-80.829	-28.	16
-20.2073	7.	28.2902	-7.	-8.0829	0	32.3316	0	80.829	-28.	-113.161	
-7.	-4.04145	-7.	20.2073	14.	-16.1658	-56.	64.6632	28.	16.1658	28.	-8
-8.0829	-14.	-8.0829	14.	16.1658	0	-64.6632	0	32.3316	56.	32.3316	-
0	-16.1658	0	-16.1658	0	32.3316	0	-129.326	0	64.6632	0	64
32.3316	56.	32.3316	-56.	-64.6632	0	258.653	0	-129.326	-224.	-129.326	2
0	64.6632	0	64.6632	0	-129.326	0	517.306	0	-258.653	0	-25
-113.161	-28.	80.829	28.	32.3316	0	-129.326	0	452.643	112.	-323.316	-
-28.	-80.829	-28.	16.1658	56.	64.6632	-224.	-258.653	112.	323.316	112.	-64
80.829	-28.	-113.161	28.	32.3316	0	-129.326	0	-323.316	112.	452.643	-
28.	16.1658	28.	-80.829	-56.	64.6632	224.	-258.653	-112.	-64.6632	-112.	32

Valores proprios matriz Ke={824.456, 824.456, 824.456, 0, 0, 0, 0, 0, 0, 0, 0, 0}

NO tenemos la suficiencia de rango para p=1

Gauss integration rule: -3

Ke=

344.678	75.	-91.7987	21.	-86.6025	-24.	-124.708	-72.	-20.7846	-36.	-20.7846	
75.	258.076	-21.	-84.8705	18.	-90.0666	96.	0	-36.	20.7846	-132.	-10
-91.7987	-21.	344.678	-75.	-86.6025	24.	-124.708	72.	-20.7846	-36.	-20.7846	
21.	-84.8705	-75.	258.076	-18.	-90.0666	-96.	0	132.	-103.923	36.	20
-86.6025	18.	-86.6025	-18.	214.774	0	41.5692	0	-41.5692	144.	-41.5692	-
-24.	-90.0666	24.	-90.0666	0	387.979	0	-41.5692	-24.	-83.1384	24.	-83
-124.708	96.	-124.708	-96.	41.5692	0	374.123	0	-83.1384	-72.	-83.1384	
-72.	0	72.	0	0	-41.5692	0	374.123	-72.	-166.277	72.	-16
-20.7846	-36.	-20.7846	132.	-41.5692	-24.	-83.1384	-72.	374.123	0	-207.846	
-36.	20.7846	-36.	-103.923	144.	-83.1384	-72.	-166.277	0	374.123	0	-41
-20.7846	-132.	-20.7846	36.	-41.5692	24.	-83.1384	72.	-207.846	0	374.123	
36.	-103.923	36.	20.7846	-144.	-83.1384	72.	-166.277	0	-41.5692	0	37

Valores propios matriz Ke={702.833, 665.108, 553.472, 553.472, 481.89, 429.721, 429.721, 118.391, 118.391, 0, 0, 0

TENEMOS LA SUFICIENCIA DE RANGO PARA p=-3

Gauss integration rule: 3

Ke=

566.381	139.	129.904	21.	79.6743	8.	-364.885	-104.	-205.537	-36.	-205.537	-
139.	405.877	-21.	62.9312	50.	113.161	64.	-129.326	-36.	-163.967	-196.	-28
129.904	-21.	566.381	-139.	79.6743	-8.	-364.885	104.	-205.537	28.	-205.537	
21.	62.9312	-139.	405.877	-50.	113.161	-64.	-129.326	196.	-288.675	36.	-16
79.6743	50.	79.6743	-50.	325.626	0	-143.183	0	-170.896	176.	-170.896	-
8.	113.161	-8.	113.161	0	646.632	0	-226.321	8.	-323.316	-8.	-32
-364.885	64.	-364.885	-64.	-143.183	0	632.776	0	120.089	-104.	120.089	1
-104.	-129.326	104.	-129.326	0	-226.321	0	484.974	-104.	0	104.	
-205.537	-36.	-205.537	196.	-170.896	8.	120.089	-104.	521.925	-64.	-60.0444	
-36.	-163.967	28.	-288.675	176.	-323.316	-104.	0	-64.	595.825	0	18
-205.537	-196.	-205.537	36.	-170.896	-8.	120.089	104.	-60.0444	0	521.925	
-28.	-288.675	36.	-163.967	-176.	-323.316	104.	0	0	180.133	64.	59

Valores propios matriz Ke={1489.8, 1489.8, 702.833, 665.108, 523.866, 523.866, 481.89, 196.429, 196.429, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=3

Gauss integration rule: 6

Ke=

675.592	161.784	148.255	21.	97.7647	8.15082	-443.897	-119.139	-193.427	-28.506	-284.288	-43
161.784	488.781	-21.	80.9345	50.1508	131.425	48.8612	-190.509	-28.506	-160.512	-211.29	-35
148.255	-21.	675.592	-161.784	97.7647	-8.15082	-443.897	119.139	-284.288	43.2897	-193.427	28
21.	80.9345	-161.784	488.781	-50.1508	131.425	-48.8612	-190.509	211.29	-350.119	28.506	-16
97.7647	50.1508	97.7647	-50.1508	395.375	0	-144.054	0	-223.425	176.151	-223.425	-17
8.15082	131.425	-8.15082	131.425	0	768.998	0	-209.885	8.15082	-410.982	-8.15082	-41
-443.897	48.8612	-443.897	-48.8612	-144.054	0	737.835	0	147.007	-119.139	147.007	11
-119.139	-190.509	119.139	-190.509	0	-209.885	0	572.03	-119.139	9.43704	119.139	9.
-193.427	-28.506	-284.288	211.29	-223.425	8.15082	147.007	-119.139	613.481	-71.7957	-59.3478	
-28.506	-160.512	43.2897	-350.119	176.151	-410.982	-119.139	9.43704	-71.7957	696.384	0	21
-284.288	-211.29	-193.427	28.506	-223.425	-8.15082	147.007	119.139	-59.3478	0	613.481	71
-43.2897	-350.119	28.506	-160.512	-176.151	-410.982	119.139	9.43704	0	215.792	71.7957	69

Valores propios matriz Ke={1775.53, 1775.53, 896.833, 768.948, 533.97, 533.97, 495.57, 321.181, 321.181, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=6

Gauss integration rule: -6

Ke=

544.21	132.6	107.734	21.	63.0466	4.8	-340.868	-100.8	-187.061	-36.	-187.061	-
132.6	391.097	-21.	48.151	46.8	92.8379	67.2	-116.394	-36.	-145.492	-189.6	-2
107.734	-21.	544.21	-132.6	63.0466	-4.8	-340.868	100.8	-187.061	21.6	-187.061	
21.	48.151	-132.6	391.097	-46.8	92.8379	-67.2	-116.394	189.6	-270.2	36.	-14
63.0466	46.8	63.0466	-46.8	314.54	0	-124.708	0	-157.963	172.8	-157.963	-1
4.8	92.8379	-4.8	92.8379	0	620.767	0	-207.846	4.8	-299.298	-4.8	-29
-340.868	67.2	-340.868	-67.2	-124.708	0	606.911	0	99.7661	-100.8	99.7661	1
-100.8	-116.394	100.8	-116.394	0	-207.846	0	473.889	-100.8	-16.6277	100.8	-16
-187.061	-36.	-187.061	189.6	-157.963	4.8	99.7661	-100.8	507.144	-57.6	-74.8246	
-36.	-145.492	21.6	-270.2	172.8	-299.298	-100.8	-16.6277	-57.6	573.655	0	15
-187.061	-189.6	-187.061	36.	-157.963	-4.8	99.7661	100.8	-74.8246	0	507.144	5
-21.6	-270.2	36.	-145.492	-172.8	-299.298	100.8	-16.6277	0	157.963	57.6	57

Valores propios matriz Ke={1377.23, 1377.23, 702.833, 665.108, 523.693, 523.693, 481.89, 198.328, 198.328, 0, 0, 0

TENEMOS LA SUFICIENCIA DE RANGO PARA p=-6

Gauss integration rule: 7

Ke=

661.865	158.514	141.728	21.	92.5264	7.40665	-432.14	-117.207	-190.16	-29.0999	-273.819	-40
158.514	478.829	-21.	76.1258	49.4067	125.328	50.7931	-182.702	-29.0999	-156.558	-208.614	-34
141.728	-21.	661.865	-158.514	92.5264	-7.40665	-432.14	117.207	-273.819	40.6136	-190.16	29
21.	76.1258	-158.514	478.829	-49.4067	125.328	-50.7931	-182.702	208.614	-341.023	29.0999	-15
92.5264	49.4067	92.5264	-49.4067	387.311	0	-139.757	0	-216.303	175.407	-216.303	-17
7.40665	125.328	-7.40665	125.328	0	753.383	0	-206.96	7.40665	-398.539	-7.40665	-39
-432.14	50.7931	-432.14	-50.7931	-139.757	0	723.591	0	140.223	-117.207	140.223	11
-117.207	-182.702	117.207	-182.702	0	-206.96	0	562.595	-117.207	4.88448	117.207	4.
-190.16	-29.0999	-273.819	208.614	-216.303	7.40665	140.223	-117.207	602.844	-69.7134	-62.785	
-29.0999	-156.558	40.6136	-341.023	175.407	-398.539	-117.207	4.88448	-69.7134	683.342	0	20
-273.819	-208.614	-190.16	29.0999	-216.303	-7.40665	140.223	117.207	-62.785	0	602.844	69
-40.6136	-341.023	29.0999	-156.558	-175.407	-398.539	117.207	4.88448	0	207.893	69.7134	68

Valores propios matriz Ke={1727.11, 1727.11, 880.958, 760.719, 532.75, 532.75, 494.987, 312.123, 312.123, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=7

Gauss integration rule: 12

Ke=

685.627	163.799	149.13	21.	98.8566	8.02518	-450.806	-120.524	-191.393	-27.7505	-291.414	-44
163.799	496.488	-21.	82.099	50.0252	132.372	47.4757	-196.108	-27.7505	-159.35	-212.549	-35
149.13	-21.	685.627	-163.799	98.8566	-8.02518	-450.806	120.524	-291.414	44.5494	-191.393	27
21.	82.099	-163.799	496.488	-50.0252	132.372	-47.4757	-196.108	212.549	-355.501	27.7505	-1
98.8566	50.0252	98.8566	-50.0252	401.919	0	-143.328	0	-228.152	176.025	-228.152	-17
8.02518	132.372	-8.02518	132.372	0	780.196	0	-207.415	8.02518	-418.763	-8.02518	-41
-450.806	47.4757	-450.806	-47.4757	-143.328	0	747.288	0	148.826	-120.524	148.826	12
-120.524	-196.108	120.524	-196.108	0	-207.415	0	580.319	-120.524	9.65656	120.524	9.
-191.393	-27.7505	-291.414	212.549	-228.152	8.02518	148.826	-120.524	622.061	-72.2999	-59.9281	
-27.7505	-159.35	44.5494	-355.501	176.025	-418.763	-120.524	9.65656	-72.2999	705.546	0	21
-291.414	-212.549	-191.393	27.7505	-228.152	-8.02518	148.826	120.524	-59.9281	0	622.061	72
-44.5494	-355.501	27.7505	-159.35	-176.025	-418.763	120.524	9.65656	0	218.411	72.2999	70

Valores propios matriz Ke=
{1799.39, 1799.39, 917.093, 779.416, 535.671, 535.671, 496.248, 333.146, 333.146, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=12