ELEMENTO TRIANGULAR REGULAR - 10 NODOS
Implementación en Mathematica

v.2018

1. DATOS INICIALES

▪ INICIO

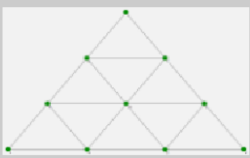
```
Off[General::"spell1"]
Off[General::"spell"]
```

```
SetDirectory[NotebookDirectory[]]
```

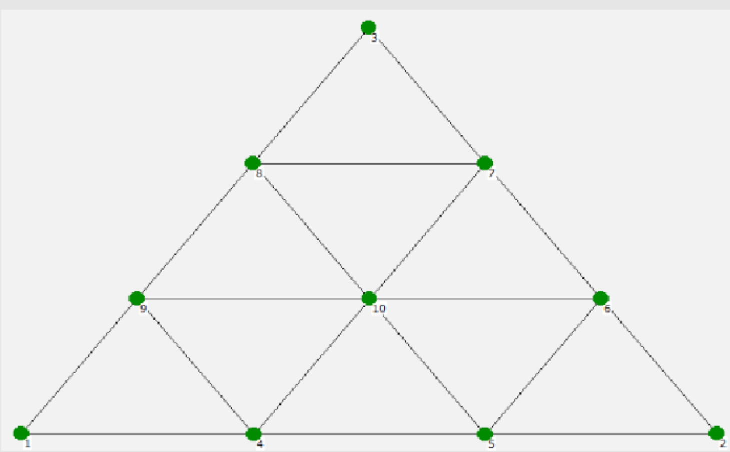
```
C:\#0-Modulos-M30x_MeF-10\#M309-m6-a6a-sws\12-I-triangulo-i
```

▪ DEFINICION ELEMENTO TRIANGULAR REGULAR DE 10 NODOS - COORDENADAS TRIANGULARES

▫ DEFINICION GRAFICA

TrianR10 =  ;

```
TrianR10r = Show[TrianR10, ImageSize → 350]
```



▫ COORDENADAS TRIANGULARES NODOS

$$Cnt = \left\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}, \left\{\frac{2}{3}, \frac{1}{3}, 0\right\}, \left\{\frac{1}{3}, \frac{2}{3}, 0\right\}, \left\{0, \frac{2}{3}, \frac{1}{3}\right\}, \left\{0, \frac{1}{3}, \frac{2}{3}\right\}, \left\{\frac{1}{3}, 0, \frac{2}{3}\right\}, \left\{\frac{2}{3}, 0, \frac{1}{3}\right\}, \left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\}\right\};$$

```
NNodos = Dimensions[Cnt][[1]]
```

```
10
```

DEFINICION ELEMENTO BASE REAL - COORDENADAS CARTESIANAS

COORDENADAS CARTESIANAS NODOS ELEMENTO BASE REAL

```
NNodosB = 3;
```

```
Cne = Table[{0, 0}, {i, NNodosB}];
```

```
Cne[[1]] = {-10, 0}; Cne[[2]] = {10, 0}; Cne[[3]] = {0, 10*Sqrt[3]};
```

FUNCION TRANSFORMACION DE COORDENADAS DE TRIANGULARES A CARTESIANAS

```
                     (       1               1               1         )   (ξ1)
TtC[ξ1_, ξ2_, ξ3_] = ( Cne[[1]][[1]]  Cne[[2]][[1]]  Cne[[3]][[1]] ) . (ξ2);
                     ( Cne[[1]][[2]]  Cne[[2]][[2]]  Cne[[3]][[2]] )   (ξ3)
```

COORDENADAS CARTESIANAS DE LOS NODOS

```
Cnc = Table[{0, 0}, {i, NNodos}];
```

```
Do[
  Cnc[[i]] = {TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[2, 1]],
    TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[3, 1]]},
  {i, NNodos}
 ];
```

IMAGEN DEL ELEMENTO

FUNCION REPRESENTACION GRAFICA ELEMENTO Y NODOS

```
ElementPlot[b_List, options___] := Module[{asa, color, nr, circles, lines},
   asa = Select[{options}, ((! SameQ[#[[1]], NodeColor]) && (! SameQ[#[[1]], NodeSize])) &];
   {color, nr} = {NodeColor, NodeSize} /. {options} /.
     {NodeColor → GrayLevel[0], NodeSize → PointSize[0.06]};
   circles = Map[Point[#] &, Partition[Flatten[b], 2]];
   lines = Line[Append[b[[1]], First[b[[1]]]]];
   Show[Graphics[{nr, color, circles}], Graphics[lines], Evaluate[Sequence[##] & @@ asa]]];
```

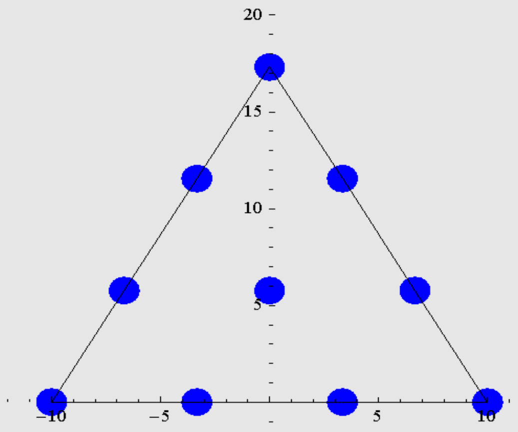
DEFINICION VECTOR DE NODOS-

```
ptsexteriores =
  {Cnc[[1]], Cnc[[4]], Cnc[[5]], Cnc[[2]], Cnc[[6]], Cnc[[7]], Cnc[[3]], Cnc[[8]], Cnc[[9]]};
```

```
ptsinteriores = {Cnc[[10]]};
```

IMAGEN DE COMPROBACION

```
Imagen = Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
   Axes → True, PlotRange → {{-12, 12}, {-2, 20}}, ImageSize → 250, NodeColor → RGBColor[0, 0, 1]]
```



3. FUNCIONES DE FORMA - METODO PRODUCTO DE CURVAS -

CURVAS A CONSIDERAR

```
Cu = Table[0, {i, 9}];
```

LADOS

```
Cu[[1]] = ξ3; Cu[[2]] = ξ1; Cu[[3]] = ξ2;
```

MEDIANAS

```
Cu[[4]] = (ξ1 - 2 / 3); Cu[[5]] = (ξ1 - 1 / 3);
```

```
Cu[[6]] = (ξ2 - 2 / 3); Cu[[7]] = (ξ2 - 1 / 3);
```

```
Cu[[8]] = (ξ3 - 2 / 3); Cu[[9]] = (ξ3 - 1 / 3);
```

DEFINICION PRODUCTOS DE CURVAS EN CADA NODO

```
Nc = Table[0, {i, NNodos}];
```

Tipo 1 - ESQUINA

```
Nc[[1]] = Cu[[2]] * Cu[[4]] * Cu[[5]];
```

```
Nc[[2]] = Cu[[3]] * Cu[[6]] * Cu[[7]];
```

```
Nc[[3]] = Cu[[1]] * Cu[[8]] * Cu[[9]];
```

Tipo 2 - LADOS

```
Nc[[4]] = Cu[[2]] * Cu[[3]] * Cu[[5]];
```

```
Nc[[5]] = Cu[[2]] * Cu[[3]] * Cu[[7]];
```

```
Nc[[6]] = Cu[[1]] * Cu[[3]] * Cu[[7]];
```

```
Nc[[7]] = Cu[[1]] * Cu[[3]] * Cu[[9]];
```

```
Nc[[8]] = Cu[[1]] * Cu[[2]] * Cu[[9]];
```

```
Nc[[9]] = Cu[[1]] * Cu[[2]] * Cu[[5]];
```

Tipo 3- INTERIORES

```
Nc[[10]] = Cu[[1]] * Cu[[2]] * Cu[[3]];
```

OBTENCION FUNCIONES DE FORMA

```
Clear[Nf]
```

```
Nfp = Table[0, {i, NNodos}];
```

```
Nf = Table[0, {i, NNodos}];
```

```
Do[
  Nfp[[i]] = a * Nc[[i]];
  eq = 1 == Nfp[[i]] /. {ξ1 -> Cnt[[i, 1]], ξ2 -> Cnt[[i, 2]], ξ3 -> Cnt[[i, 3]]};
  as = a /. Solve[eq, a][[1]]; Print["Nodo ", i];
  Nf[[i]] = Simplify[Nfp[[i]] /. {a → as}],
  {i, NNodos}
 ];
```

```
Nodo 1
Nodo 2
Nodo 3
Nodo 4
Nodo 5
Nodo 6
Nodo 7
Nodo 8
Nodo 9
Nodo 10
```

```
MatrixForm[Nf]
```

$$\begin{pmatrix} \frac{1}{2}\,\xi 1\,(-2+3\,\xi 1)\,(-1+3\,\xi 1) \\ \frac{1}{2}\,\xi 2\,(-2+3\,\xi 2)\,(-1+3\,\xi 2) \\ \frac{1}{2}\,\xi 3\,(-2+3\,\xi 3)\,(-1+3\,\xi 3) \\ \frac{9}{2}\,\xi 1\,(-1+3\,\xi 1)\,\xi 2 \\ \frac{9}{2}\,\xi 1\,\xi 2\,(-1+3\,\xi 2) \\ \frac{9}{2}\,\xi 2\,(-1+3\,\xi 2)\,\xi 3 \\ \frac{9}{2}\,\xi 2\,\xi 3\,(-1+3\,\xi 3) \\ \frac{9}{2}\,\xi 1\,\xi 3\,(-1+3\,\xi 3) \\ \frac{9}{2}\,\xi 1\,(-1+3\,\xi 1)\,\xi 3 \\ 27\,\xi 1\,\xi 2\,\xi 3 \end{pmatrix}$$

- **COMPROBACION SUMA UNIDAD**

$$\text{Suma} = \sum_{i=1}^{\text{NNodos}} \text{Nf}[[i]]$$

$$\frac{1}{2}\,\xi 1\,(-2+3\,\xi 1)\,(-1+3\,\xi 1) + \frac{9}{2}\,\xi 1\,(-1+3\,\xi 1)\,\xi 2 + \frac{9}{2}\,\xi 1\,\xi 2\,(-1+3\,\xi 2) + \frac{1}{2}\,\xi 2\,(-2+3\,\xi 2)\,(-1+3\,\xi 2) + \frac{9}{2}\,\xi 1\,(-1+3\,\xi 1)\,\xi 3 + 27\,\xi 1\,\xi 2\,\xi 3 + \frac{9}{2}\,\xi 2\,(-1+3\,\xi 2)\,\xi 3 + \frac{9}{2}\,\xi 1\,\xi 3\,(-1+3\,\xi 3) + \frac{9}{2}\,\xi 2\,\xi 3\,(-1+3\,\xi 3) + \frac{1}{2}\,\xi 3\,(-2+3\,\xi 3)\,(-1+3\,\xi 3)$$

```
Simplify[Suma /. {ξ1 → 1 − ξ2 − ξ3}]
```

```
1
```

OK.

- **Representación Gráfica.**

- **Función Representación Gráfica Funciones de Forma**

```
PlotTriangleShapeFunction[xytrig_, f_, Nsub_, aspect_] :=
  Module[{Ni, line3D = {}, poly3D = {}, zc1, zc2, zc3, xyf1, xyf2, xyf3, xc, yc, x1, x2, x3, y1, y2,
    y3, z1, z2, z3, iz1, iz2, iz3, d}, {{x1, y1, z1}, {x2, y2, z2}, {x3, y3, z3}} = Take[xytrig, 3];
   xc = {x1, x2, x3}; yc = {y1, y2, y3}; Ni = Nsub * 3; Do[
    Do[iz3 = Ni − iz1 − iz2; If[iz3 ≤ 0, Continue[]]; d = 0; If[Mod[iz1 + 2, 3] == 0 && Mod[iz2 − 1, 3] == 0, d = 1];
     If[Mod[iz1 − 2, 3] == 0 && Mod[iz2 + 1, 3] == 0, d = −1]; If[d == 0, Continue[]];
     zc1 = N[{iz1 + d + d, iz2 − d, iz3 − d} / Ni]; zc2 = N[{iz1 − d, iz2 + d + d, iz3 − d} / Ni];
     zc3 = N[{iz1 − d, iz2 − d, iz3 + d + d} / Ni]; xyf1 = {xc.zc1, yc.zc1, f[zc1[[1]], zc1[[2]], zc1[[3]]]};
     xyf2 = {xc.zc2, yc.zc2, f[zc2[[1]], zc2[[2]], zc2[[3]]]}; xyf3 =
      {xc.zc3, yc.zc3, f[zc3[[1]], zc3[[2]], zc3[[3]]]}; AppendTo[poly3D, Polygon[{xyf1, xyf2, xyf3}]];
     AppendTo[line3D, Line[{xyf1, xyf2, xyf3, xyf1}]], {iz2, 1, Ni − iz1}], {iz1, 1, Ni}];
   Show[Graphics3D[RGBColor[1, 0, 0]], Graphics3D[poly3D], Graphics3D[Thickness[.002]],
    Graphics3D[line3D], Graphics3D[RGBColor[0, 0, 0]], Graphics3D[Thickness[.005]],
    Graphics3D[Line[xytrig]], PlotRange → All, BoxRatios → {1, 1, aspect}, Boxed → False]];
```

▫ **Representación Gráfica Funciones Forma Elemento.**

```
Ng = Table[0, {i, NNodos}];
```

```
xyc1 = {0, 0, 0}; xyc2 = {3, 0, 0}; xyc3 = {Sqrt[3], 3 / 2, 0}; xytrig = N[{xyc1, xyc2, xyc3, xyc1}];
```

Control de Cuadricula

```
Nsub = 15;
```

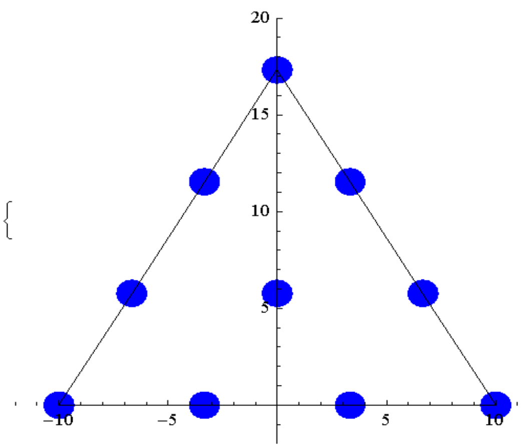
```
Do[
  fi[ξ1_, ξ2_, ξ3_] = Nf[[i]];
  Ng[[i]] = PlotTriangleShapeFunction[xytrig, fi, Nsub, 1 / 2],
  {i, NNodos}
 ];
```

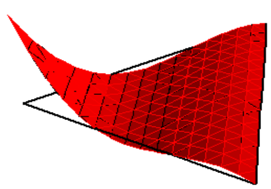
4. RESULTADOS INTERACTIVOS -

```
Manipulate[{Imagen, Ng[[n]], Nf[[n]]}, {n, 1, Dimensions[Nf][[1]], 1}, {n, Range[Dimensions[Nf][[1]]]},
 FrameLabel → {"FUNCION DE FORMA EN NODO n - TRIANGULO REGULAR 10 NODOS"}, SaveDefinitions → True]
```

n

n 1

{ 

, 
, $\frac{1}{2}$ ξ1 (−2 + 3 ξ1) (−1 + 3 ξ1) }

FUNCION DE FORMA EN NODO n – TRIANGULO REGULAR 10 NODOS

5. DERIVADAS FUNCIONES DE FORMA Y JACOBIANO -

▪ **Inicializaciones Necesarias - #**

```
ClearAll[x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, y1, y2, y3, y4, y5, y6, y7, y8, y9, y10, ξ1, ξ2, ξ3];
```

▪ 1 - Defnción Isoparamétrica del Elemento -

```
IsoP = 
```

$$\begin{bmatrix}1\\x\\y\\u_x\\u_y\end{bmatrix}=\begin{bmatrix}1&1&\dots&1\\x_1&x_2&\dots&x_n\\y_1&y_2&\dots&y_n\\u_{x1}&u_{x2}&\dots&u_{xn}\\u_{y1}&u_{y2}&\dots&u_{yn}\end{bmatrix}\begin{bmatrix}N_1^{(e)}\\N_2^{(e)}\\\vdots\\N_n^{(e)}\end{bmatrix}. \qquad (16.6)$$

```
;
```

```
IsoPr = Show[IsoP, ImageSize → 650]
```

$$\begin{bmatrix}1\\x\\y\\u_x\\u_y\end{bmatrix}=\begin{bmatrix}1&1&\dots&1\\x_1&x_2&\dots&x_n\\y_1&y_2&\dots&y_n\\u_{x1}&u_{x2}&\dots&u_{xn}\\u_{y1}&u_{y2}&\dots&u_{yn}\end{bmatrix}\begin{bmatrix}N_1^{(e)}\\N_2^{(e)}\\\vdots\\N_n^{(e)}\end{bmatrix}. \qquad (16.6)$$

```
x = {x1, x2, x3, x4, x5, x6, x7, x8, x9, x10};
y = {y1, y2, y3, y4, y5, y6, y7, y8, y9, y10};
```

▪ 2 - Funciones de Forma <<< -----

```
Nf
```

$$\left\{\frac{1}{2}\xi1\,(-2+3\,\xi1)\,(-1+3\,\xi1),\ \frac{1}{2}\xi2\,(-2+3\,\xi2)\,(-1+3\,\xi2),\ \frac{1}{2}\xi3\,(-2+3\,\xi3)\,(-1+3\,\xi3),\ \frac{9}{2}\xi1\,(-1+3\,\xi1)\,\xi2,\ \frac{9}{2}\xi1\,\xi2\,(-1+3\,\xi2),\ \frac{9}{2}\xi2\,(-1+3\,\xi2)\,\xi3,\ \frac{9}{2}\xi2\,\xi3\,(-1+3\,\xi3),\ \frac{9}{2}\xi1\,\xi3\,(-1+3\,\xi3),\ \frac{9}{2}\xi1\,(-1+3\,\xi1)\,\xi3,\ 27\,\xi1\,\xi2\,\xi3\right\}$$

▪ 3 - Derivadas Funciones de Forma respecto Coordenadas Naturales.

```
Nf1 = D[Nf, ξ1];
Nf2 = D[Nf, ξ2];
Nf3 = D[Nf, ξ3];
{Nf1, Nf2, Nf3} = Simplify[{Nf1, Nf2, Nf3}];
```

```
Nf1 // MatrixForm
```

$$\begin{pmatrix}1-9\,\xi1+\frac{27\,\xi1^2}{2}\\0\\0\\\frac{9}{2}(-1+6\,\xi1)\,\xi2\\\frac{9}{2}\xi2\,(-1+3\,\xi2)\\0\\0\\\frac{9}{2}\xi3\,(-1+3\,\xi3)\\\frac{9}{2}(-1+6\,\xi1)\,\xi3\\27\,\xi2\,\xi3\end{pmatrix}$$

```
Nf2 // MatrixForm
```

$$\begin{pmatrix} 0 \\ 1 - 9\,\xi 2 + \frac{27\,\xi 2^2}{2} \\ 0 \\ \frac{9}{2}\,\xi 1\,(-1 + 3\,\xi 1) \\ \frac{9}{2}\,\xi 1\,(-1 + 6\,\xi 2) \\ \frac{9}{2}\,(-1 + 6\,\xi 2)\,\xi 3 \\ \frac{9}{2}\,\xi 3\,(-1 + 3\,\xi 3) \\ 0 \\ 0 \\ 27\,\xi 1\,\xi 3 \end{pmatrix}$$

```
Nf3 // MatrixForm
```

$$\begin{pmatrix} 0 \\ 0 \\ 1 - 9\,\xi 3 + \frac{27\,\xi 3^2}{2} \\ 0 \\ 0 \\ \frac{9}{2}\,\xi 2\,(-1 + 3\,\xi 2) \\ \frac{9}{2}\,\xi 2\,(-1 + 6\,\xi 3) \\ \frac{9}{2}\,\xi 1\,(-1 + 6\,\xi 3) \\ \frac{9}{2}\,\xi 1\,(-1 + 3\,\xi 1) \\ 27\,\xi 1\,\xi 2 \end{pmatrix}$$

▪ 4 - Derivadas Coordenadas Triangulares respecto a las Cartesianas - Desarrollo

▫ Calculo Elementos Matriz Jacobiana - Elemento Considerado

```
SistemA = [image];
```

```
SistemAr = Show[SistemA, ImageSize → 650]
```

$$\begin{bmatrix} \sum^{1} x_i \frac{\partial N_i}{\partial \zeta_1} & \sum^{1} x_i \frac{\partial N_i}{\partial \zeta_2} & \sum^{1} x_i \frac{\partial N_i}{\partial \zeta_3} \\ \sum y_i \frac{\partial N_i}{\partial \zeta_1} & \sum y_i \frac{\partial N_i}{\partial \zeta_2} & \sum y_i \frac{\partial N_i}{\partial \zeta_3} \end{bmatrix} \begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (24.18)$$

```
{Nf1x, Nf2x, Nf3x} = Simplify[{x.Nf1, x.Nf2, x.Nf3}];
{Nf1y, Nf2y, Nf3y} = Simplify[{y.Nf1, y.Nf2, y.Nf3}];
```

```
{Nf1x, Nf2x, Nf3x} // MatrixForm
```

$$\begin{pmatrix} \frac{1}{2}\left(x1\left(2 - 18\,\xi 1 + 27\,\xi 1^2\right) + 9\,(x4\,(-1 + 6\,\xi 1)\,\xi 2 + x5\,\xi 2\,(-1 + 3\,\xi 2) + \xi 3\,(-x8 - x9 + 6\,x9\,\xi 1 + 6\,x10\,\xi 2 + 3\,x8\,\xi 3))\right) \\ \frac{1}{2}\left(x2\left(2 - 18\,\xi 2 + 27\,\xi 2^2\right) + 9\,(x4\,\xi 1\,(-1 + 3\,\xi 1) + x5\,\xi 1\,(-1 + 6\,\xi 2) + \xi 3\,(-x6 - x7 + 6\,x10\,\xi 1 + 6\,x6\,\xi 2 + 3\,x7\,\xi 3))\right) \\ \frac{1}{2}\left(x3\left(2 - 18\,\xi 3 + 27\,\xi 3^2\right) + 9\,(x9\,\xi 1\,(-1 + 3\,\xi 1) + x8\,\xi 1\,(-1 + 6\,\xi 3) + \xi 2\,(-x6 - x7 + 6\,x10\,\xi 1 + 3\,x6\,\xi 2 + 6\,x7\,\xi 3))\right) \end{pmatrix}$$

```
{Nf1y, Nf2y, Nf3y} // MatrixForm
```

$$\begin{pmatrix} \frac{1}{2}\left(y1\left(2-18\,\xi 1+27\,\xi 1^2\right)+9\left(y4\left(-1+6\,\xi 1\right)\xi 2+y5\,\xi 2\left(-1+3\,\xi 2\right)+\xi 3\left(-y8-y9+6\,y9\,\xi 1+6\,y10\,\xi 2+3\,y8\,\xi 3\right)\right)\right) \\ \frac{1}{2}\left(y2\left(2-18\,\xi 2+27\,\xi 2^2\right)+9\left(y4\,\xi 1\left(-1+3\,\xi 1\right)+y5\,\xi 1\left(-1+6\,\xi 2\right)+\xi 3\left(-y6-y7+6\,y10\,\xi 1+6\,y6\,\xi 2+3\,y7\,\xi 3\right)\right)\right) \\ \frac{1}{2}\left(y3\left(2-18\,\xi 3+27\,\xi 3^2\right)+9\left(y9\,\xi 1\left(-1+3\,\xi 1\right)+y8\,\xi 1\left(-1+6\,\xi 3\right)+\xi 2\left(-y6-y7+6\,y10\,\xi 1+3\,y6\,\xi 2+6\,y7\,\xi 3\right)\right)\right) \end{pmatrix}$$

JacobianA = rewritten

$$\mathbf{J\,P} = \begin{bmatrix} 1 & 1 & 1 \\ J_{x1} & J_{x2} & J_{x3} \\ J_{y1} & J_{y2} & J_{y3} \end{bmatrix} \begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (24.19)$$

;

```
JacobianAr = Show[JacobianA, ImageSize → 650]
```

rewritten

$$\mathbf{J\,P} = \begin{bmatrix} 1 & 1 & 1 \\ J_{x1} & J_{x2} & J_{x3} \\ J_{y1} & J_{y2} & J_{y3} \end{bmatrix} \begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (24.19)$$

Para mostrar el planteamiento genérico, de momento no aplicamos estas definiciones:

```
Clear[Jx1, Jx2, Jx3, Jy1, Jy2, Jy3]
```

```
(*Jx1=Nf1x; Jx2=Nf2x; Jx3=Nf3x;*)
```

```
(*Jy1=Nf1y; Jy2=Nf2y; Jy3=Nf3y;*)
```

Definición Matriz Jacobiana y Sistema de Ecuaciones a Resolver - Planteamiento Genérico

$$J = \begin{pmatrix} 1 & 1 & 1 \\ Jx1 & Jx2 & Jx3 \\ Jy1 & Jy2 & Jy3 \end{pmatrix};$$

Determinante de la Matriz Jacobiana - J

```
Jdet = Det[J]
```

```
-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3
```

Defincion de Jc

```
Jc = 1/2*Jdet
```

$$\frac{1}{2}\left(-Jx2\,Jy1+Jx3\,Jy1+Jx1\,Jy2-Jx3\,Jy2-Jx1\,Jy3+Jx2\,Jy3\right)$$

Matriz Derivadas Coordenadas Triangulares respecto a las Cartesianas - Matriz Incognitas

$$P = \begin{pmatrix} f1x & f1y \\ f2x & f2y \\ f3x & f3y \end{pmatrix};$$

Matriz de Terminos Independientes

```
Ti = (0 0
      1 0
      0 1);
```

Solución del Sistema de Ecuaciones

```
P = Inverse[J].Ti
```

```
{{(Jy2 - Jy3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (-Jx2 + Jx3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)},
 {(-Jy1 + Jy3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (Jx1 - Jx3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)},
 {(Jy1 - Jy2) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (-Jx1 + Jx2) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)}}
```

Defínción Matriz Jacobiana Inversa Modificada - Jim

JacobianaModificadA = $\begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J} \begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \quad (24.20)$;

```
JacobianaModificadAr = Show[JacobianaModificadA, ImageSize → 650]
```

$$\begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J} \begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \quad (24.20)$$

```
Jc
```

$\frac{1}{2}$ (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)

```
P
```

```
{{(Jy2 - Jy3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (-Jx2 + Jx3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)},
 {(-Jy1 + Jy3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (Jx1 - Jx3) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)},
 {(Jy1 - Jy2) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3),
  (-Jx1 + Jx2) / (-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3)}}
```

```
Jim = 2 * Jc * P
```

```
{{Jy2 - Jy3, -Jx2 + Jx3}, {-Jy1 + Jy3, Jx1 - Jx3}, {Jy1 - Jy2, -Jx1 + Jx2}}
```

```
Jim // MatrixForm
```

```
( Jy2 - Jy3    -Jx2 + Jx3
 -Jy1 + Jy3     Jx1 - Jx3
  Jy1 - Jy2    -Jx1 + Jx2 )
```

```
(Jy23  Jx32)
(Jy31  Jx13) = Jim;
(Jy12  Jx21)
```

```
Jx13
```

```
Jx1 - Jx3
```

Necesario para mantener el planteamiento genérico:

```
Clear[Jdet, Jc, Jy23, Jx32, Jy31, Jx13, Jy12, Jx21]
```

```
Jc = 1/2*Jdet;
```

```
JacobianaModificadAr
```

$$\begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J} \begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \qquad (24.20)$$

```
(f1x  f1y)      1     (Jy23  Jx32)
(f2x  f2y) = ------ * (Jy31  Jx13);
(f3x  f3y)   2*Jc     (Jy12  Jx21)
```

```
f1x
```

$$\frac{Jy23}{Jdet}$$

■ 5 - Derivadas de las Funciones de Forma respecto a las Coordenadas Cartesianas - Desarrollo

▫ Definición de las Derivadas de las Funciones de Forma respecto a las Coordenadas Cartesianas - Planteamiento Genérico

DerivadasFuncionesFormaCartesianaS =

$$\frac{\partial N_i}{\partial x} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{y23} + \frac{\partial N_i}{\partial \zeta_2} J_{y31} + \frac{\partial N_i}{\partial \zeta_3} J_{y12}\right), \quad \frac{\partial N_i}{\partial y} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{x32} + \frac{\partial N_i}{\partial \zeta_2} J_{x13} + \frac{\partial N_i}{\partial \zeta_3} J_{x21}\right). \qquad (24.22)$$

;

```
DerivadasFuncionesFormaCartesianaSr = Show[DerivadasFuncionesFormaCartesianas, ImageSize → 650]
```

$$\frac{\partial N_i}{\partial x} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{y23} + \frac{\partial N_i}{\partial \zeta_2} J_{y31} + \frac{\partial N_i}{\partial \zeta_3} J_{y12}\right),$$
$$\frac{\partial N_i}{\partial y} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{x32} + \frac{\partial N_i}{\partial \zeta_2} J_{x13} + \frac{\partial N_i}{\partial \zeta_3} J_{x21}\right). \qquad (24.22)$$

DerivadasFuncionesFormaCartesianasM =

In matrix form:

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = \mathbf{P}^T \left[\frac{\partial N_i}{\partial \zeta_1} \quad \frac{\partial N_i}{\partial \zeta_2} \quad \frac{\partial N_i}{\partial \zeta_3} \right]^T, \qquad (24.23)$$

;

```
DerivadasFuncionesFormaCartesianasMr = Show[DerivadasFuncionesFormaCartesianasM, ImageSize → 650]
```

In matrix form:

$$\begin{bmatrix} \dfrac{\partial N_i}{\partial x} \\ \dfrac{\partial N_i}{\partial y} \end{bmatrix} = \mathbf{P}^T \left[\dfrac{\partial N_i}{\partial \zeta_1} \quad \dfrac{\partial N_i}{\partial \zeta_2} \quad \dfrac{\partial N_i}{\partial \zeta_3} \right]^T, \tag{24.23}$$

$$\begin{pmatrix} dNx \\ dNy \end{pmatrix} = \text{Transpose}\left[\begin{pmatrix} f1x & f1y \\ f2x & f2y \\ f3x & f3y \end{pmatrix}\right].\begin{pmatrix} Nf1 \\ Nf2 \\ Nf3 \end{pmatrix};$$

dNx & dNy <<< -----

```
dNx
```

$$\Big\{\frac{Jy23\left(1-9\,\xi1+\frac{27\,\xi1^2}{2}\right)}{Jdet},\ \frac{Jy31\left(1-9\,\xi2+\frac{27\,\xi2^2}{2}\right)}{Jdet},$$
$$\frac{Jy12\left(1-9\,\xi3+\frac{27\,\xi3^2}{2}\right)}{Jdet},\ \frac{9\,Jy31\,\xi1\,(-1+3\,\xi1)}{2\,Jdet}+\frac{9\,Jy23\,(-1+6\,\xi1)\,\xi2}{2\,Jdet},$$
$$\frac{9\,Jy23\,\xi2\,(-1+3\,\xi2)}{2\,Jdet}+\frac{9\,Jy31\,\xi1\,(-1+6\,\xi2)}{2\,Jdet},\ \frac{9\,Jy12\,\xi2\,(-1+3\,\xi2)}{2\,Jdet}+\frac{9\,Jy31\,(-1+6\,\xi2)\,\xi3}{2\,Jdet},$$
$$\frac{9\,Jy31\,\xi3\,(-1+3\,\xi3)}{2\,Jdet}+\frac{9\,Jy12\,\xi2\,(-1+6\,\xi3)}{2\,Jdet},\ \frac{9\,Jy23\,\xi3\,(-1+3\,\xi3)}{2\,Jdet}+\frac{9\,Jy12\,\xi1\,(-1+6\,\xi3)}{2\,Jdet},$$
$$\frac{9\,Jy12\,\xi1\,(-1+3\,\xi1)}{2\,Jdet}+\frac{9\,Jy23\,(-1+6\,\xi1)\,\xi3}{2\,Jdet},\ \frac{27\,Jy12\,\xi1\,\xi2}{Jdet}+\frac{27\,Jy31\,\xi1\,\xi3}{Jdet}+\frac{27\,Jy23\,\xi2\,\xi3}{Jdet}\Big\}$$

```
dNy
```

$$\Big\{\frac{Jx32\left(1-9\,\xi1+\frac{27\,\xi1^2}{2}\right)}{Jdet},\ \frac{Jx13\left(1-9\,\xi2+\frac{27\,\xi2^2}{2}\right)}{Jdet},$$
$$\frac{Jx21\left(1-9\,\xi3+\frac{27\,\xi3^2}{2}\right)}{Jdet},\ \frac{9\,Jx13\,\xi1\,(-1+3\,\xi1)}{2\,Jdet}+\frac{9\,Jx32\,(-1+6\,\xi1)\,\xi2}{2\,Jdet},$$
$$\frac{9\,Jx32\,\xi2\,(-1+3\,\xi2)}{2\,Jdet}+\frac{9\,Jx13\,\xi1\,(-1+6\,\xi2)}{2\,Jdet},\ \frac{9\,Jx21\,\xi2\,(-1+3\,\xi2)}{2\,Jdet}+\frac{9\,Jx13\,(-1+6\,\xi2)\,\xi3}{2\,Jdet},$$
$$\frac{9\,Jx13\,\xi3\,(-1+3\,\xi3)}{2\,Jdet}+\frac{9\,Jx21\,\xi2\,(-1+6\,\xi3)}{2\,Jdet},\ \frac{9\,Jx32\,\xi3\,(-1+3\,\xi3)}{2\,Jdet}+\frac{9\,Jx21\,\xi1\,(-1+6\,\xi3)}{2\,Jdet},$$
$$\frac{9\,Jx21\,\xi1\,(-1+3\,\xi1)}{2\,Jdet}+\frac{9\,Jx32\,(-1+6\,\xi1)\,\xi3}{2\,Jdet},\ \frac{27\,Jx21\,\xi1\,\xi2}{Jdet}+\frac{27\,Jx13\,\xi1\,\xi3}{Jdet}+\frac{27\,Jx32\,\xi2\,\xi3}{Jdet}\Big\}$$

Se observa quedan en función de los elementos de la Matriz Jacobiana Inversa Modificada - Jim

---

Matriz Jacobiana Inversa Modificada - Jim - Planteamiento Genérico

```
JacobianaModificadAr
```

$$\begin{bmatrix} \dfrac{\partial \zeta_1}{\partial x} & \dfrac{\partial \zeta_1}{\partial y} \\ \dfrac{\partial \zeta_2}{\partial x} & \dfrac{\partial \zeta_2}{\partial y} \\ \dfrac{\partial \zeta_3}{\partial x} & \dfrac{\partial \zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J}\begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \tag{24.20}$$

$$\begin{pmatrix} Jy23 & Jx32 \\ Jy31 & Jx13 \\ Jy12 & Jx21 \end{pmatrix} = Jim;$$

```
Jx13
```

Jx1 – Jx3

Elemetos Matriz Jacobiana - Elemento Considerado

```
JacobianAr
```

rewritten

$$\mathbf{J\,P} = \begin{bmatrix} 1 & 1 & 1 \\ J_{x1} & J_{x2} & J_{x3} \\ J_{y1} & J_{y2} & J_{y3} \end{bmatrix} \begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (24.19)$$

Aplicamos las deiniciones correspondientes al alemento considerado:

```
Jx1 = Nf1x; Jx2 = Nf2x; Jx3 = Nf3x;
```

```
Jy1 = Nf1y; Jy2 = Nf2y; Jy3 = Nf3y;
```

Matriz Jacobiana Inversa Modificada - Jim - Elemento Considerado dNx & dNy <<< -----

$$\begin{pmatrix} Jy23 & Jx32 \\ Jy31 & Jx13 \\ Jy12 & Jx21 \end{pmatrix};$$

```
Jy23
```

$$\frac{1}{2}\left(y2\left(2-18\,\xi2+27\,\xi2^2\right)+9\left(y4\,\xi1\,(-1+3\,\xi1)+y5\,\xi1\,(-1+6\,\xi2)+\xi3\,(-y6-y7+6\,y10\,\xi1+6\,y6\,\xi2+3\,y7\,\xi3)\right)\right)+$$
$$\frac{1}{2}\left(-y3\left(2-18\,\xi3+27\,\xi3^2\right)-9\left(y9\,\xi1\,(-1+3\,\xi1)+y8\,\xi1\,(-1+6\,\xi3)+\xi2\,(-y6-y7+6\,y10\,\xi1+3\,y6\,\xi2+6\,y7\,\xi3)\right)\right)$$

```
Jx32
```

$$\frac{1}{2}\left(-x2\left(2-18\,\xi2+27\,\xi2^2\right)-9\left(x4\,\xi1\,(-1+3\,\xi1)+x5\,\xi1\,(-1+6\,\xi2)+\xi3\,(-x6-x7+6\,x10\,\xi1+6\,x6\,\xi2+3\,x7\,\xi3)\right)\right)+$$
$$\frac{1}{2}\left(x3\left(2-18\,\xi3+27\,\xi3^2\right)+9\left(x9\,\xi1\,(-1+3\,\xi1)+x8\,\xi1\,(-1+6\,\xi3)+\xi2\,(-x6-x7+6\,x10\,\xi1+3\,x6\,\xi2+6\,x7\,\xi3)\right)\right)$$

```
Jy31
```

$$\frac{1}{2}\left(y3\left(2-18\,\xi3+27\,\xi3^2\right)+9\left(y9\,\xi1\,(-1+3\,\xi1)+y8\,\xi1\,(-1+6\,\xi3)+\xi2\,(-y6-y7+6\,y10\,\xi1+3\,y6\,\xi2+6\,y7\,\xi3)\right)\right)+$$
$$\frac{1}{2}\left(-y1\left(2-18\,\xi1+27\,\xi1^2\right)-9\left(y4\,(-1+6\,\xi1)\,\xi2+y5\,\xi2\,(-1+3\,\xi2)+\xi3\,(-y8-y9+6\,y9\,\xi1+6\,y10\,\xi2+3\,y8\,\xi3)\right)\right)$$

Jx13

$$\frac{1}{2}\left(-x3\left(2-18\,\xi3+27\,\xi3^2\right)-9\,(x9\,\xi1\,(-1+3\,\xi1)+x8\,\xi1\,(-1+6\,\xi3)+\xi2\,(-x6-x7+6\,x10\,\xi1+3\,x6\,\xi2+6\,x7\,\xi3))\right)+\frac{1}{2}\left(x1\left(2-18\,\xi1+27\,\xi1^2\right)+9\,(x4\,(-1+6\,\xi1)\,\xi2+x5\,\xi2\,(-1+3\,\xi2)+\xi3\,(-x8-x9+6\,x9\,\xi1+6\,x10\,\xi2+3\,x8\,\xi3))\right)$$

Jy12

$$\frac{1}{2}\left(-y2\left(2-18\,\xi2+27\,\xi2^2\right)-9\,(y4\,\xi1\,(-1+3\,\xi1)+y5\,\xi1\,(-1+6\,\xi2)+\xi3\,(-y6-y7+6\,y10\,\xi1+6\,y6\,\xi2+3\,y7\,\xi3))\right)+\frac{1}{2}\left(y1\left(2-18\,\xi1+27\,\xi1^2\right)+9\,(y4\,(-1+6\,\xi1)\,\xi2+y5\,\xi2\,(-1+3\,\xi2)+\xi3\,(-y8-y9+6\,y9\,\xi1+6\,y10\,\xi2+3\,y8\,\xi3))\right)$$

Jx21

$$\frac{1}{2}\left(x2\left(2-18\,\xi2+27\,\xi2^2\right)+9\,(x4\,\xi1\,(-1+3\,\xi1)+x5\,\xi1\,(-1+6\,\xi2)+\xi3\,(-x6-x7+6\,x10\,\xi1+6\,x6\,\xi2+3\,x7\,\xi3))\right)+\frac{1}{2}\left(-x1\left(2-18\,\xi1+27\,\xi1^2\right)-9\,(x4\,(-1+6\,\xi1)\,\xi2+x5\,\xi2\,(-1+3\,\xi2)+\xi3\,(-x8-x9+6\,x9\,\xi1+6\,x10\,\xi2+3\,x8\,\xi3))\right)$$

MODULO CALCULO FUNCIONES DE FORMA Y JACOBIANO

MODULO GENERICO A COMPLETAR -

MODULO GENERICO A COMPLETAR: XX = No. Nodos, {xI,yI}= nodo i-esimo

```
(*TrigXXIsoPShapeFunDer[ncoor_,tcoor_]:=Module[{ξ1,ξ2,ξ3,x1,x2,x3,x4,x5,x6,x7,x8,x9,xI,y1,y2,y3,
    y4,y5,y6,y7,y8,y9,yI,Jx21,Jx32,Jx13,Jy12,Jy23,Jy31,Nf,dNx,dNy,Jdet},{ξ1,ξ2,ξ3}=tcoor;
   {{x1,y1},{x2,y2},{x3,y3},{x4,y4},{x5,y5},{x6,y6},{x7,y7},{x8,y8},{x9,y9},{xI,yI}}=ncoor;
   Nf=(*Nf*);
   Jx21=(*Jx21*);
   Jy12=(*Jy12*);
   Jx13=(*Jx13*);
   Jy31=(*Jy31*);
   Jx32=(*Jx32*);
   Jy23=(*Jy23*);
   Jdet=Jx21*Jy31-Jy12*Jx13;
   dNx=(*dNx*);
   dNy=(*dNy*);
   Return[Simplify[{Nf,dNx,dNy,Jdet}]]];*)
```

MODULO COMPLETADO -

```
Trig10IsoPShapeFunDer[ncoor_, tcoor_] :=
  Module[{ξ1, ξ2, ξ3, x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, y1, y2, y3, y4, y5, y6, y7, y8,
    y9, y10, Jx21, Jx32, Jx13, Jy12, Jy23, Jy31, Nf, dNx, dNy, Jdet}, {ξ1, ξ2, ξ3} = tcoor;
   {{x1, y1}, {x2, y2}, {x3, y3}, {x4, y4}, {x5, y5}, {x6, y6},
     {x7, y7}, {x8, y8}, {x9, y9}, {x10, y10}} = ncoor;
   Nf = {1/2 ξ1 (-2 + 3 ξ1) (-1 + 3 ξ1), 1/2 ξ2 (-2 + 3 ξ2) (-1 + 3 ξ2), 1/2 ξ3 (-2 + 3 ξ3) (-1 + 3 ξ3),
     9/2 ξ1 (-1 + 3 ξ1) ξ2, 9/2 ξ1 ξ2 (-1 + 3 ξ2), 9/2 ξ2 (-1 + 3 ξ2) ξ3,
```

$$\frac{9}{2}\xi2\,\xi3\,(-1+3\,\xi3),\ \frac{9}{2}\xi1\,\xi3\,(-1+3\,\xi3),\ \frac{9}{2}\xi1\,(-1+3\,\xi1)\,\xi3,\ 27\,\xi1\,\xi2\,\xi3\Big\};$$

$$\text{Jx21}=\frac{1}{2}\left(\text{x2}\left(2-18\,\xi2+27\,\xi2^{2}\right)+9\,(\text{x4}\,\xi1\,(-1+3\,\xi1)+\text{x5}\,\xi1\,(-1+6\,\xi2)+\xi3\,(-\text{x6}-\text{x7}+6\,\text{x10}\,\xi1+6\,\text{x6}\,\xi2+3\,\text{x7}\,\xi3))\right)+\frac{1}{2}\left(-\text{x1}\left(2-18\,\xi1+27\,\xi1^{2}\right)-9\,(\text{x4}\,(-1+6\,\xi1)\,\xi2+\text{x5}\,\xi2\,(-1+3\,\xi2)+\xi3\,(-\text{x8}-\text{x9}+6\,\text{x9}\,\xi1+6\,\text{x10}\,\xi2+3\,\text{x8}\,\xi3))\right);$$

$$\text{Jy12}=\frac{1}{2}\left(-\text{y2}\left(2-18\,\xi2+27\,\xi2^{2}\right)-9\,(\text{y4}\,\xi1\,(-1+3\,\xi1)+\text{y5}\,\xi1\,(-1+6\,\xi2)+\xi3\,(-\text{y6}-\text{y7}+6\,\text{y10}\,\xi1+6\,\text{y6}\,\xi2+3\,\text{y7}\,\xi3))\right)+\frac{1}{2}\left(\text{y1}\left(2-18\,\xi1+27\,\xi1^{2}\right)+9\,(\text{y4}\,(-1+6\,\xi1)\,\xi2+\text{y5}\,\xi2\,(-1+3\,\xi2)+\xi3\,(-\text{y8}-\text{y9}+6\,\text{y9}\,\xi1+6\,\text{y10}\,\xi2+3\,\text{y8}\,\xi3))\right);$$

$$\text{Jx13}=\frac{1}{2}\left(-\text{x3}\left(2-18\,\xi3+27\,\xi3^{2}\right)-9\,(\text{x9}\,\xi1\,(-1+3\,\xi1)+\text{x8}\,\xi1\,(-1+6\,\xi3)+\xi2\,(-\text{x6}-\text{x7}+6\,\text{x10}\,\xi1+3\,\text{x6}\,\xi2+6\,\text{x7}\,\xi3))\right)+\frac{1}{2}\left(\text{x1}\left(2-18\,\xi1+27\,\xi1^{2}\right)+9\,(\text{x4}\,(-1+6\,\xi1)\,\xi2+\text{x5}\,\xi2\,(-1+3\,\xi2)+\xi3\,(-\text{x8}-\text{x9}+6\,\text{x9}\,\xi1+6\,\text{x10}\,\xi2+3\,\text{x8}\,\xi3))\right);$$

$$\text{Jy31}=\frac{1}{2}\left(\text{y3}\left(2-18\,\xi3+27\,\xi3^{2}\right)+9\,(\text{y9}\,\xi1\,(-1+3\,\xi1)+\text{y8}\,\xi1\,(-1+6\,\xi3)+\xi2\,(-\text{y6}-\text{y7}+6\,\text{y10}\,\xi1+3\,\text{y6}\,\xi2+6\,\text{y7}\,\xi3))\right)+\frac{1}{2}\left(-\text{y1}\left(2-18\,\xi1+27\,\xi1^{2}\right)-9\,(\text{y4}\,(-1+6\,\xi1)\,\xi2+\text{y5}\,\xi2\,(-1+3\,\xi2)+\xi3\,(-\text{y8}-\text{y9}+6\,\text{y9}\,\xi1+6\,\text{y10}\,\xi2+3\,\text{y8}\,\xi3))\right);$$

$$\text{Jx32}=\frac{1}{2}\left(-\text{x2}\left(2-18\,\xi2+27\,\xi2^{2}\right)-9\,(\text{x4}\,\xi1\,(-1+3\,\xi1)+\text{x5}\,\xi1\,(-1+6\,\xi2)+\xi3\,(-\text{x6}-\text{x7}+6\,\text{x10}\,\xi1+6\,\text{x6}\,\xi2+3\,\text{x7}\,\xi3))\right)+\frac{1}{2}\left(\text{x3}\left(2-18\,\xi3+27\,\xi3^{2}\right)+9\,(\text{x9}\,\xi1\,(-1+3\,\xi1)+\text{x8}\,\xi1\,(-1+6\,\xi3)+\xi2\,(-\text{x6}-\text{x7}+6\,\text{x10}\,\xi1+3\,\text{x6}\,\xi2+6\,\text{x7}\,\xi3))\right);$$

$$\text{Jy23}=\frac{1}{2}\left(\text{y2}\left(2-18\,\xi2+27\,\xi2^{2}\right)+9\,(\text{y4}\,\xi1\,(-1+3\,\xi1)+\text{y5}\,\xi1\,(-1+6\,\xi2)+\xi3\,(-\text{y6}-\text{y7}+6\,\text{y10}\,\xi1+6\,\text{y6}\,\xi2+3\,\text{y7}\,\xi3))\right)+\frac{1}{2}\left(-\text{y3}\left(2-18\,\xi3+27\,\xi3^{2}\right)-9\,(\text{y9}\,\xi1\,(-1+3\,\xi1)+\text{y8}\,\xi1\,(-1+6\,\xi3)+\xi2\,(-\text{y6}-\text{y7}+6\,\text{y10}\,\xi1+3\,\text{y6}\,\xi2+6\,\text{y7}\,\xi3))\right);$$

$$\text{Jdet}=\text{Jx21}*\text{Jy31}-\text{Jy12}*\text{Jx13};$$

$$\text{dNx}=\Big\{\frac{\text{Jy23}\left(1-9\,\xi1+\frac{27\,\xi1^{2}}{2}\right)}{\text{Jdet}},\ \frac{\text{Jy31}\left(1-9\,\xi2+\frac{27\,\xi2^{2}}{2}\right)}{\text{Jdet}},\ \frac{\text{Jy12}\left(1-9\,\xi3+\frac{27\,\xi3^{2}}{2}\right)}{\text{Jdet}},\ \frac{9\,\text{Jy31}\,\xi1\,(-1+3\,\xi1)}{2\,\text{Jdet}}+\frac{9\,\text{Jy23}\,(-1+6\,\xi1)\,\xi2}{2\,\text{Jdet}},$$
$$\frac{9\,\text{Jy23}\,\xi2\,(-1+3\,\xi2)}{2\,\text{Jdet}}+\frac{9\,\text{Jy31}\,\xi1\,(-1+6\,\xi2)}{2\,\text{Jdet}},\ \frac{9\,\text{Jy12}\,\xi2\,(-1+3\,\xi2)}{2\,\text{Jdet}}+\frac{9\,\text{Jy31}\,(-1+6\,\xi2)\,\xi3}{2\,\text{Jdet}},$$
$$\frac{9\,\text{Jy31}\,\xi3\,(-1+3\,\xi3)}{2\,\text{Jdet}}+\frac{9\,\text{Jy12}\,\xi2\,(-1+6\,\xi3)}{2\,\text{Jdet}},\ \frac{9\,\text{Jy23}\,\xi3\,(-1+3\,\xi3)}{2\,\text{Jdet}}+\frac{9\,\text{Jy12}\,\xi1\,(-1+6\,\xi3)}{2\,\text{Jdet}},$$
$$\frac{9\,\text{Jy12}\,\xi1\,(-1+3\,\xi1)}{2\,\text{Jdet}}+\frac{9\,\text{Jy23}\,(-1+6\,\xi1)\,\xi3}{2\,\text{Jdet}},\ \frac{27\,\text{Jy12}\,\xi1\,\xi2}{\text{Jdet}}+\frac{27\,\text{Jy31}\,\xi1\,\xi3}{\text{Jdet}}+\frac{27\,\text{Jy23}\,\xi2\,\xi3}{\text{Jdet}}\Big\};$$

$$\text{dNy}=\Big\{\frac{\text{Jx32}\left(1-9\,\xi1+\frac{27\,\xi1^{2}}{2}\right)}{\text{Jdet}},\ \frac{\text{Jx13}\left(1-9\,\xi2+\frac{27\,\xi2^{2}}{2}\right)}{\text{Jdet}},\ \frac{\text{Jx21}\left(1-9\,\xi3+\frac{27\,\xi3^{2}}{2}\right)}{\text{Jdet}},$$
$$\frac{9\,\text{Jx13}\,\xi1\,(-1+3\,\xi1)}{2\,\text{Jdet}}+\frac{9\,\text{Jx32}\,(-1+6\,\xi1)\,\xi2}{2\,\text{Jdet}},\ \frac{9\,\text{Jx32}\,\xi2\,(-1+3\,\xi2)}{2\,\text{Jdet}}+\frac{9\,\text{Jx13}\,\xi1\,(-1+6\,\xi2)}{2\,\text{Jdet}},$$
$$\frac{9\,\text{Jx21}\,\xi2\,(-1+3\,\xi2)}{2\,\text{Jdet}}+\frac{9\,\text{Jx13}\,(-1+6\,\xi2)\,\xi3}{2\,\text{Jdet}},\ \frac{9\,\text{Jx13}\,\xi3\,(-1+3\,\xi3)}{2\,\text{Jdet}}+\frac{9\,\text{Jx21}\,\xi2\,(-1+6\,\xi3)}{2\,\text{Jdet}},$$
$$\frac{9\,\text{Jx32}\,\xi3\,(-1+3\,\xi3)}{2\,\text{Jdet}}+\frac{9\,\text{Jx21}\,\xi1\,(-1+6\,\xi3)}{2\,\text{Jdet}},\ \frac{9\,\text{Jx21}\,\xi1\,(-1+3\,\xi1)}{2\,\text{Jdet}}+\frac{9\,\text{Jx32}\,(-1+6\,\xi1)\,\xi3}{2\,\text{Jdet}},$$

```
      27 Jx21 ξ1 ξ2   27 Jx13 ξ1 ξ3   27 Jx32 ξ2 ξ3
      ------------- + ------------- + -------------};
          Jdet            Jdet            Jdet
    Return[Simplify[{Nf, dNx, dNy, Jdet}]]];
```

6. MATRIZ DE RIGIDEZ - INTEGRACION NUMERICA -

MODULO DE CALCULO DE LA MATRIZ DE RIGIDEZ

MODULO GENERICO A COMPLETAR -

MODULO GENERICO: XX = No. Nodos, YY = Grados Libertad

```
(*TrigXXIsoPMembraneStiffness[ncoor_,mprop_,fprop_,opt_]:=
  Module[{i,k,l,p=3,numer=False,Emat,th={fprop},h,tcoor,w,c,Nf,dNx,dNy,Jdet,B,Ke=Table[0,{YY},{YY}]},
   Emat=mprop[[1]];If[Length[fprop]>0,th=fprop[[1]]];
   If[Length[opt]>0,numer=opt[[1]]];
   If[Length[opt]>1,p=opt[[2]]];
   If[p≠1&&p≠-3&&p≠3&&p≠6&&p≠-6&&p≠7&&p≠12,Print["Illegal p"];Return[Null]];
   For[k=1,k≤Abs[p],k++,{tcoor,w}=TrigGaussRuleInfo[{p,numer},k];
    {Nf,dNx,dNy,Jdet}=TrigXXIsoPShapeFunDer[ncoor,tcoor];
    If[Length[th]==0,h=th,h=th.Nf];c=w*Jdet*h/2;
    B={Flatten[Table[{dNx[[i]],0},{i,XX}]],
      Flatten[Table[{0,dNy[[i]]},{i,XX}]],Flatten[Table[{dNy[[i]],dNx[[i]]},{i,XX}]]};
    Ke+=c*Transpose[B].(Emat.B);];If[!numer,Ke=Simplify[Ke]];Return[Ke]];*)
```

MODULO COMPLETADO -

```
Trig10IsoPMembraneStiffness[ncoor_, mprop_, fprop_, opt_] :=
  Module[{i, k, l, p = 3, numer = False, Emat, th = {fprop}, h, tcoor, w, c, Nf, dNx, dNy, Jdet,
    B, Ke = Table[0, {20}, {20}]}, Emat = mprop[[1]]; If[Length[fprop] > 0, th = fprop[[1]]];
   If[Length[opt] > 0, numer = opt[[1]]];
   If[Length[opt] > 1, p = opt[[2]]];
   If[p ≠ 1 && p ≠ -3 && p ≠ 3 && p ≠ 6 && p ≠ -6 && p ≠ 7 && p ≠ 12, Print["Illegal p"]; Return[Null]];
   For[k = 1, k ≤ Abs[p], k++, {tcoor, w} = TrigGaussRuleInfo[{p, numer}, k];
    {Nf, dNx, dNy, Jdet} = Trig10IsoPShapeFunDer[ncoor, tcoor];
    If[Length[th] == 0, h = th, h = th.Nf]; c = w*Jdet*h/2;
    B = {Flatten[Table[{dNx[[i]], 0}, {i, 10}]],
      Flatten[Table[{0, dNy[[i]]}, {i, 10}]], Flatten[Table[{dNy[[i]], dNx[[i]]}, {i, 10}]]};
    Ke += c*Transpose[B].(Emat.B);]; If[! numer, Ke = Simplify[Ke]]; Return[Ke]];
```

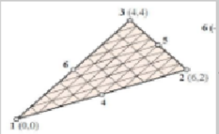
■ MODULO REGLAS DE CUADRATURA DE GAUSS

□ OPCION UNICA: DEFINICION DE CARLOS FELIPPA

```
TrigGaussRuleInfo[{rule_, numer_}, point_] :=
  Module[{zeta, p = rule, i = point, g1, g2, g3, g4, w1, w2, w3, eps = 10. ^ (-24),
    jkl = {{1, 2, 3}, {2, 1, 3}, {1, 3, 2}, {3, 1, 2}, {2, 3, 1}, {3, 2, 1}}, info = {{Null, Null, Null}, 0}},
   If[p == 1, info = {{1 / 3, 1 / 3, 1 / 3}, 1}]; If[p == 3, info = {{1, 1, 1} / 6, 1 / 3}; info[[1, i]] = 2 / 3];
   If[p == -3, info = {{1, 1, 1} / 2, 1 / 3}; info[[1, i]] = 0]; If[p == 6,
    g1 = (8 - Sqrt[10] + Sqrt[38 - 44 * Sqrt[2 / 5]]) / 18; g2 = (8 - Sqrt[10] - Sqrt[38 - 44 * Sqrt[2 / 5]]) / 18;
    If[i < 4, info = {{g1, g1, g1}, (620 + Sqrt[213 125 - 53 320 * Sqrt[10]]) / 3720}; info[[1, i]] = 1 - 2 * g1];
    If[i > 3, info = {{g2, g2, g2}, (620 - Sqrt[213 125 - 53 320 * Sqrt[10]]) / 3720};
     info[[1, i - 3]] = 1 - 2 * g2]]; If[p == -6, If[i < 4, info = {{1, 1, 1} / 6, 3 / 10}; info[[1, i]] = 2 / 3];
    If[i > 3, info = {{1, 1, 1} / 2, 1 / 30}; info[[1, i - 3]] = 0]]; If[p == 7, g1 = (6 - Sqrt[15]) / 21;
    g2 = (6 + Sqrt[15]) / 21; If[i < 4, info = {{g1, g1, g1}, (155 - Sqrt[15]) / 1200}; info[[1, i]] = 1 - 2 * g1];
    If[i > 3 && i < 7, info = {{g2, g2, g2}, (155 + Sqrt[15]) / 1200}; info[[1, i - 3]] = 1 - 2 * g2];
    If[i == 7, info = {{1 / 3, 1 / 3, 1 / 3}, 9 / 40}]];
   If[p == 12, g1 = 0.063089014491502228340331602870819157; g2 = 0.24928674517091042129163855310701907;
    g3 = 0.053145049844816947353249671631398147; g4 = 0.31035245103378440541660773395655215;
    If[! numer, {g1, g2, g3, g4} = Rationalize[{g1, g2, g3, g4}, eps]];
    w1 = (30 * g2 ^ 3 * (4 * g3 ^ 2 + (1 - 2 * g4) ^ 2 + 4 * g3 * (-1 + g4)) + g3 ^ 2 * (1 - 15 * g4) +
         (-1 + g4) * g4 - g3 * (-1 + g4) * (-1 + 15 * g4) + 2 * g2 * (1 + 60 * g3 * g4 * (-1 + g3 + g4)) -
         6 * g2 ^ 2 * (3 + 10 * (-1 + g4) * g4 + 10 * g3 ^ 2 * (1 + 3 * g4) + 10 * g3 * (-1 + g4) * (1 + 3 * g4))) /
       (180 * (g1 - g2) * (-(g2 * (-1 + 2 * g2) * (-1 + g3) * g3) + (-1 + g3) * (g2 - 2 * g2 ^ 2 - 2 * g3 + 3 * g2 * g3) * g4 -
            (g2 * (-1 + 2 * g2 - 3 * g3) + 2 * g3) * g4 ^ 2 + 2 * g1 ^ 2 * (g2 * (-2 + 3 * g2) + g3 - g3 ^ 2 + g4 - g3 * g4 - g4 ^ 2) +
            g1 * (-4 * g2 ^ 2 + (-1 + g3) * g3 + (-1 + g3) * (1 + 3 * g3) * g4 +
               (1 + 3 * g3) * g4 ^ 2 - 2 * g2 * (-1 + g3 ^ 2 + g3 * (-1 + g4) + (-1 + g4) * g4))));
    w2 = (-1 + 12 * (2 - 3 * g1) * g1 * w1 + 4 * g3 ^ 2 * (-1 + 3 * w1) + 4 * g3 * (-1 + g4) * (-1 + 3 * w1) +
         4 * (-1 + g4) * g4 * (-1 + 3 * w1)) / (12 * (g2 * (-2 + 3 * g2) + g3 - g3 ^ 2 + g4 - g3 * g4 - g4 ^ 2));
    w3 = (1 - 3 * w1 - 3 * w2) / 6; If[i < 4, info = {{g1, g1, g1}, w1}; info[[1, i]] = 1 - 2 * g1];
    If[i > 3 && i < 7, info = {{g2, g2, g2}, w2}; info[[1, i - 3]] = 1 - 2 * g2];
    If[i > 6, {j, k, l} = jkl[[i - 6]]; info = {{0, 0, 0}, w3};
     info[[1, j]] = g3; info[[1, k]] = g4; info[[1, l]] = 1 - g3 - g4]];
   If[numer, Return[N[info]], Return[Simplify[info]]];];
```

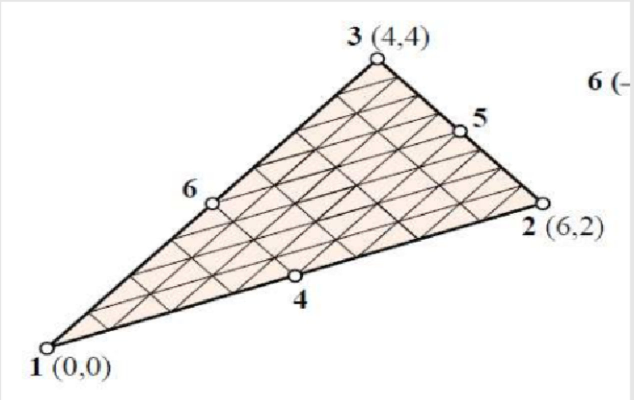
7. TEST DEL ELEMENTO DE LADOS RECTOS SUPERPARAMETRICO.

■ DEFINICION DE LA GEOMETRIA

LadosRectosTesT = 

;

```
LadosRectosTesTr = Show[LadosRectosTesT, ImageSize → 300]
```



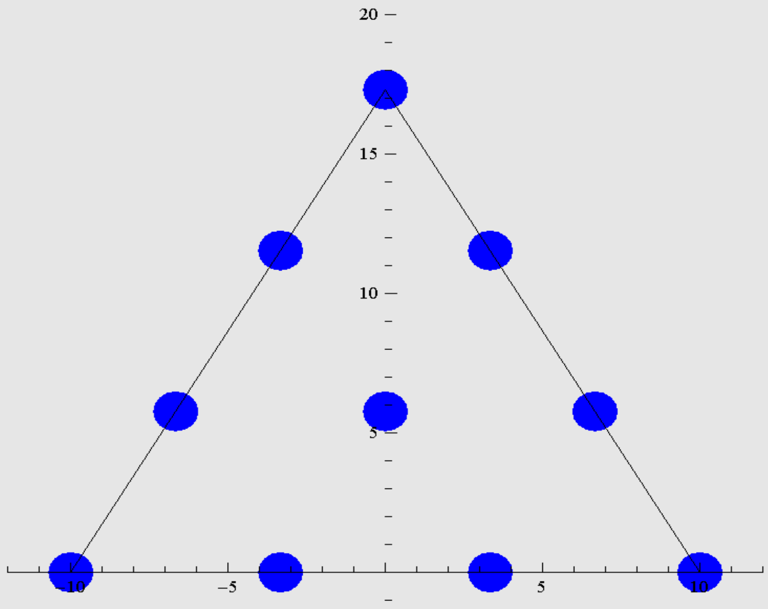
- **DEFINICION DE LOS NODOS ELEMENTO - #**

```
ncoor = Cnc;
```

```
ptsexteriores =
  {Cnc[[1]], Cnc[[4]], Cnc[[5]], Cnc[[2]], Cnc[[6]], Cnc[[7]], Cnc[[3]], Cnc[[8]], Cnc[[9]]};
```

```
ptsinteriores = {Cnc[[10]]};
```

```
Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
  Axes → True, PlotRange → {{-12, 12}, {-2, 20}}, NodeColor → RGBColor[0, 0, 1]]
```



- **DEFINICION DEL MATERIAL**

```
ClearAll[Em, nu, h];
```

```
h = 1; Em = 288; nu = 1 / 3;
```

```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

```
Print["Emat=", Emat // MatrixForm]
```

$$\text{Emat=}\begin{pmatrix} 324 & 108 & 0 \\ 108 & 324 & 0 \\ 0 & 0 & 108 \end{pmatrix}$$

VERIFICACION DE LA MATRIZ DE RIGIDEZ -

NUMERO DE PUNTOS DE GAUSS MINIMO PARA CONSEGUIR SUFICIENCIA DE RANGO -

```
NF = NNodos * 2.;
```

$$NG = \frac{NF - 3}{3}$$

```
5.66667
```

Se necesitan como mínimo 6 Puntos -- Regla 6 minima

BUCLE GENERICO A COMPLETAR -

BUCLE GENERICO: XX = No. Nodos, ZZ = GRADOS DE LIBERTAD - 3

```
(*For [i=1, i≤7,i++,p={1,-3,3,6,-6,7,12}[[i]];
  Ke=TrigXXIsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
  Print["Gauss integration rule: ",p];
  Print["Ke=",Chop[Simplify[Ke]]//MatrixForm];
  Valores=Chop[Eigenvalues[N[Ke]]];
  If[Valores[[ZZ]]!=0,Print["Valores propios matriz Ke=",Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=",p];Break[],Print[
    "Valores proprios matriz Ke=",Valores];Print["NO tenemos la suficiencia de rango para p=",p]]
 ];*)
```

DESARROLLO DE LA MATRIZ DE RIGIDEZ -

```
For [i = 1, i ≤ 7, i++, p = {1, -3, 3, 6, -6, 7, 12}[[i]];
  Ke = Trig10IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
  Print["Gauss integration rule: ", p];
  Print["Ke=", Chop[Simplify[Ke]] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]]];
  If[Valores[[17]] != 0, Print["Valores propios matriz Ke=", Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=", p] (*;Break[]*), Print[
    "Valores proprios matriz Ke=", Valores]; Print["NO tenemos la suficiencia de rango para p=", p]]
 ];
```

Gauss integration rule: 1

Ke=

38.9711	13.5	-31.1769	0	-7.79423	-13.5	-116.913	-40.5	93.5307	0	93.5307
13.5	23.3827	0	0	-13.5	-23.3827	-40.5	-70.1481	0	0	0
-31.1769	0	38.9711	-13.5	-7.79423	13.5	93.5307	0	-116.913	40.5	-116.913
0	0	-13.5	23.3827	13.5	-23.3827	0	0	40.5	-70.1481	40.5
-7.79423	-13.5	-7.79423	13.5	15.5885	0	23.3827	40.5	23.3827	-40.5	23.3827
-13.5	-23.3827	13.5	-23.3827	0	46.7654	40.5	70.1481	-40.5	70.1481	-40.5
-116.913	-40.5	93.5307	0	23.3827	40.5	350.74	121.5	-280.592	0	-280.592
-40.5	-70.1481	0	0	40.5	70.1481	121.5	210.444	0	0	0
93.5307	0	-116.913	40.5	23.3827	-40.5	-280.592	0	350.74	-121.5	350.74
0	0	40.5	-70.1481	-40.5	70.1481	0	0	-121.5	210.444	-121.5
93.5307	0	-116.913	40.5	23.3827	-40.5	-280.592	0	350.74	-121.5	350.74
0	0	40.5	-70.1481	-40.5	70.1481	0	0	-121.5	210.444	-121.5
23.3827	40.5	23.3827	-40.5	-46.7654	0	-70.1481	-121.5	-70.1481	121.5	-70.1481
40.5	70.1481	-40.5	70.1481	0	-140.296	-121.5	-210.444	121.5	-210.444	121.5
23.3827	40.5	23.3827	-40.5	-46.7654	0	-70.1481	-121.5	-70.1481	121.5	-70.1481
40.5	70.1481	-40.5	70.1481	0	-140.296	-121.5	-210.444	121.5	-210.444	121.5
-116.913	-40.5	93.5307	0	23.3827	40.5	350.74	121.5	-280.592	0	-280.592
-40.5	-70.1481	0	0	40.5	70.1481	121.5	210.444	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

Valores proprios matriz Ke={1777.08, 888.542, 888.542, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

NO tenemos la suficiencia de rango para p=1

Gauss integration rule: -3

Ke=

53.5853	18.5625	9.74279	0	2.4357	4.21875	-134.45	-50.625	62.841	17.7188	-208.983
18.5625	32.1512	0	0	4.21875	7.30709	-50.625	-87.6851	17.7188	30.6898	-17.7188
9.74279	0	53.5853	-18.5625	2.4357	-4.21875	62.841	-17.7188	-134.45	50.625	-143.219
0	0	-18.5625	32.1512	-4.21875	7.30709	-17.7188	30.6898	50.625	-87.6851	45.5625
2.4357	4.21875	2.4357	-4.21875	21.4341	0	-59.9181	-68.3438	-59.9181	68.3438	23.3827
4.21875	7.30709	-4.21875	7.30709	0	64.3024	-68.3438	-179.754	68.3438	-179.754	-5.0625
-134.45	-50.625	62.841	-17.7188	-59.9181	-68.3438	1025.92	410.063	-631.333	0	249.902
-50.625	-87.6851	-17.7188	30.6898	-68.3438	-179.754	410.063	868.082	0	157.833	22.7812
62.841	17.7188	-134.45	50.625	-59.9181	68.3438	-631.333	0	1025.92	-410.063	-26.3055
17.7188	30.6898	50.625	-87.6851	68.3438	-179.754	0	157.833	-410.063	868.082	-45.5625
-208.983	-17.7188	-143.219	45.5625	23.3827	-5.0625	249.902	22.7813	-26.3055	-45.5625	1262.67
-17.7188	-30.6898	45.5625	-78.9166	-5.0625	70.1481	22.7813	-197.291	-45.5625	-78.9166	-273.375
-90.6079	-86.0625	54.0725	-22.7813	-55.5339	-5.0625	407.736	296.156	-105.222	-182.25	-39.4583
-86.0625	-149.065	-22.7813	39.4583	-5.0625	-166.602	296.156	749.707	-182.25	157.833	341.719
54.0725	22.7813	-90.6079	86.0625	-55.5339	5.0625	-105.222	182.25	407.736	-296.156	-65.7638
22.7813	39.4583	86.0625	-149.065	5.0625	-166.602	182.25	157.833	-296.156	749.707	-205.031
-143.219	-45.5625	-208.983	17.7188	23.3827	5.0625	-26.3055	45.5625	249.902	-22.7813	920.693
-45.5625	-78.9166	17.7188	-30.6898	5.0625	70.1481	45.5625	-78.9166	-22.7813	-197.291	0
394.583	136.688	394.583	-136.688	157.833	0	-789.166	-820.125	-789.166	820.125	-1972.91
136.688	236.75	-136.688	236.75	0	473.499	-820.125	-1420.5	820.125	-1420.5	136.687

Valores proprios matriz Ke={7963.35, 7963.35, 1737.62, 1657.25,
1447.68, 1447.68, 363.898, 202.168, 202.168, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

NO tenemos la suficiencia de rango para p=-3

Gauss integration rule: 3

Ke=

53.5853	18.5625	9.74279	0	2.4357	4.21875	-11.6913	40.5	-24.8441	-12.6563	18.9984
18.5625	32.1512	0	0	4.21875	7.30709	40.5	70.1481	-12.6563	-21.9213	12.6563
9.74279	0	53.5853	-18.5625	2.4357	-4.21875	-24.8441	12.6562	-11.6913	-40.5	84.7622
0	0	-18.5625	32.1512	-4.21875	7.30709	12.6562	-21.9213	-40.5	70.1481	15.1875
2.4357	4.21875	2.4357	-4.21875	21.4341	0	10.2299	-7.59375	10.2299	7.59375	-11.6913
4.21875	7.30709	-4.21875	7.30709	0	64.3024	-7.59375	30.6898	7.59375	30.6898	-5.0625
-11.6913	40.5	-24.8441	12.6562	10.2299	-7.59375	184.139	-75.9375	0	0	-13.1528
40.5	70.1481	12.6562	-21.9213	-7.59375	30.6898	-75.9375	447.194	0	-52.611	-7.59375
-24.8441	-12.6563	-11.6913	-40.5	10.2299	7.59375	0	0	184.139	75.9375	-184.139
-12.6563	-21.9213	-40.5	70.1481	7.59375	30.6898	0	-52.611	75.9375	447.194	-136.688
18.9984	12.6563	84.7622	15.1875	-11.6913	-5.0625	-13.1528	-7.59375	-184.139	-136.688	315.666
12.6563	21.9213	15.1875	-26.3055	-5.0625	-35.074	-7.59375	65.7638	-136.688	-341.972	151.875
32.1512	5.0625	-33.6126	7.59375	14.6142	55.6875	-65.7638	22.7812	52.611	30.375	-39.4583
5.0625	8.76851	7.59375	-13.1528	55.6875	43.8425	22.7812	-39.4583	30.375	0	-22.7813
-33.6126	-7.59375	32.1512	-5.0625	14.6142	-55.6875	52.611	-30.375	-65.7638	-22.7813	39.4583
-7.59375	-13.1528	-5.0625	8.76851	-55.6875	43.8425	-30.375	0	-22.7813	-39.4583	37.9688
84.7622	-15.1875	18.9984	-12.6563	-11.6913	5.0625	-184.139	136.687	-13.1528	7.59375	-26.3055
-15.1875	-26.3055	-12.6563	21.9213	5.0625	-35.074	136.687	-341.972	7.59375	65.7638	0
-131.528	-45.5625	-131.528	45.5625	-52.611	0	52.611	-91.125	52.611	91.125	-184.139
-45.5625	-78.9166	45.5625	-78.9166	0	-157.833	-91.125	-157.833	91.125	-157.833	-45.5625

Valores proprios matriz Ke={1145.15, 1145.15, 710.249, 710.249, 693.368, 394.583, 181.818, 181.818, 145.486, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

NO tenemos la suficiencia de rango para p=3

Gauss integration rule: 6

Ke=

132.502	45.9	-21.8238	0	-5.45596	-9.45	-166.017	4.05	86.5159	4.05	-11.6913
45.9	79.5011	0	0	-9.45	-16.3679	4.05	7.01481	4.05	7.01481	-4.05
-21.8238	0	132.502	-45.9	-5.45596	9.45	86.5159	-4.05	-166.017	-4.05	-32.7358
0	0	-45.9	79.5011	9.45	-16.3679	-4.05	7.01481	-4.05	7.01481	72.9
-5.45596	-9.45	-5.45596	9.45	53.0008	0	-4.67654	0	-4.67654	0	23.3827
-9.45	-16.3679	9.45	-16.3679	0	159.002	0	-14.0296	0	-14.0296	-32.4
-166.017	4.05	86.5159	-4.05	-4.67654	0	631.333	0	-378.8	0	21.0444
4.05	7.01481	-4.05	7.01481	0	-14.0296	0	631.333	0	-126.267	36.45
86.5159	4.05	-166.017	-4.05	-4.67654	0	-378.8	0	631.333	0	-105.222
4.05	7.01481	-4.05	7.01481	0	-14.0296	0	-126.267	0	631.333	-182.25
-11.6913	-4.05	-32.7358	72.9	23.3827	-32.4	21.0444	36.45	-105.222	-182.25	631.333
-4.05	-7.01481	72.9	-126.267	-32.4	70.1481	36.45	63.1333	-182.25	-315.666	0
-11.6913	-4.05	30.3975	-36.45	-39.7506	76.95	21.0444	36.45	21.0444	36.45	-189.4
-4.05	-7.01481	-36.45	63.1333	76.95	-119.252	36.45	63.1333	36.45	63.1333	109.35
30.3975	36.45	-11.6913	4.05	-39.7506	-76.95	21.0444	-36.45	21.0444	-36.45	84.1777
36.45	63.1333	4.05	-7.01481	-76.95	-119.252	-36.45	63.1333	-36.45	63.1333	0
-32.7358	-72.9	-11.6913	4.05	23.3827	32.4	-105.222	182.25	21.0444	-36.45	84.1777
-72.9	-126.267	4.05	-7.01481	32.4	70.1481	182.25	-315.666	-36.45	63.1333	0
0	0	0	0	0	0	-126.267	-218.7	-126.267	218.7	-505.066
0	0	0	0	0	0	-218.7	-378.8	218.7	-378.8	0

Valores propios matriz Ke={2125.27, 2125.27, 1301.53, 1211.27, 1211.27, 559.68, 553.525, 553.525, 531.307, 297.77, 297.77, 116.014, 116.014, 100.025, 98.2603, 21.9542, 21.9542, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=6

Gauss integration rule: -6

Ke=

53.5853	18.5625	9.74279	0	2.4357	4.21875	-23.9673	31.3875	-16.0756	-9.61875	-3.79969
18.5625	32.1512	0	0	4.21875	7.30709	31.3875	54.3647	-9.61875	-16.6602	9.61875
9.74279	0	53.5853	-18.5625	2.4357	-4.21875	-16.0756	9.61875	-23.9673	-31.3875	61.9641
0	0	-18.5625	32.1512	-4.21875	7.30709	9.61875	-16.6602	-31.3875	54.3647	18.225
2.4357	4.21875	2.4357	-4.21875	21.4341	0	3.21512	-13.6688	3.21512	13.6688	-8.18394
4.21875	7.30709	-4.21875	7.30709	0	64.3024	-13.6688	9.64536	13.6688	9.64536	-5.0625
-23.9673	31.3875	-16.0756	9.61875	3.21512	-13.6688	268.316	-27.3375	-63.1333	0	13.1528
31.3875	54.3647	9.61875	-16.6602	-13.6688	9.64536	-27.3375	489.283	0	-31.5666	-4.55625
-16.0756	-9.61875	-23.9673	-31.3875	3.21512	13.6688	-63.1333	0	268.316	27.3375	-168.355
-9.61875	-16.6602	-31.3875	54.3647	13.6688	9.64536	0	-31.5666	27.3375	489.283	-127.575
-3.79969	9.61875	61.9641	18.225	-8.18394	-5.0625	13.1528	-4.55625	-168.355	-127.575	410.366
9.61875	16.6602	18.225	-31.5666	-5.0625	-24.5518	-4.55625	39.4583	-127.575	-315.666	109.35
19.8753	-4.05	-24.8441	4.55625	7.59937	49.6125	-18.4139	50.1187	36.8277	9.1125	-39.4583
-4.05	-7.01481	4.55625	-7.89166	49.6125	22.7981	50.1187	39.4583	9.1125	15.7833	13.6688
-24.8441	-4.55625	19.8753	4.05	7.59937	-49.6125	36.8277	-9.1125	-18.4139	-50.1188	28.9361
-4.55625	-7.89166	4.05	-7.01481	-49.6125	22.7981	-9.1125	15.7833	-50.1188	39.4583	13.6688
61.9641	-18.225	-3.79969	-9.61875	-8.18394	5.0625	-168.355	127.575	13.1528	4.55625	68.3944
-18.225	-31.5666	-9.61875	16.6602	5.0625	-24.5518	127.575	-315.666	4.55625	39.4583	0
-78.9166	-27.3375	-78.9166	27.3375	-31.5666	0	-31.5666	-164.025	-31.5666	164.025	-363.016
-27.3375	-47.3499	27.3375	-47.3499	0	-94.6999	-164.025	-284.1	164.025	-284.1	-27.3375

Valores proprios matriz Ke={1584.68, 1584.68, 812.197, 812.197, 737.476, 479.267, 239.562, 239.562, 196.826, 85.7875, 85.7875, 72.5921, 72.5921, 41.5823, 30.8193, 0, 0, 0, 0, 0}

NO tenemos la suficiencia de rango para p=-6

Gauss integration rule: 7

Ke=

132.502	45.9	-21.8238	0	-5.45596	-9.45	-166.017	4.05	86.5159	4.05	-11.6913
45.9	79.5011	0	0	-9.45	-16.3679	4.05	7.01481	4.05	7.01481	-4.05
-21.8238	0	132.502	-45.9	-5.45596	9.45	86.5159	-4.05	-166.017	-4.05	-32.7358
0	0	-45.9	79.5011	9.45	-16.3679	-4.05	7.01481	-4.05	7.01481	72.9
-5.45596	-9.45	-5.45596	9.45	53.0008	0	-4.67654	0	-4.67654	0	23.3827
-9.45	-16.3679	9.45	-16.3679	0	159.002	0	-14.0296	0	-14.0296	-32.4
-166.017	4.05	86.5159	-4.05	-4.67654	0	631.333	0	-378.8	0	21.0444
4.05	7.01481	-4.05	7.01481	0	-14.0296	0	631.333	0	-126.267	36.45
86.5159	4.05	-166.017	-4.05	-4.67654	0	-378.8	0	631.333	0	-105.222
4.05	7.01481	-4.05	7.01481	0	-14.0296	0	-126.267	0	631.333	-182.25
-11.6913	-4.05	-32.7358	72.9	23.3827	-32.4	21.0444	36.45	-105.222	-182.25	631.333
-4.05	-7.01481	72.9	-126.267	-32.4	70.1481	36.45	63.1333	-182.25	-315.666	0
-11.6913	-4.05	30.3975	-36.45	-39.7506	76.95	21.0444	36.45	21.0444	36.45	-189.4
-4.05	-7.01481	-36.45	63.1333	76.95	-119.252	36.45	63.1333	36.45	63.1333	109.35
30.3975	36.45	-11.6913	4.05	-39.7506	-76.95	21.0444	-36.45	21.0444	-36.45	84.1777
36.45	63.1333	4.05	-7.01481	-76.95	-119.252	-36.45	63.1333	-36.45	63.1333	0
-32.7358	-72.9	-11.6913	4.05	23.3827	32.4	-105.222	182.25	21.0444	-36.45	84.1777
-72.9	-126.267	4.05	-7.01481	32.4	70.1481	182.25	-315.666	-36.45	63.1333	0
0	0	0	0	0	0	-126.267	-218.7	-126.267	218.7	-505.066
0	0	0	0	0	0	-218.7	-378.8	218.7	-378.8	0

Valores propios matriz Ke={2125.27, 2125.27, 1301.53, 1211.27, 1211.27, 559.68, 553.525, 553.525, 531.307, 297.77, 297.77, 116.014, 116.014, 100.025, 98.2603, 21.9542, 21.9542, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=7

Gauss integration rule: 12

Ke=

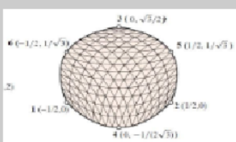
132.502	45.9	-21.8238	0	-5.45596	-9.45	-166.017	4.05	86.5159	4.05	-11.6913
45.9	79.5011	0	0	-9.45	-16.3679	4.05	7.01481	4.05	7.01481	-4.05
-21.8238	0	132.502	-45.9	-5.45596	9.45	86.5159	-4.05	-166.017	-4.05	-32.7358
0	0	-45.9	79.5011	9.45	-16.3679	-4.05	7.01481	-4.05	7.01481	72.9
-5.45596	-9.45	-5.45596	9.45	53.0008	0	-4.67654	0	-4.67654	0	23.3827
-9.45	-16.3679	9.45	-16.3679	0	159.002	0	-14.0296	0	-14.0296	-32.4
-166.017	4.05	86.5159	-4.05	-4.67654	0	631.333	0	-378.8	0	21.0444
4.05	7.01481	-4.05	7.01481	0	-14.0296	0	631.333	0	-126.267	36.45
86.5159	4.05	-166.017	-4.05	-4.67654	0	-378.8	0	631.333	0	-105.222
4.05	7.01481	-4.05	7.01481	0	-14.0296	0	-126.267	0	631.333	-182.25
-11.6913	-4.05	-32.7358	72.9	23.3827	-32.4	21.0444	36.45	-105.222	-182.25	631.333
-4.05	-7.01481	72.9	-126.267	-32.4	70.1481	36.45	63.1333	-182.25	-315.666	0
-11.6913	-4.05	30.3975	-36.45	-39.7506	76.95	21.0444	36.45	21.0444	36.45	-189.4
-4.05	-7.01481	-36.45	63.1333	76.95	-119.252	36.45	63.1333	36.45	63.1333	109.35
30.3975	36.45	-11.6913	4.05	-39.7506	-76.95	21.0444	-36.45	21.0444	-36.45	84.1777
36.45	63.1333	4.05	-7.01481	-76.95	-119.252	-36.45	63.1333	-36.45	63.1333	0
-32.7358	-72.9	-11.6913	4.05	23.3827	32.4	-105.222	182.25	21.0444	-36.45	84.1777
-72.9	-126.267	4.05	-7.01481	32.4	70.1481	182.25	-315.666	-36.45	63.1333	0
0	0	0	0	0	0	-126.267	-218.7	-126.267	218.7	-505.066
0	0	0	0	0	0	-218.7	-378.8	218.7	-378.8	0

Valores propios matriz Ke={2125.27, 2125.27, 1301.53, 1211.27, 1211.27, 559.68, 553.525, 553.525, 531.307, 297.77, 297.77, 116.014, 116.014, 100.025, 98.2603, 21.9542, 21.9542, 0, 0, 0}

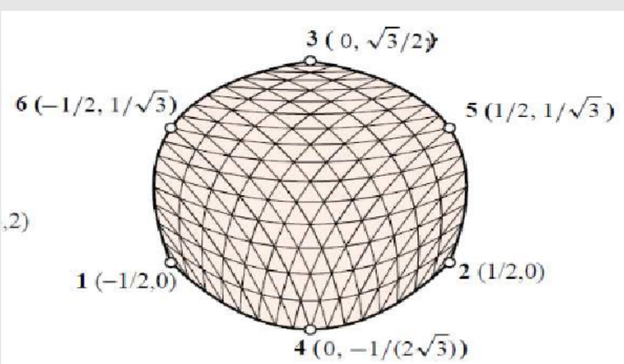
TENEMOS LA SUFICIENCIA DE RANGO PARA p=12

8. TEST DEL ELEMENTO DE LADOS CURVOS.

▪ DEFINICION DE LA GEOMETRIA

LadosRectosTesT =  ;

```
LadosRectosTesTr = Show[LadosRectosTesT, ImageSize → 300]
```



▪ DEFINICION DE LOS NODOS ELEMENTO -

▫ ELEMENTO BASE REAL

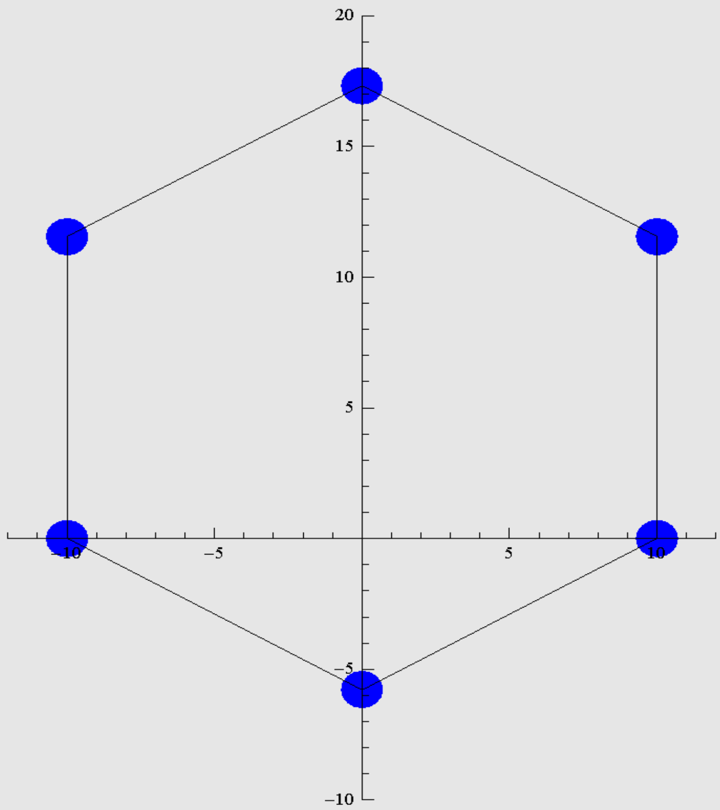
```
Cng = Table[{0, 0}, {i, 6}];
```

```
Cng[[1]] = {-10, 0}; Cng[[2]] = {10, 0}; Cng[[3]] = {0, 10*Sqrt[3]}; Cng[[4]] = {0, -10*Sqrt[3]*1/3};
Cng[[5]] = {10, 10*Sqrt[3]*2/3}; Cng[[6]] = {-10, 10*Sqrt[3]*2/3};
```

```
ptsexteriores = {Cng[[1]], Cng[[4]], Cnc[[2]], Cng[[5]], Cng[[3]], Cng[[6]]};
```

```
ptsinteriores = {};
```

```
Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
  Axes → True, PlotRange → {{-12, 12}, {-10, 20}}, NodeColor → RGBColor[0, 0, 1]]
```



- **TRANSFORMACION DE COORDENADAS TRIANGULARES A CARTESIANAS**

```
Nf6 = {ξ1 * (2 * ξ1 - 1), ξ2 * (2 * ξ2 - 1), ξ3 * (2 * ξ3 - 1), 4 * ξ1 * ξ2, 4 * ξ2 * ξ3, 4 * ξ3 * ξ1};
```

```
TtC[ξ1_, ξ2_, ξ3_] =
  (      1               1               1               1               1               1
   Cng[[1]][[1]]  Cng[[2]][[1]]  Cng[[3]][[1]]  Cng[[4]][[1]]  Cng[[5]][[1]]  Cng[[6]][[1]]
   Cng[[1]][[2]]  Cng[[2]][[2]]  Cng[[3]][[2]]  Cng[[4]][[2]]  Cng[[5]][[2]]  Cng[[6]][[2]] ).Nf6;
```

- **COORDENADAS TRIANGULARES DEL ELEMENTO CONSIDERADO**

```
Cnt
```

$$\left\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}, \left\{\frac{2}{3}, \frac{1}{3}, 0\right\}, \left\{\frac{1}{3}, \frac{2}{3}, 0\right\}, \left\{0, \frac{2}{3}, \frac{1}{3}\right\}, \left\{0, \frac{1}{3}, \frac{2}{3}\right\}, \left\{\frac{1}{3}, 0, \frac{2}{3}\right\}, \left\{\frac{2}{3}, 0, \frac{1}{3}\right\}, \left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\}\right\}$$

- **COORDENADAS CARTESIANAS NODOS ELEMENTO REAL CONSIDERADO Y COMPROBACION GRAFICA - #**

```
Cnc = Table[{0, 0}, {i, NNodos}];
```

```
Do[
  Cnc[[i]] = {TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[2]],
    TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[3]]},
  {i, NNodos}
 ];
```

```
Cnc
```

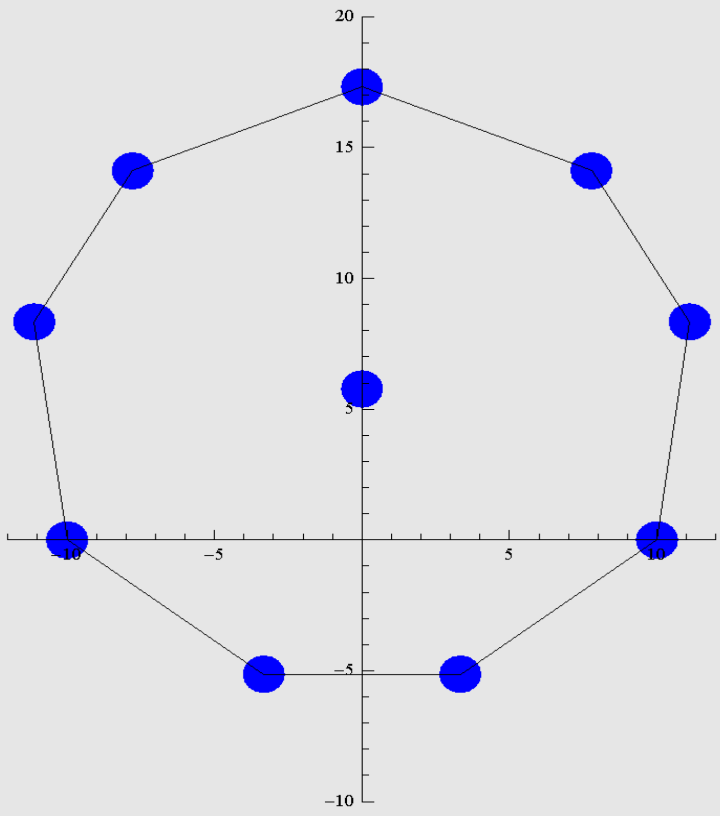
$$\left\{\{-10, 0\}, \{10, 0\}, \left\{0, 10\sqrt{3}\right\}, \left\{-\frac{10}{3}, -\frac{80}{9\sqrt{3}}\right\}, \left\{\frac{10}{3}, -\frac{80}{9\sqrt{3}}\right\}, \left\{\frac{100}{9}, \frac{130}{9\sqrt{3}}\right\}, \left\{\frac{70}{9}, \frac{220}{9\sqrt{3}}\right\}, \left\{-\frac{70}{9}, \frac{220}{9\sqrt{3}}\right\}, \left\{-\frac{100}{9}, \frac{130}{9\sqrt{3}}\right\}, \left\{0, \frac{10}{\sqrt{3}}\right\}\right\}$$

```
ncoor = Cnc;
```

```
ptsexteriores =
  {Cnc[[1]], Cnc[[4]], Cnc[[5]], Cnc[[2]], Cnc[[6]], Cnc[[7]], Cnc[[3]], Cnc[[8]], Cnc[[9]]};
```

```
ptsinteriores = {Cnc[[10]]};
```

```
Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
  Axes → True, PlotRange → {{-12, 12}, {-10, 20}}, NodeColor → RGBColor[0, 0, 1]]
```



- **DEFINICION DEL MATERIAL**

```
ClearAll[Em, nu, h];
```

```
h = 1; Em = 7 * 72; nu = 0; h = 1;
```

```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

▪ VERIFICACION DE LA MATRIZ DE RIGIDEZ -

▫ NUMERO DE PUNTOS DE GAUSS MINIMO PARA CONSEGUIR SUFICIENCIA DE RANGO -

```
NF = NNodos * 2.;
```

```
NG = (NF - 3)/3
```

```
5.66667
```

Se necesitan como mínimo 6 Puntos -- Regla 6 minima

▫ BUCLE GENERICO A COMPLETAR -

BUCLE GENERICO: XX = No. Nodos, ZZ = GRADOS DE LIBERTAD - 3

```
(*For [i=1, i≤7,i++,p={1,-3,3,6,-6,7,12}[[i]];
  Ke=TrigXXIsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
  Print["Gauss integration rule: ",p];
  Print["Ke=",Chop[Simplify[Ke]]//MatrixForm];
  Valores=Chop[Eigenvalues[N[Ke]]];
  If[Valores[[ZZ]]!=0,Print["Valores propios matriz Ke=",Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=",p];Break[],Print[
    "Valores proprios matriz Ke=",Valores];Print["NO tenemos la suficiencia de rango para p=",p]]
 ];*)
```

▫ DESARROLLO DE LA MATRIZ DE RIGIDEZ -

```
For [i = 1, i ≤ 7, i++, p = {1, -3, 3, 6, -6, 7, 12}[[i]];
  Ke = Trig10IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
  Print["Gauss integration rule: ", p];
  Print["Ke=", Chop[Simplify[Ke]] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]]];
  If[Valores[[17]] != 0, Print["Valores propios matriz Ke=", Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=", p] (*;Break[]*), Print[
    "Valores proprios matriz Ke=", Valores]; Print["NO tenemos la suficiencia de rango para p=", p]]
 ];
```

Gauss integration rule: 1

Ke=

63.6529	15.75	-45.4663	-15.75	-18.1865	0	-190.959	-47.25	136.399	47.25	136.399
15.75	45.4663	15.75	-9.09327	-31.5	-36.3731	-47.25	-136.399	-47.25	27.2798	-47.25
-45.4663	15.75	63.6529	-15.75	-18.1865	0	136.399	-47.25	-190.959	47.25	-190.959
-15.75	-9.09327	-15.75	45.4663	31.5	-36.3731	47.25	27.2798	47.25	-136.399	47.25
-18.1865	-31.5	-18.1865	31.5	36.3731	0	54.5596	94.5	54.5596	-94.5	54.5596
0	-36.3731	0	-36.3731	0	72.7461	0	109.119	0	109.119	0
-190.959	-47.25	136.399	47.25	54.5596	0	572.876	141.75	-409.197	-141.75	-409.197
-47.25	-136.399	-47.25	27.2798	94.5	109.119	141.75	409.197	141.75	-81.8394	141.75
136.399	-47.25	-190.959	47.25	54.5596	0	-409.197	141.75	572.876	-141.75	572.876
47.25	27.2798	47.25	-136.399	-94.5	109.119	-141.75	-81.8394	-141.75	409.197	-141.75
136.399	-47.25	-190.959	47.25	54.5596	0	-409.197	141.75	572.876	-141.75	572.876
47.25	27.2798	47.25	-136.399	-94.5	109.119	-141.75	-81.8394	-141.75	409.197	-141.75
54.5596	94.5	54.5596	-94.5	-109.119	0	-163.679	-283.5	-163.679	283.5	-163.679
0	109.119	0	109.119	0	-218.238	0	-327.358	0	-327.358	0
54.5596	94.5	54.5596	-94.5	-109.119	0	-163.679	-283.5	-163.679	283.5	-163.679
0	109.119	0	109.119	0	-218.238	0	-327.358	0	-327.358	0
-190.959	-47.25	136.399	47.25	54.5596	0	572.876	141.75	-409.197	-141.75	-409.197
-47.25	-136.399	-47.25	27.2798	94.5	109.119	141.75	409.197	141.75	-81.8394	141.75
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

Valores proprios matriz Ke={2073.26, 2073.26, 2073.26, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

NO tenemos la suficiencia de rango para p=1

Gauss integration rule: -3

Ke=

41.1903	10.0313	6.52225	4.92187	-0.216506	-1.03125	-151.987	-25.3125	98.6457	11.3906	-172
10.0313	29.6072	-4.92188	-2.46276	8.8125	4.276	-37.125	-102.299	-0.421875	59.1874	91.
6.52225	-4.92188	41.1903	-10.0313	-0.216506	1.03125	98.6457	-11.3906	-151.987	25.3125	-141
4.92187	-2.46276	-10.0313	29.6072	-8.8125	4.276	0.421875	59.1874	37.125	-102.299	31.
-0.216506	8.8125	-0.216506	-8.8125	23.8157	0	-58.4567	-156.937	-58.4567	156.937	64.
-1.03125	4.276	1.03125	4.276	0	46.9819	2.53125	-121.298	-2.53125	-121.298	-8.
-151.987	-37.125	98.6457	0.421875	-58.4567	2.53125	2328.04	432.844	-1953.18	-398.672	210
-25.3125	-102.299	-11.3906	59.1874	-156.937	-121.298	432.844	1459.96	398.672	-769.437	-318
98.6457	-0.421875	-151.987	37.125	-58.4567	-2.53125	-1953.18	398.672	2328.04	-432.844	-65.
11.3906	59.1874	25.3125	-102.299	156.937	-121.298	-398.672	-769.437	-432.844	1459.96	205
-172.447	91.125	-141.758	31.2187	64.3024	-8.4375	210.444	-318.937	-65.7638	205.031	205
-68.3437	-7.30709	43.0313	-112.529	-20.25	93.5307	159.469	-170.986	-113.906	-13.1528	-592
-38.7276	-145.547	73.8016	-25.7344	-87.6851	11.8125	269.632	603.703	-6.57638	-444.234	-106
13.9219	-141.027	-13.9219	84.0315	0	-166.602	-34.1719	598.451	34.1719	46.0347	113
73.8016	25.7344	-38.7276	145.547	-87.6851	-11.8125	-6.57638	444.234	269.632	-603.703	-14
13.9219	84.0315	-13.9219	-141.027	0	-166.602	-34.1719	46.0347	34.1719	598.451	113
-141.758	-31.2187	-172.447	-91.125	64.3024	8.4375	-65.7638	-205.031	210.444	318.937	762
-43.0312	-112.529	68.3438	-7.30709	20.25	93.5307	113.906	-13.1528	-159.469	-170.986	-318
284.976	83.5313	284.976	-83.5313	140.296	0	-670.791	-1161.84	-670.791	1161.84	-149
83.5313	188.523	-83.5313	188.523	0	333.203	-205.031	-986.457	205.031	-986.457	683

**Valores proprios matriz Ke={6355.33, 6355.33, 4674.29,
4195.57, 4142.38, 4142.38, 614.907, 614.907, 368.25, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}**

NO tenemos la suficiencia de rango para p=-3

Gauss integration rule: 3

Ke=

200.323	49.4063	29.2554	4.92188	17.6453	1.78125	88.3346	3.9375	-71.2035	-17.2969	49.3634
49.4063	143.273	-4.92188	13.7752	11.625	25.3854	133.875	160.106	-29.1094	-51.5556	24.1875
29.2554	-4.92188	200.323	-49.4063	17.6453	-1.78125	-71.2035	17.2969	88.3346	-3.9375	201.838
4.92188	13.7752	-49.4063	143.273	-11.625	25.3854	29.1094	-51.5556	-133.875	160.106	61.5938
17.6453	11.625	17.6453	-11.625	114.748	0	28.9036	-12.375	28.9036	12.375	-36.373
1.78125	25.3854	-1.78125	25.3854	0	228.847	5.34375	60.2429	-5.34375	60.2429	2.8125
88.3346	133.875	-71.2035	29.1094	28.9036	5.34375	241.134	4.21875	-70.8788	-44.2969	-49.688
3.9375	160.106	17.2969	-51.5556	-12.375	60.2429	4.21875	436.964	44.2969	-114.721	-13.5
-71.2035	-29.1094	88.3346	-133.875	28.9036	-5.34375	-70.8788	44.2969	241.134	-4.21875	-83.300
-17.2969	-51.5556	-3.9375	160.106	12.375	60.2429	-44.2969	-114.721	-4.21875	436.964	-7.5937
49.3634	24.1875	201.838	61.5938	-36.3731	2.8125	-49.6882	-13.5	-83.3008	-7.59375	391.66
6.46875	39.783	-68.3437	46.603	-9.	-86.386	39.6563	30.6898	-184.781	-194.369	82.6875
55.4527	2.95312	-76.5621	14.2031	82.4889	0.5625	9.49922	-18.9844	-0.730709	14.7656	-103.76
20.6719	33.6938	26.0156	-46.197	130.5	165.952	51.8906	28.4976	67.9219	-18.2677	-63.281
-76.5621	-14.2031	55.4527	-2.95312	82.4889	-0.5625	-0.730709	-14.7656	9.49922	18.9844	21.921
-26.0156	-46.197	-20.6719	33.6938	-130.5	165.952	-67.9219	-18.2677	-51.8906	28.4976	54.843
201.838	-61.5938	49.3634	-24.1875	-36.3731	-2.8125	-83.3008	7.59375	-49.6882	13.5	37.996
68.3437	46.603	-6.46875	39.783	9.	-86.386	184.781	-194.369	-39.6563	30.6898	35.4375
-494.446	-112.219	-494.446	112.219	-300.078	0	-92.0693	-17.7188	-92.0693	17.7188	-429.65
-112.219	-364.867	112.219	-364.867	0	-559.236	-336.656	-337.588	336.656	-337.588	-177.18

Valores propios matriz Ke={3087.38, 3087.38, 775.937, 766.108,
732.426, 732.426, 359.848, 359.848, 353.901, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

NO tenemos la suficiencia de rango para p=3

Gauss integration rule: 6

Ke=

767.026	188.726	-94.5897	-11.025	-61.1236	-8.29666	-492.37	-136.819	270.733	60.6446	65.5106
188.726	549.104	11.025	-49.9682	-30.3467	-83.4343	42.731	-208.075	-14.9554	124.257	11.4068
-94.5897	11.025	767.026	-188.726	-61.1236	8.29666	270.733	-60.6446	-492.37	136.819	-319.89
-11.025	-49.9682	-188.726	549.104	30.3467	-83.4343	14.9554	124.257	-42.731	-208.075	236.4
-61.1236	-30.3467	-61.1236	30.3467	440.143	0	45.0144	0.42668	45.0144	-0.42668	141.092
-8.29666	-83.4343	8.29666	-83.4343	0	875.987	0.42668	71.85	-0.42668	71.85	-14.2034
-492.37	42.731	270.733	14.9554	45.0144	0.42668	1084.68	117.121	-800.369	-127.575	-223.866
-136.819	-208.075	-60.6446	124.257	0.42668	71.85	117.121	742.009	127.575	-333.863	18.5297
270.733	-14.9554	-492.37	-42.731	45.0144	-0.42668	-800.369	127.575	1084.68	-117.121	314.915
60.6446	124.257	136.819	-208.075	-0.42668	71.85	-127.575	-333.863	-117.121	742.009	-136.805
65.5106	11.4068	-319.89	236.4	141.092	-14.2034	-223.866	18.5297	314.915	-136.805	929.107
11.4068	51.3538	56.8503	-380.555	-89.8034	253.898	18.5297	-142.88	-136.805	156.946	-206.941
64.7716	11.8335	180.66	-112.648	-238.407	9.80628	80.5482	22.2133	-179.174	44.3329	-450.489
11.8335	52.0928	-37.048	214.33	189.356	-462.038	22.2133	106.198	44.3329	-187.572	74.428
180.66	112.648	64.7716	-11.8335	-238.407	-9.80628	-179.174	-44.3329	80.5482	-22.2133	-147.079
37.048	214.33	-11.8335	52.0928	-189.356	-462.038	-44.3329	-187.572	-22.2133	106.198	25.8032
-319.89	-236.4	65.5106	-11.4068	141.092	14.2034	314.915	136.805	-223.866	-18.5297	119.023
-56.8503	-380.555	-11.4068	51.3538	89.8034	253.898	136.805	156.946	-18.5297	-142.88	0
-380.728	-96.6682	-380.728	96.6682	-213.294	0	-100.111	-180.875	-100.111	180.875	-428.323
-96.6682	-269.105	96.6682	-269.105	0	-436.539	-180.875	-328.871	180.875	-328.871	-8.61849

Valores propios matriz Ke={2889.45, 2787.76, 2787.76, 2252.23, 2252.23, 1310.5, 906.282, 906.282,
669.171, 600.489, 363.985, 363.985, 339.745, 297.773, 297.773, 203.237, 203.237, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=6

Gauss integration rule: -6

Ke=

184.409	45.4688	26.9821	4.92188	15.8591	1.5	64.3024	1.0125	-54.2186	-14.4281	27.1824
45.4688	131.906	-4.92188	12.1514	11.3438	23.2744	116.775	133.866	-26.2406	-40.4813	30.8813
26.9821	-4.92188	184.409	-45.4688	15.8591	-1.5	-54.2186	14.4281	64.3024	-1.0125	167.478
4.92187	12.1514	-45.4688	131.906	-11.3438	23.2744	26.2406	-40.4813	-116.775	133.866	58.5563
15.8591	11.3438	15.8591	-11.3438	105.655	0	20.1676	-26.8312	20.1676	26.8312	-26.3055
1.5	23.2744	-1.5	23.2744	0	210.661	5.0625	42.0888	-5.0625	42.0888	1.6875
64.3024	116.775	-54.2186	26.2406	20.1676	5.0625	449.824	47.0813	-259.109	-79.7344	-23.675
1.0125	133.866	14.4281	-40.4813	-26.8312	42.0888	47.0813	539.263	79.7344	-180.193	-44.0437
-54.2186	-26.2406	64.3024	-116.775	20.1676	-5.0625	-259.109	79.7344	449.824	-47.0813	-81.5471
-14.4281	-40.4813	-1.0125	133.866	26.8312	42.0888	-79.7344	-180.193	-47.0813	539.263	13.6688
27.1824	30.8813	167.478	58.5563	-26.3055	1.6875	-23.675	-44.0437	-81.5471	13.6688	557.677
-1.0125	35.074	-57.2062	30.6898	-10.125	-68.3944	51.6375	10.5222	-177.694	-176.247	15.1875
46.0347	-11.8969	-61.5257	10.2094	65.4715	1.6875	35.5125	43.2844	-1.31528	-31.1344	-199.922
19.9969	16.2217	22.0219	-33.1742	117.45	132.697	43.2844	85.4929	64.5469	-11.8375	-45.5625
-61.5257	-10.2094	46.0347	11.8969	65.4715	-1.6875	-1.31528	31.1344	35.5125	-43.2844	5.2611
-22.0219	-33.1742	-19.9969	16.2217	-117.45	132.697	-64.5469	-11.8375	-43.2844	85.4929	60.75
167.478	-58.5563	27.1824	-30.8812	-26.3055	-1.6875	-81.5471	-13.6688	-23.675	44.0437	110.483
57.2062	30.6898	1.0125	35.074	10.125	-68.3944	177.694	-176.247	-51.6375	10.5222	0
-416.504	-92.6438	-416.504	92.6438	-256.04	0	-149.941	-132.131	-149.941	132.131	-536.633
-92.6438	-309.528	92.6438	-309.528	0	-469.992	-323.494	-402.474	323.494	-402.474	-91.125

Valores propios matriz Ke={3257.32, 3257.32, 1057.9, 1012.23, 823.998, 823.998, 436.671, 436.671, 360.245, 248.886, 248.886, 102.965, 98.8426, 98.8426, 96.8205, 7.23316, 7.23316, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=-6

Gauss integration rule: 7

Ke=

681.728	168.053	-78.2378	-11.025	-48.6504	-6.05735	-402.495	-117.735	217.545	52.8955	66.1232
168.053	487.677	11.025	-38.7879	-28.1073	-68.3753	61.8153	-146.482	-22.7045	87.6848	14.2637
-78.2378	11.025	681.728	-168.053	-48.6504	6.05735	217.545	-52.8955	-402.495	117.735	-234.699
-11.025	-38.7879	-168.053	487.677	28.1073	-68.3753	22.7045	87.6848	-61.8153	-146.482	214.612
-48.6504	-28.1073	-48.6504	28.1073	390.651	0	44.0749	-1.53414	44.0749	1.53414	107.077
-6.05735	-68.3753	6.05735	-68.3753	0	778.753	-1.53414	75.2441	1.53414	75.2441	-10.8834
-402.495	61.8153	217.545	22.7045	44.0749	-1.53414	1007.56	103.121	-743.815	-127.575	-157.83
-117.735	-146.482	-52.8955	87.6848	-1.53414	75.2441	103.121	693.418	127.575	-299.858	7.00515
217.545	-22.7045	-402.495	-61.8153	44.0749	1.53414	-743.815	127.575	1007.56	-103.121	206.567
52.8955	87.6848	117.735	-146.482	1.53414	75.2441	-127.575	-299.858	-103.121	693.418	-113.944
66.1232	14.2637	-234.699	214.612	107.077	-10.8834	-157.83	7.00515	206.567	-113.944	861.26
14.2637	53.1958	35.0619	-314.278	-86.4834	198.153	7.00515	-88.0005	-113.944	74.9958	-187.589
68.7804	12.7296	133.223	-101.579	-186.271	7.10216	25.728	25.1847	-111.525	33.7398	-410.847
12.7296	50.5386	-25.9789	172.007	186.652	-362.705	25.1847	54.8088	33.7398	-134.306	64.664
133.223	101.579	68.7804	-12.7296	-186.271	-7.10216	-111.525	-33.7398	25.728	-25.1847	-99.3913
25.9789	172.007	-12.7296	50.5386	-186.652	-362.705	-33.7398	-134.306	-25.1847	54.8088	26.7346
-234.699	-214.612	66.1232	-14.2637	107.077	10.8834	206.567	113.944	-157.83	-7.00515	69.3492
-35.0619	-314.278	-14.2637	53.1958	86.4834	198.153	113.944	74.9958	-7.00515	-88.0005	0
-403.317	-104.042	-403.317	104.042	-223.112	0	-85.8122	-170.926	-85.8122	170.926	-407.608
-104.042	-283.18	104.042	-283.18	0	-463.386	-170.926	-317.505	170.926	-317.505	-14.8631

Valores propios matriz Ke={2702.26, 2702.26, 2444.97, 2005.96, 2005.96, 1121., 977.655, 977.655, 669.597, 567.497, 353.869, 353.869, 338.806, 275.339, 275.339, 210.733, 210.733, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=7

Gauss integration rule: 12

Ke=

809.209	197.118	-102.142	-11.025	-69.1701	-8.01108	-528.95	-145.786	288.617	63.7823	66.6
197.118	581.596	11.025	-58.1796	-30.0611	-91.1511	33.7636	-243.226	-11.8177	145.635	10.5
-102.142	11.025	809.209	-197.118	-69.1701	8.01108	288.617	-63.7823	-528.95	145.786	-363.
-11.025	-58.1796	-197.118	581.596	30.0611	-91.1511	11.8177	145.635	-33.7636	-243.226	241.
-69.1701	-30.0611	-69.1701	30.0611	467.79	0	46.8816	0.823023	46.8816	-0.823023	158.
-8.01108	-91.1511	8.01108	-91.1511	0	923.015	0.823023	72.2472	-0.823023	72.2472	-11.
-528.95	33.7636	288.617	11.8177	46.8816	0.823023	1178.8	127.326	-883.073	-127.575	-189.
-145.786	-243.226	-63.7823	145.635	0.823023	72.2472	127.326	831.811	127.575	-414.363	18.9
288.617	-11.8177	-528.95	-33.7636	46.8816	-0.823023	-883.073	127.575	1178.8	-127.326	298.
63.7823	145.635	145.786	-243.226	-0.823023	72.2472	-127.575	-414.363	-127.326	831.811	-138.
66.6186	10.5721	-363.164	241.503	158.879	-11.122	-189.176	18.9104	298.622	-138.984	1028.
10.5721	52.5102	61.9527	-409.012	-86.722	275.373	18.9104	-113.458	-138.984	138.138	-213.
65.193	11.3952	203.882	-112.704	-266.15	5.94129	43.8827	23.7507	-148.765	42.2421	-531.
11.3952	53.9357	-37.1043	230.37	185.491	-506.026	23.7507	71.3076	42.2421	-153.87	75.3
203.882	112.704	65.193	-11.3952	-266.15	-5.94129	-148.765	-42.2421	43.8827	-23.7507	-116.
37.1043	230.37	-11.3952	53.9357	-185.491	-506.026	-42.2421	-153.87	-23.7507	71.3076	23.3
-363.164	-241.503	66.6186	-10.5721	158.879	11.122	298.622	138.984	-189.176	-18.9104	85.0
-61.9527	-409.012	-10.5721	52.5102	86.722	275.373	138.984	138.138	-18.9104	-113.458	0
-370.094	-93.1971	-370.094	93.1971	-208.672	0	-106.843	-185.558	-106.843	185.558	-438.
-93.1971	-262.479	93.1971	-262.479	0	-423.901	-185.558	-334.222	185.558	-334.222	-5.67

Valores propios matriz Ke={3057.97, 2860.25, 2860.25, 2356.38, 2356.38, 1447.54, 1132.64, 1132.64, 678.157, 615.775, 421.968, 421.968, 339.997, 331.271, 331.271, 217.876, 217.876, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=12