ELEMENTO TRIANGULAR DE TRANSICION - 4 NODOS

Implementación en Mathematica

v.2018

1. DATOS INICIALES

INICIO

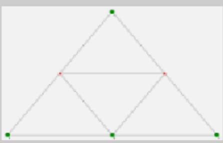
```
Off[General::"spell1"]
Off[General::"spell"]
```

```
SetDirectory[NotebookDirectory[]]
```

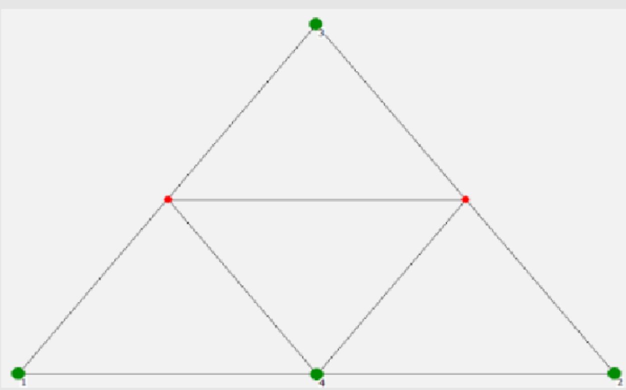
```
C:\#0-Modulos-M30x_MeF-10\#M309-m6-a6a-sws\12-I-triangulo-i
```

DEFINICION ELEMENTO TRIANGULAR DE TRANSICION DE 4 NODOS

DEFINICION GRAFICA

TriT4 =  ;

```
TriT4r = Show[TriT4, ImageSize → 300]
```



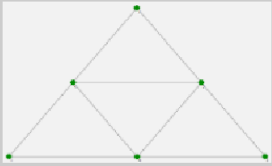
COORDENADAS TRIANGULARES NODOS

$$Cnt = \left\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}, \left\{\frac{1}{2}, \frac{1}{2}, 0\right\}\right\};$$

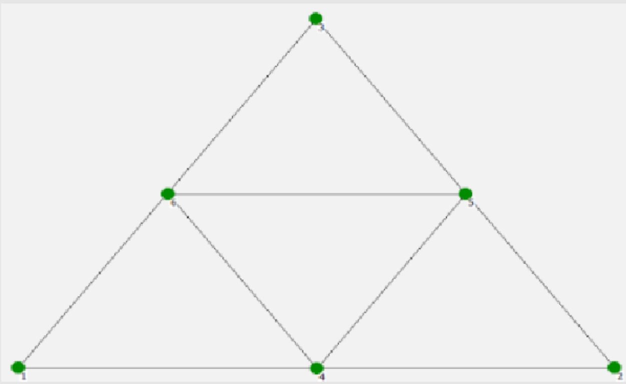
```
NNodos = Dimensions[Cnt][[1]]
```

```
4
```

▪ ELEMENTO COMPLETO NECESARIO - 2 DIVISIONES POR LADO

TriR6 =  ;

```
TriR6r = Show[TriR6, ImageSize → 300]
```



▪ DEFINICION ELEMENTO BASE REAL - COORDENADAS CARTESIANAS

▫ COORDENADAS CARTESIANAS NODOS ELEMENTO BASE REAL

```
NNodosB = 3;
```

```
Cne = Table[{0, 0}, {i, NNodosB}];
```

```
Cne[[1]] = {-10, 0}; Cne[[2]] = {10, 0}; Cne[[3]] = {0, 10*Sqrt[3]};
```

▫ FUNCION TRANSFORMACION DE COORDENADAS DE TRIANGULARES A CARTESIANAS

$$TtC[\xi 1\_, \xi 2\_, \xi 3\_] = \begin{pmatrix} 1 & 1 & 1 \\ Cne[[1]][[1]] & Cne[[2]][[1]] & Cne[[3]][[1]] \\ Cne[[1]][[2]] & Cne[[2]][[2]] & Cne[[3]][[2]] \end{pmatrix} . \begin{pmatrix} \xi 1 \\ \xi 2 \\ \xi 3 \end{pmatrix};$$

▫ COORDENADAS CARTESIANAS DE LOS NODOS

```
Cnc = Table[{0, 0}, {i, NNodos}];
```

```
Do[
  Cnc[[i]] = {TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[2, 1]],
    TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[3, 1]]},
  {i, NNodos}
 ];
```

IMAGEN DEL ELEMENTO -

FUNCION REPRESENTACION GRAFICA ELEMENTO Y NODOS

```
ElementPlot[b_List, options___] := Module[{asa, color, nr, circles, lines},
    asa = Select[{options}, ((! SameQ[#[[1]], NodeColor]) && (! SameQ[#[[1]], NodeSize])) &];
    {color, nr} = {NodeColor, NodeSize} /. {options} /.
      {NodeColor → GrayLevel[0], NodeSize → PointSize[0.06]};
    circles = Map[Point[#] &, Partition[Flatten[b], 2]];
    lines = Line[Append[b[[1]], First[b[[1]]]]];
    Show[Graphics[{nr, color, circles}], Graphics[lines], Evaluate[Sequence[##] & @@ asa]]];
```

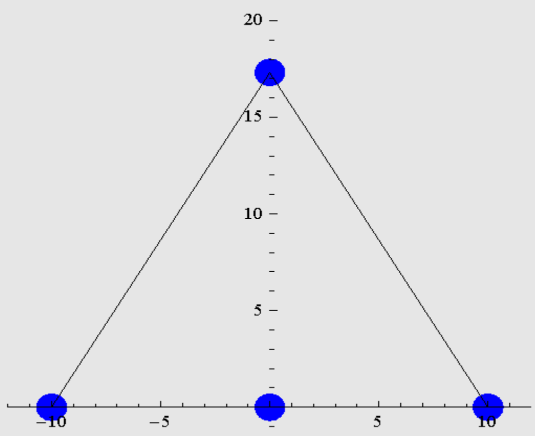
DEFINICION VECTOR DE NODOS-

```
ptsexteriores = {Cnc[[1]], Cnc[[4]], Cnc[[2]], Cnc[[3]]};
```

```
ptsinteriores = {};
```

IMAGEN DE COMPROBACION

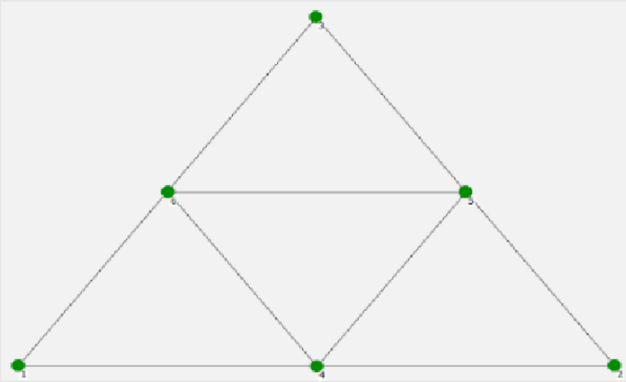
```
Imagen = Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
    Axes → True, PlotRange → {{-12, 12}, {-2, 20}}, ImageSize → 250, NodeColor → RGBColor[0, 0, 1]]
```



3. FUNCIONES DE FORMA - METODO PRODUCTO DE CURVAS -

■ CURVAS A CONSIDERAR - TRIANGULO REGULAR DE 6 NODOS

```
TriR6r
```



```
Cu = Table[0, {i, 9}];
```

▫ LADOS

```
Cu[[1]] = ξ3; Cu[[2]] = ξ1; Cu[[3]] = ξ2;
```

▫ MEDIANAS

```
Cu[[4]] = (ξ1 - 1 / 2);
```

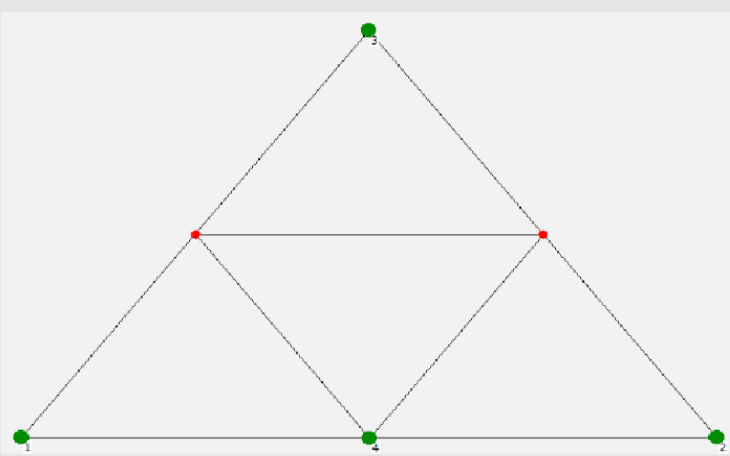
```
Cu[[5]] = (ξ2 - 1 / 2);
```

```
Cu[[6]] = (ξ3 - 1 / 2);
```

■ DEFINICION PRODUCTO DE CURVAS EN CADA NODO - NODOS NO ESQUINA

```
Nc = Table[0, {i, NNodos}];
```

```
Show[TriT4, ImageSize → 350]
```



Tipo 2 - LADOS

```
Nc[[4]] = Cu[[2]] * Cu[[3]];
```

OBTENCION FUNCIONES DE FORMA - NODOS NO ESQUINA

```
Clear[Nf]
```

```
Nfp = Table[0, {i, NNodos}];
```

```
Nf = Table[0, {i, NNodos}];
```

```
Do[
  Nfp[[i]] = a * Nc[[i]];
  eq = 1 == Nfp[[i]] /. {ξ1 -> Cnt[[i, 1]], ξ2 -> Cnt[[i, 2]], ξ3 -> Cnt[[i, 3]]};
  as = a /. Solve[eq, a][[1]]; Print["Nodo ", i];
  Nf[[i]] = Simplify[Nfp[[i]] /. {a → as}],
  {i, 4, NNodos}
 ];
```

Nodo 4

```
MatrixForm[Nf]
```

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 4\,\xi 1\,\xi 2 \end{pmatrix}$$

OBTENCION FUNCIONES DE FORMA - NODOS ESQUINA

Utilizamos las Funciones de Forma del Triangulo de 3 Nodos.

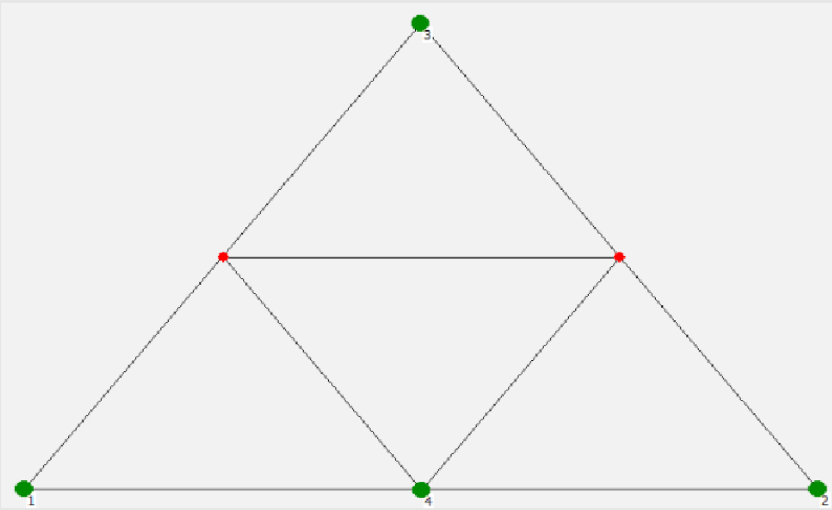
```
Nf3 = {ξ1, ξ2, ξ3};
```

NODO 1

```
Clear[a4]
```

```
Nf[[1]] = Nf3[[1]] + a4 * Nf[[4]];
```

```
Show[TriT4, ImageSize → 400]
```




```
eq = 0 == Nf[[1]] /. {ξ1 -> Cnt[[4]][[1]], ξ2 -> Cnt[[4]][[2]], ξ3 -> Cnt[[4]][[3]]}
a4s = a4 /. Solve[eq, a4][[1]]
```

$0 == \frac{1}{2} + a4$

$-\frac{1}{2}$

```
Nf[[1]] = Simplify[Nf[[1]] /. {a4 → a4s}]
```

ξ1 − 2 ξ1 ξ2

ξ1 − 2 ξ1 ξ2

ξ1 − 2 ξ1 ξ2

ξ1 − 2 ξ1 ξ2

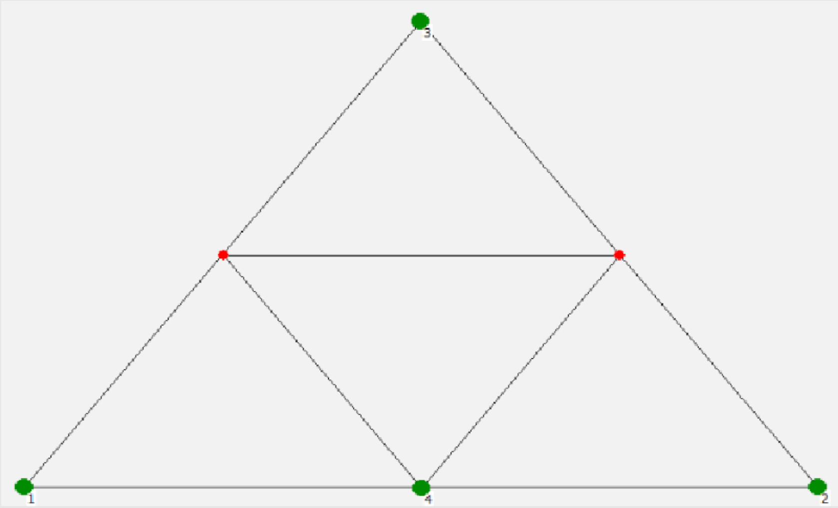
ξ1 − 2 ξ1 ξ2

NODO 2

```
Clear[a4]
```

```
Nf[[2]] = Nf3[[2]] + a4 * Nf[[4]];
```

```
Show[TriT4, ImageSize → 400]
```




```
eq = 0 == Nf[[2]] /. {ξ1 -> Cnt[[4]][[1]], ξ2 -> Cnt[[4]][[2]], ξ3 -> Cnt[[4]][[3]]}
a4s = a4 /. Solve[eq, a4][[1]]
```

$0 == \frac{1}{2} + a4$

$-\frac{1}{2}$

```
Nf[[2]] = Simplify[Nf[[2]] /. {a4 → a4s}]
```

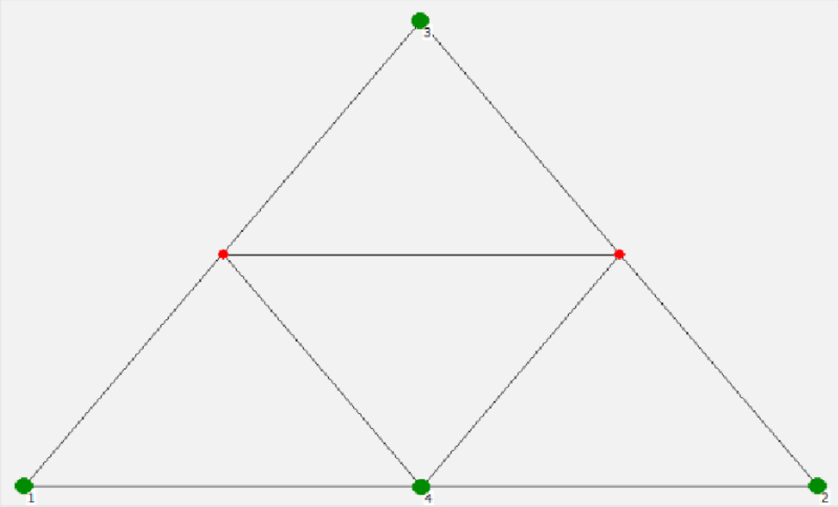
$\xi 2 - 2\,\xi 1\,\xi 2$

NODO 3

```
Clear[a4]
```

```
Nf[[3]] = Nf3[[3]] + a4 * Nf[[4]];
```

```
Show[TriT4, ImageSize → 400]
```



```
eq = 0 == Nf[[3]] /. {ξ1 -> Cnt[[4]][[1]], ξ2 -> Cnt[[4]][[2]], ξ3 -> Cnt[[4]][[3]]}
a4s = a4 /. Solve[eq, a4][[1]]
```

```
0 == a4
```

```
0
```

```
Nf[[3]] = Simplify[Nf[[3]] /. {a4 → a4s}]
```

```
ξ3
```

- FUNCIONES DE FORMA DE TODOS LOS NODOS

```
MatrixForm[Nf]
```

$$\begin{pmatrix} \xi1 - 2\,\xi1\,\xi2 \\ \xi2 - 2\,\xi1\,\xi2 \\ \xi3 \\ 4\,\xi1\,\xi2 \end{pmatrix}$$

- COMPROBACION SUMA UNIDAD

$$\text{Suma} = \sum_{i=1}^{\text{NNodos}} \text{Nf}[[i]]$$

```
ξ1 + ξ2 + ξ3
```

```
Simplify[Suma /. {ξ1 → 1 - ξ2 - ξ3}]
```

```
1
```

OK - SE CUMPLE LA CONDICION DE COMPLETITUD

- Proceso para comprobar Valor Funciones de Forman en Nodos - en caso de Error

```
(*Do[
  Print["NODO ",j];
  Do[
   Print[i,"  ",Simplify[Nf[[j]]/.{ξ1 ->Cnt[[i,1]],ξ2->Cnt[[i,2]],ξ3->Cnt[[i,3]]}]/.{ξ1→1-ξ2-ξ3}],
   {i,NNodos}
     ],
  {j,NNodos}
 ];*)
```

▪ REPRESENTACION GRAFICA

▫ Función Representación Gráfica Funciones de Forma

```
PlotTriangleShapeFunction[xytrig_, f_, Nsub_, aspect_] :=
  Module[{Ni, line3D = {}, poly3D = {}, zc1, zc2, zc3, xyf1, xyf2, xyf3, xc, yc, x1, x2, x3, y1, y2,
    y3, z1, z2, z3, iz1, iz2, iz3, d}, {{x1, y1, z1}, {x2, y2, z2}, {x3, y3, z3}} = Take[xytrig, 3];
   xc = {x1, x2, x3}; yc = {y1, y2, y3}; Ni = Nsub * 3; Do[Do[iz3 = Ni - iz1 - iz2; If[iz3 ≤ 0, Continue[]]; d = 0;
     If[Mod[iz1 + 2, 3] == 0 && Mod[iz2 - 1, 3] == 0, d = 1]; If[Mod[iz1 - 2, 3] == 0 && Mod[iz2 + 1, 3] == 0, d = -1];
     If[d == 0, Continue[]]; zc1 = N[{iz1 + d + d, iz2 - d, iz3 - d} / Ni]; zc2 = N[{iz1 - d, iz2 + d + d, iz3 - d} / Ni];
     zc3 = N[{iz1 - d, iz2 - d, iz3 + d + d} / Ni]; xyf1 = {xc.zc1, yc.zc1, f[zc1[[1]], zc1[[2]], zc1[[3]]]};
     xyf2 = {xc.zc2, yc.zc2, f[zc2[[1]], zc2[[2]], zc2[[3]]]}; xyf3 =
      {xc.zc3, yc.zc3, f[zc3[[1]], zc3[[2]], zc3[[3]]]}; AppendTo[poly3D, Polygon[{xyf1, xyf2, xyf3}]];
     AppendTo[line3D, Line[{xyf1, xyf2, xyf3, xyf1}]], {iz2, 1, Ni - iz1}], {iz1, 1, Ni}];
   Show[Graphics3D[RGBColor[1, 0, 0]], Graphics3D[poly3D], Graphics3D[Thickness[.002]],
    Graphics3D[line3D], Graphics3D[RGBColor[0, 0, 0]], Graphics3D[Thickness[.005]],
    Graphics3D[Line[xytrig]], PlotRange → All, BoxRatios → {1, 1, aspect}, Boxed → False]];
```

▫ Representación Gráfica Funciones Forma Elemento.

```
Ng = Table[0, {i, NNodos}];
```

```
xyc1 = {0, 0, 0}; xyc2 = {3, 0, 0}; xyc3 = {Sqrt[3], 3 / 2, 0}; xytrig = N[{xyc1, xyc2, xyc3, xyc1}];
```

Control de Cuadricula

```
Nsub = 15;
```

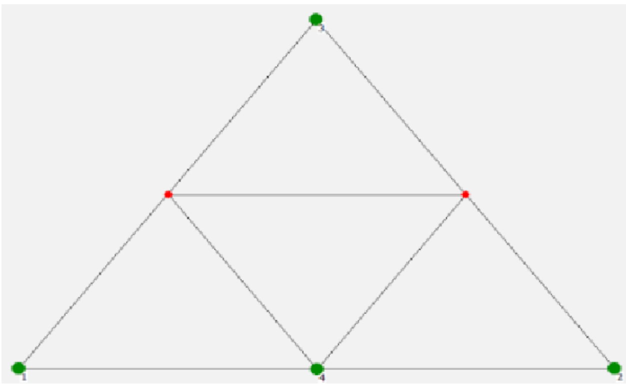
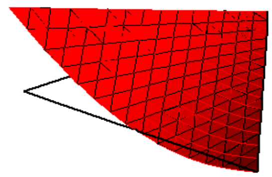
```
Do[
  fi[ξ1_, ξ2_, ξ3_] = Nf[[i]];
  Ng[[i]] = PlotTriangleShapeFunction[xytrig, fi, Nsub, 1 / 2],
  {i, NNodos}
 ];
```

4. RESULTADOS INTERACTIVOS -

```
Manipulate[{TriT4r, Ng[[n]], Nf[[n]]}, {n, 1, Dimensions[Nf][[1]], 1}, {n, Range[Dimensions[Nf][[1]]]},
 FrameLabel→{"FUNCION DE FORMA EN NODO n – TRIANGULO TRANSICION 4 NODOS"}, SaveDefinitions→True]
```

n

n 1 2 3 4

{  ,  , $\xi1 - 2\,\xi1\,\xi2$ }

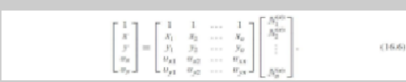
FUNCION DE FORMA EN NODO n – TRIANGULO TRANSICION 4 NODOS

5. DERIVADAS FUNCIONES DE FORMA Y JACOBIANO -

Inicializaciones Necesarias -

```
ClearAll[x1, x2, x3, x4, y1, y2, y3, y4, ξ1, ξ2, ξ3];
```

1 - Defincíón Isoparamétrica del Elemento -

IsoP =  ;

```
IsoPr = Show[IsoP, ImageSize→650]
```

$$\begin{bmatrix} 1 \\ x \\ y \\ u_x \\ u_y \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \\ u_{x1} & u_{x2} & \dots & u_{xn} \\ u_{y1} & u_{y2} & \dots & u_{yn} \end{bmatrix} \begin{bmatrix} N_1^{(e)} \\ N_2^{(e)} \\ \vdots \\ N_n^{(e)} \end{bmatrix}. \qquad (16.6)$$

```
x = {x1, x2, x3, x4};
y = {y1, y2, y3, y4};
```

- **2 - Funciones de Forma <<< -----**

```
Nf
```

```
{ξ1 - 2 ξ1 ξ2, ξ2 - 2 ξ1 ξ2, ξ3, 4 ξ1 ξ2}
```

- **3 - Derivadas Funciones de Forma respecto Coordenadas Naturales.**

```
Nf1 = D[Nf, ξ1];
Nf2 = D[Nf, ξ2];
Nf3 = D[Nf, ξ3];
{Nf1, Nf2, Nf3} = Simplify[{Nf1, Nf2, Nf3}];
```

```
Nf1 // MatrixForm
```

$$\begin{pmatrix} 1 - 2\,\xi 2 \\ -2\,\xi 2 \\ 0 \\ 4\,\xi 2 \end{pmatrix}$$

```
Nf2 // MatrixForm
```

$$\begin{pmatrix} -2\,\xi 1 \\ 1 - 2\,\xi 1 \\ 0 \\ 4\,\xi 1 \end{pmatrix}$$

```
Nf3 // MatrixForm
```

$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

- **4 - Derivadas Coordenadas Triangulares respecto a las Cartesianas - Desarrollo**

- **Calculo Elementos Matriz Jacobiana - Elemento Considerado**

```
SistemA = ;
```

```
SistemAr = Show[SistemA, ImageSize → 650]
```

$$\begin{bmatrix} \sum^{1} x_i \frac{\partial N_i}{\partial \zeta_1} & \sum^{1} x_i \frac{\partial N_i}{\partial \zeta_2} & \sum^{1} x_i \frac{\partial N_i}{\partial \zeta_3} \\ \sum y_i \frac{\partial N_i}{\partial \zeta_1} & \sum y_i \frac{\partial N_i}{\partial \zeta_2} & \sum y_i \frac{\partial N_i}{\partial \zeta_3} \end{bmatrix} \begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (24.18)$$

```
{Nf1x, Nf2x, Nf3x} = Simplify[{x.Nf1, x.Nf2, x.Nf3}];
{Nf1y, Nf2y, Nf3y} = Simplify[{y.Nf1, y.Nf2, y.Nf3}];
```

```
{Nf1x, Nf2x, Nf3x} // MatrixForm
```

$$\begin{pmatrix} x1 - 2\,x1\,\xi 2 - 2\,x2\,\xi 2 + 4\,x4\,\xi 2 \\ x2 - 2\,x1\,\xi 1 - 2\,x2\,\xi 1 + 4\,x4\,\xi 1 \\ x3 \end{pmatrix}$$

```
{Nf1y, Nf2y, Nf3y} // MatrixForm
```

$$\begin{pmatrix} y1 - 2\,y1\,\xi 2 - 2\,y2\,\xi 2 + 4\,y4\,\xi 2 \\ y2 - 2\,y1\,\xi 1 - 2\,y2\,\xi 1 + 4\,y4\,\xi 1 \\ y3 \end{pmatrix}$$

JacobianA = rewritten

$$\mathbf{J}\,\mathbf{P} = \begin{bmatrix} 1 & 1 & 1 \\ J_{x1} & J_{x2} & J_{x3} \\ J_{y1} & J_{y2} & J_{y3} \end{bmatrix} \begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (24.19)$$

;

```
JacobianAr = Show[JacobianA, ImageSize → 650]
```

rewritten

$$\mathbf{J}\,\mathbf{P} = \begin{bmatrix} 1 & 1 & 1 \\ J_{x1} & J_{x2} & J_{x3} \\ J_{y1} & J_{y2} & J_{y3} \end{bmatrix} \begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (24.19)$$

Para mostrar el planteamiento genérico, de momento no aplicamos estas definiciones:

```
Clear[Jx1, Jx2, Jx3, Jy1, Jy2, Jy3]
```

```
(*Jx1=Nf1x; Jx2=Nf2x; Jx3=Nf3x;*)
```

```
(*Jy1=Nf1y; Jy2=Nf2y; Jy3=Nf3y;*)
```

Definición Matriz Jacobiana y Sistema de Ecuaciones a Resolver - Planteamiento Genérico

$$J = \begin{pmatrix} 1 & 1 & 1 \\ Jx1 & Jx2 & Jx3 \\ Jy1 & Jy2 & Jy3 \end{pmatrix};$$

Determinante de la Matriz Jacobiana - J

```
Jdet = Det[J]
```

-Jx2 Jy1 + Jx3 Jy1 + Jx1 Jy2 - Jx3 Jy2 - Jx1 Jy3 + Jx2 Jy3

Defincin de Jc

```
Jc = 1 / 2 * Jdet
```

$$\frac{1}{2}\,(-Jx2\,Jy1 + Jx3\,Jy1 + Jx1\,Jy2 - Jx3\,Jy2 - Jx1\,Jy3 + Jx2\,Jy3)$$

Matriz Derivadas Coordenadas Triangulares respecto a las Cartesianas - Matriz Incognitas

```
P = ( f1x  f1y
      f2x  f2y
      f3x  f3y );
```

Matriz de Terminos Independientes

```
Ti = ( 0  0
       1  0
       0  1 );
```

Solución del Sistema de Ecuaciones

```
P = Inverse[J].Ti
```

$$\left\{\left\{\frac{Jy2 - Jy3}{-Jx2\,Jy1 + Jx3\,Jy1 + Jx1\,Jy2 - Jx3\,Jy2 - Jx1\,Jy3 + Jx2\,Jy3}, \frac{-Jx2 + Jx3}{-Jx2\,Jy1 + Jx3\,Jy1 + Jx1\,Jy2 - Jx3\,Jy2 - Jx1\,Jy3 + Jx2\,Jy3}\right\}, \left\{\frac{-Jy1 + Jy3}{-Jx2\,Jy1 + Jx3\,Jy1 + Jx1\,Jy2 - Jx3\,Jy2 - Jx1\,Jy3 + Jx2\,Jy3}, \frac{Jx1 - Jx3}{-Jx2\,Jy1 + Jx3\,Jy1 + Jx1\,Jy2 - Jx3\,Jy2 - Jx1\,Jy3 + Jx2\,Jy3}\right\}, \left\{\frac{Jy1 - Jy2}{-Jx2\,Jy1 + Jx3\,Jy1 + Jx1\,Jy2 - Jx3\,Jy2 - Jx1\,Jy3 + Jx2\,Jy3}, \frac{-Jx1 + Jx2}{-Jx2\,Jy1 + Jx3\,Jy1 + Jx1\,Jy2 - Jx3\,Jy2 - Jx1\,Jy3 + Jx2\,Jy3}\right\}\right\}$$

Defnción Matriz Jacobiana Inversa Modificada - Jim

JacobianaModificadA = $\begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J}\begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \quad (24.20)$;

```
JacobianaModificadAr = Show[JacobianaModificadA, ImageSize → 650]
```

$$\begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J}\begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \qquad (24.20)$$

```
Jc
```

$$\frac{1}{2}\left(-Jx2\,Jy1 + Jx3\,Jy1 + Jx1\,Jy2 - Jx3\,Jy2 - Jx1\,Jy3 + Jx2\,Jy3\right)$$

```
P
```

$$\left\{\left\{\frac{Jy2-Jy3}{-Jx2\,Jy1+Jx3\,Jy1+Jx1\,Jy2-Jx3\,Jy2-Jx1\,Jy3+Jx2\,Jy3},\ \frac{-Jx2+Jx3}{-Jx2\,Jy1+Jx3\,Jy1+Jx1\,Jy2-Jx3\,Jy2-Jx1\,Jy3+Jx2\,Jy3}\right\},\ \left\{\frac{-Jy1+Jy3}{-Jx2\,Jy1+Jx3\,Jy1+Jx1\,Jy2-Jx3\,Jy2-Jx1\,Jy3+Jx2\,Jy3},\ \frac{Jx1-Jx3}{-Jx2\,Jy1+Jx3\,Jy1+Jx1\,Jy2-Jx3\,Jy2-Jx1\,Jy3+Jx2\,Jy3}\right\},\ \left\{\frac{Jy1-Jy2}{-Jx2\,Jy1+Jx3\,Jy1+Jx1\,Jy2-Jx3\,Jy2-Jx1\,Jy3+Jx2\,Jy3},\ \frac{-Jx1+Jx2}{-Jx2\,Jy1+Jx3\,Jy1+Jx1\,Jy2-Jx3\,Jy2-Jx1\,Jy3+Jx2\,Jy3}\right\}\right\}$$

```
Jim = 2 * Jc * P
```

```
{{Jy2 - Jy3, -Jx2 + Jx3}, {-Jy1 + Jy3, Jx1 - Jx3}, {Jy1 - Jy2, -Jx1 + Jx2}}
```

```
Jim // MatrixForm
```

$$\begin{pmatrix} Jy2-Jy3 & -Jx2+Jx3 \\ -Jy1+Jy3 & Jx1-Jx3 \\ Jy1-Jy2 & -Jx1+Jx2 \end{pmatrix}$$

$$\begin{pmatrix} Jy23 & Jx32 \\ Jy31 & Jx13 \\ Jy12 & Jx21 \end{pmatrix} = Jim;$$

```
Jx13
```

```
Jx1 - Jx3
```

Necesario para mantener el planteamiento genérico:

```
Clear[Jdet, Jc, Jy23, Jx32, Jy31, Jx13, Jy12, Jx21]
```

```
Jc = 1 / 2 * Jdet;
```

```
JacobianaModificadAr
```

$$\begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J}\begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \qquad (24.20)$$

$$\begin{pmatrix} f1x & f1y \\ f2x & f2y \\ f3x & f3y \end{pmatrix} = \frac{1}{2*Jc} * \begin{pmatrix} Jy23 & Jx32 \\ Jy31 & Jx13 \\ Jy12 & Jx21 \end{pmatrix};$$

```
f1x
```

$$\frac{Jy23}{Jdet}$$

■ 5 - Derivadas de las Funciones de Forma respecto a las Coordenadas Cartesianas - Desarrollo

▫ Definición de las Derivadas de las Funciones de Forma respecto a las Coordenadas Cartesianas - Planteamiento Genérico

```
DerivadasFuncionesFormaCartesianaS =
```

$$\frac{\partial N_i}{\partial x} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{y23} + \frac{\partial N_i}{\partial \zeta_2} J_{y31} + \frac{\partial N_i}{\partial \zeta_3} J_{y12}\right), \qquad \frac{\partial N_i}{\partial y} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{x32} + \frac{\partial N_i}{\partial \zeta_2} J_{x13} + \frac{\partial N_i}{\partial \zeta_3} J_{x21}\right). \quad (24.22)$$

;

```
DerivadasFuncionesFormaCartesianaSr = Show[DerivadasFuncionesFormaCartesianaS, ImageSize → 650]
```

$$\frac{\partial N_i}{\partial x} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{y23} + \frac{\partial N_i}{\partial \zeta_2} J_{y31} + \frac{\partial N_i}{\partial \zeta_3} J_{y12}\right),$$
$$\frac{\partial N_i}{\partial y} = \frac{1}{2J}\left(\frac{\partial N_i}{\partial \zeta_1} J_{x32} + \frac{\partial N_i}{\partial \zeta_2} J_{x13} + \frac{\partial N_i}{\partial \zeta_3} J_{x21}\right). \quad (24.22)$$

```
DerivadasFuncionesFormaCartesianasM =
```

In matrix form:

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = \mathbf{P}^T \left[\frac{\partial N_i}{\partial \zeta_1} \quad \frac{\partial N_i}{\partial \zeta_2} \quad \frac{\partial N_i}{\partial \zeta_3} \right]^T, \quad (24.23)$$

;

```
DerivadasFuncionesFormaCartesianasMr = Show[DerivadasFuncionesFormaCartesianasM, ImageSize → 650]
```

In matrix form:

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = \mathbf{P}^T \left[\frac{\partial N_i}{\partial \zeta_1} \quad \frac{\partial N_i}{\partial \zeta_2} \quad \frac{\partial N_i}{\partial \zeta_3} \right]^T, \quad (24.23)$$

$$\begin{pmatrix} dNx \\ dNy \end{pmatrix} = \text{Transpose}\left[\begin{pmatrix} f1x & f1y \\ f2x & f2y \\ f3x & f3y \end{pmatrix}\right].\begin{pmatrix} Nf1 \\ Nf2 \\ Nf3 \end{pmatrix};$$

▫ dNx & dNy <<< -----

```
dNx
```

$$\left\{-\frac{2\,Jy31\,\xi1}{Jdet} + \frac{Jy23\,(1 - 2\,\xi2)}{Jdet}, \frac{Jy31\,(1 - 2\,\xi1)}{Jdet} - \frac{2\,Jy23\,\xi2}{Jdet}, \frac{Jy12}{Jdet}, \frac{4\,Jy31\,\xi1}{Jdet} + \frac{4\,Jy23\,\xi2}{Jdet}\right\}$$

```
dNy
```

$$\left\{-\frac{2\,Jx13\,\xi1}{Jdet} + \frac{Jx32\,(1 - 2\,\xi2)}{Jdet}, \frac{Jx13\,(1 - 2\,\xi1)}{Jdet} - \frac{2\,Jx32\,\xi2}{Jdet}, \frac{Jx21}{Jdet}, \frac{4\,Jx13\,\xi1}{Jdet} + \frac{4\,Jx32\,\xi2}{Jdet}\right\}$$

Se observa quedan en función de los elementos de la Matriz Jacobiana Inversa Modificada - Jim

---

Matriz Jacobiana Inversa Modificada - Jim - Planteamiento Genérico

```
JacobianaModificadAr
```

$$\begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \frac{1}{2J} \begin{bmatrix} J_{y23} & J_{x32} \\ J_{y31} & J_{x13} \\ J_{y12} & J_{x21} \end{bmatrix} = \mathbf{P}, \qquad (24.20)$$

$$\begin{pmatrix} Jy23 & Jx32 \\ Jy31 & Jx13 \\ Jy12 & Jx21 \end{pmatrix} = Jim;$$

```
Jx13
```

Jx1 − Jx3

Elemetos Matriz Jacobiana - Elemento Considerado

```
JacobianAr
```

rewritten

$$\mathbf{J}\,\mathbf{P} = \begin{bmatrix} 1 & 1 & 1 \\ J_{x1} & J_{x2} & J_{x3} \\ J_{y1} & J_{y2} & J_{y3} \end{bmatrix} \begin{bmatrix} \frac{\partial \zeta_1}{\partial x} & \frac{\partial \zeta_1}{\partial y} \\ \frac{\partial \zeta_2}{\partial x} & \frac{\partial \zeta_2}{\partial y} \\ \frac{\partial \zeta_3}{\partial x} & \frac{\partial \zeta_3}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (24.19)$$

Aplicamos las deiniciones correspondientes al alemento considerado:

```
Jx1 = Nf1x; Jx2 = Nf2x; Jx3 = Nf3x;
```

```
Jy1 = Nf1y; Jy2 = Nf2y; Jy3 = Nf3y;
```

Matriz Jacobiana Inversa Modificada - Jim - Elemento Considerado

$$\begin{pmatrix} Jy23 & Jx32 \\ Jy31 & Jx13 \\ Jy12 & Jx21 \end{pmatrix};$$

```
Jy23
```

y2 − y3 − 2 y1 ξ1 − 2 y2 ξ1 + 4 y4 ξ1

```
Jx32
```

−x2 + x3 + 2 x1 ξ1 + 2 x2 ξ1 − 4 x4 ξ1

```
Jy31
```

−y1 + y3 + 2 y1 ξ2 + 2 y2 ξ2 − 4 y4 ξ2

```
Jx13
x1 - x3 - 2 x1 ξ2 - 2 x2 ξ2 + 4 x4 ξ2
```

```
Jy12
y1 - y2 + 2 y1 ξ1 + 2 y2 ξ1 - 4 y4 ξ1 - 2 y1 ξ2 - 2 y2 ξ2 + 4 y4 ξ2
```

```
Jx21
-x1 + x2 - 2 x1 ξ1 - 2 x2 ξ1 + 4 x4 ξ1 + 2 x1 ξ2 + 2 x2 ξ2 - 4 x4 ξ2
```

- **MODULO CALCULO FUNCIONES DE FORMA Y JACOBIANO**

MODULO GENERICO A COMPLETAR -

MODULO GENERICO A COMPLETAR: XX = No. Nodos, {xI,yI}= nodo i-esimo

```
(*TrigXXIsoPShapeFunDer[ncoor_,tcoor_]:=Module[{ξ1,ξ2,ξ3,x1,x2,x3,x4,x5,x6,x7,x8,x9,xI,y1,y2,y3,
    y4,y5,y6,y7,y8,y9,yI,Jx21,Jx32,Jx13,Jy12,Jy23,Jy31,Nf,dNx,dNy,Jdet},{ξ1,ξ2,ξ3}=tcoor;
   {{x1,y1},{x2,y2},{x3,y3},{x4,y4},{x5,y5},{x6,y6},{x7,y7},{x8,y8},{x9,y9},{xI,yI}}=ncoor;
   Nf=(*Nf*);
   Jx21=(*Jx21*);
   Jy12=(*Jy12*);
   Jx13=(*Jx13*);
   Jy31=(*Jy31*);
   Jx32=(*Jx32*);
   Jy23=(*Jy23*);
   Jdet=Jx21*Jy31-Jy12*Jx13;
   dNx=(*dNx*);
   dNy=(*dNy*);
   Return[Simplify[{Nf,dNx,dNy,Jdet}]]];*)
```

MODULO COMPLETADO -

```
Trig4TIsoPShapeFunDer[ncoor_, tcoor_] := Module[{ξ1, ξ2, ξ3, x1, x2, x3, x4, y1, y2,
    y3, y4, Jx21, Jx32, Jx13, Jy12, Jy23, Jy31, Nf, dNx, dNy, Jdet}, {ξ1, ξ2, ξ3} = tcoor;
   {{x1, y1}, {x2, y2}, {x3, y3}, {x4, y4}} = ncoor;
   Nf = {ξ1 - 2 ξ1 ξ2, ξ2 - 2 ξ1 ξ2, ξ3, 4 ξ1 ξ2};
   Jx21 = -x1 + x2 - 2 x1 ξ1 - 2 x2 ξ1 + 4 x4 ξ1 + 2 x1 ξ2 + 2 x2 ξ2 - 4 x4 ξ2;
   Jy12 = y1 - y2 + 2 y1 ξ1 + 2 y2 ξ1 - 4 y4 ξ1 - 2 y1 ξ2 - 2 y2 ξ2 + 4 y4 ξ2;
   Jx13 = x1 - x3 - 2 x1 ξ2 - 2 x2 ξ2 + 4 x4 ξ2;
   Jy31 = -y1 + y3 + 2 y1 ξ2 + 2 y2 ξ2 - 4 y4 ξ2;
   Jx32 = -x2 + x3 + 2 x1 ξ1 + 2 x2 ξ1 - 4 x4 ξ1;
   Jy23 = y2 - y3 - 2 y1 ξ1 - 2 y2 ξ1 + 4 y4 ξ1;
   Jdet = Jx21 * Jy31 - Jy12 * Jx13;
```

$$dNx = \left\{-\frac{2\,Jy31\,\xi1}{Jdet} + \frac{Jy23\,(1-2\,\xi2)}{Jdet},\ \frac{Jy31\,(1-2\,\xi1)}{Jdet} - \frac{2\,Jy23\,\xi2}{Jdet},\ \frac{Jy12}{Jdet},\ \frac{4\,Jy31\,\xi1}{Jdet} + \frac{4\,Jy23\,\xi2}{Jdet}\right\};$$

$$dNy = \left\{-\frac{2\,Jx13\,\xi1}{Jdet} + \frac{Jx32\,(1-2\,\xi2)}{Jdet},\ \frac{Jx13\,(1-2\,\xi1)}{Jdet} - \frac{2\,Jx32\,\xi2}{Jdet},\ \frac{Jx21}{Jdet},\ \frac{4\,Jx13\,\xi1}{Jdet} + \frac{4\,Jx32\,\xi2}{Jdet}\right\};$$

```
   Return[Simplify[{Nf, dNx, dNy, Jdet}]]];
```

6. MATRIZ DE RIGIDEZ - INTEGRACION NUMERICA -

MODULO DE CALCULO DE LA MATRIZ DE RIGIDEZ

MODULO GENERICO A COMPLETAR -

MODULO GENERICO: XX = No. Nodos, YY = Grados Libertad

```
(*TrigXXIsoPMembraneStiffness[ncoor_,mprop_,fprop_,opt_]:=
  Module[{i,k,l,p=3,numer=False,Emat,th={fprop},h,tcoor,w,c,Nf,dNx,dNy,Jdet,B,Ke=Table[0,{YY},{YY}]},
   Emat=mprop[[1]];If[Length[fprop]>0,th=fprop[[1]]];
   If[Length[opt]>0,numer=opt[[1]]];
   If[Length[opt]>1,p=opt[[2]]];
   If[p≠1&&p≠-3&&p≠3&&p≠6&&p≠-6&&p≠7&&p≠12,Print["Illegal p"];Return[Null]];
   For[k=1,k≤Abs[p],k++,{tcoor,w}=TrigGaussRuleInfo[{p,numer},k];
    {Nf,dNx,dNy,Jdet}=TrigXXIsoPShapeFunDer[ncoor,tcoor];
    If[Length[th]==0,h=th,h=th.Nf];c=w*Jdet*h/2;
    B={Flatten[Table[{dNx[[i]],0},{i,XX}]],
      Flatten[Table[{0,dNy[[i]]},{i,XX}]],Flatten[Table[{dNy[[i]],dNx[[i]]},{i,XX}]]};
    Ke+=c*Transpose[B].(Emat.B);];If[!numer,Ke=Simplify[Ke]];Return[Ke]];*)
```

MODULO COMPLETADO -

```
Trig4TIsoPMembraneStiffness[ncoor_, mprop_, fprop_, opt_] :=
  Module[{i, k, l, p = 3, numer = False, Emat, th = {fprop}, h, tcoor, w, c, Nf, dNx, dNy, Jdet,
    B, Ke = Table[0, {8}, {8}]}, Emat = mprop[[1]]; If[Length[fprop] > 0, th = fprop[[1]]];
   If[Length[opt] > 0, numer = opt[[1]]];
   If[Length[opt] > 1, p = opt[[2]]];
   If[p ≠ 1 && p ≠ -3 && p ≠ 3 && p ≠ 6 && p ≠ -6 && p ≠ 7 && p ≠ 12, Print["Illegal p"]; Return[Null]];
   For[k = 1, k ≤ Abs[p], k++, {tcoor, w} = TrigGaussRuleInfo[{p, numer}, k];
    {Nf, dNx, dNy, Jdet} = Trig4TIsoPShapeFunDer[ncoor, tcoor];
    If[Length[th] == 0, h = th, h = th.Nf]; c = w*Jdet*h/2;
    B = {Flatten[Table[{dNx[[i]], 0}, {i, 4}]],
      Flatten[Table[{0, dNy[[i]]}, {i, 4}]], Flatten[Table[{dNy[[i]], dNx[[i]]}, {i, 4}]]};
    Ke += c*Transpose[B].(Emat.B);]; If[! numer, Ke = Simplify[Ke]]; Return[Ke]];
```

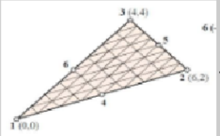
■ MODULO REGLAS DE CUADRATURA DE GAUSS

□ OPCION UNICA: DEFINICION DE CARLOS FELIPPA

```
TrigGaussRuleInfo[{rule_, numer_}, point_] :=
  Module[{zeta, p = rule, i = point, g1, g2, g3, g4, w1, w2, w3, eps = 10.^(-24),
     jkl = {{1, 2, 3}, {2, 1, 3}, {1, 3, 2}, {3, 1, 2}, {2, 3, 1}, {3, 2, 1}}, info = {{Null, Null, Null}, 0}},
    If[p == 1, info = {{1/3, 1/3, 1/3}, 1}]; If[p == 3, info = {{1, 1, 1}/6, 1/3}; info[[1, i]] = 2/3];
    If[p == -3, info = {{1, 1, 1}/2, 1/3}; info[[1, i]] = 0]; If[p == 6,
     g1 = (8 - Sqrt[10] + Sqrt[38 - 44*Sqrt[2/5]])/18; g2 = (8 - Sqrt[10] - Sqrt[38 - 44*Sqrt[2/5]])/18;
     If[i < 4, info = {{g1, g1, g1}, (620 + Sqrt[213 125 - 53 320*Sqrt[10]])/3720}; info[[1, i]] = 1 - 2*g1];
     If[i > 3, info = {{g2, g2, g2}, (620 - Sqrt[213 125 - 53 320*Sqrt[10]])/3720}; info[[1, i - 3]] = 1 - 2*g2]];
    If[p == -6, If[i < 4, info = {{1, 1, 1}/6, 3/10}; info[[1, i]] = 2/3];
     If[i > 3, info = {{1, 1, 1}/2, 1/30}; info[[1, i - 3]] = 0]]; If[p == 7, g1 = (6 - Sqrt[15])/21;
     g2 = (6 + Sqrt[15])/21; If[i < 4, info = {{g1, g1, g1}, (155 - Sqrt[15])/1200}; info[[1, i]] = 1 - 2*g1];
     If[i > 3 && i < 7, info = {{g2, g2, g2}, (155 + Sqrt[15])/1200}; info[[1, i - 3]] = 1 - 2*g2];
     If[i == 7, info = {{1/3, 1/3, 1/3}, 9/40}]];
    If[p == 12, g1 = 0.063089014491502228340331602870819157; g2 = 0.24928674517091042129163855310701907 6;
     g3 = 0.053145049844816947353249671631398147; g4 = 0.31035245103378440541660773395655215 3;
     If[! numer, {g1, g2, g3, g4} = Rationalize[{g1, g2, g3, g4}, eps]];
     w1 = (30*g2^3*(4*g3^2 + (1 - 2*g4)^2 + 4*g3*(-1 + g4)) + g3^2*(1 - 15*g4) +
          (-1 + g4)*g4 - g3*(-1 + g4)*(-1 + 15*g4) + 2*g2*(1 + 60*g3*g4*(-1 + g3 + g4)) -
          6*g2^2*(3 + 10*(-1 + g4)*g4 + 10*g3^2*(1 + 3*g4) + 10*g3*(-1 + g4)*(1 + 3*g4)))/
        (180*(g1 - g2)*(-(g2*(-1 + 2*g2)*(-1 + g3)*g3) + (-1 + g3)*(g2 - 2*g2^2 - 2*g3 + 3*g2*g3)*g4 -
            (g2*(-1 + 2*g2 - 3*g3) + 2*g3)*g4^2 + 2*g1^2*(g2*(-2 + 3*g2) + g3 - g3^2 + g4 - g3*g4 - g4^2) +
            g1*(-4*g2^2 + (-1 + g3)*g3 + (-1 + g3)*(1 + 3*g3)*g4 +
               (1 + 3*g3)*g4^2 - 2*g2*(-1 + g3^2 + g3*(-1 + g4) + (-1 + g4)*g4))));
     w2 = (-1 + 12*(2 - 3*g1)*g1*w1 + 4*g3^2*(-1 + 3*w1) + 4*g3*(-1 + g4)*(-1 + 3*w1) +
          4*(-1 + g4)*g4*(-1 + 3*w1))/(12*(g2*(-2 + 3*g2) + g3 - g3^2 + g4 - g3*g4 - g4^2));
     w3 = (1 - 3*w1 - 3*w2)/6; If[i < 4, info = {{g1, g1, g1}, w1}; info[[1, i]] = 1 - 2*g1];
     If[i > 3 && i < 7, info = {{g2, g2, g2}, w2}; info[[1, i - 3]] = 1 - 2*g2];
     If[i > 6, {j, k, l} = jkl[[i - 6]]; info = {{0, 0, 0}, w3}; info[[1, j]] = g3; info[[1, k]] = g4;
      info[[1, l]] = 1 - g3 - g4]]; If[numer, Return[N[info]], Return[Simplify[info]]];];
```

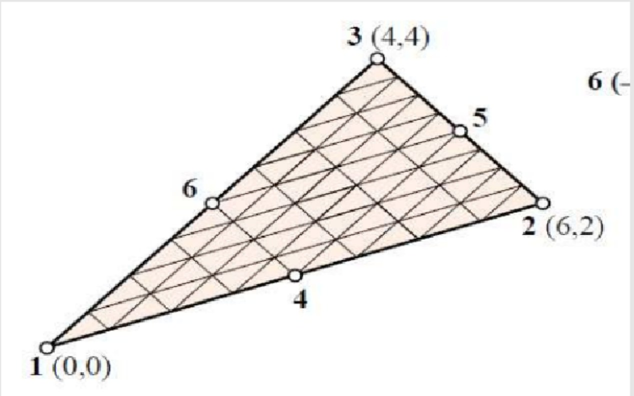
7. TEST DEL ELEMENTO DE LADOS RECTOS SUPERPARAMETRICO.

■ DEFINICION DE LA GEOMETRIA

LadosRectosTesT = 

;

```
LadosRectosTesTr = Show[LadosRectosTesT, ImageSize → 300]
```



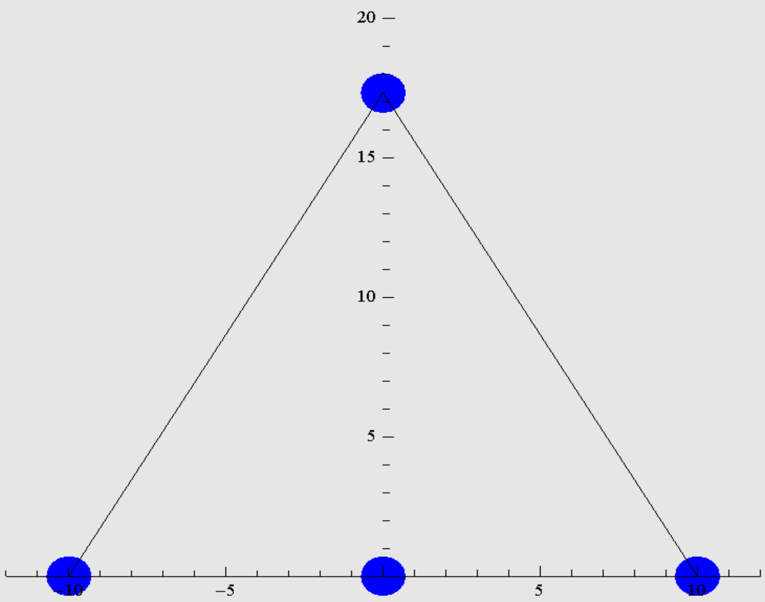
■ **DEFINICION DE LOS NODOS ELEMENTO - #**

```
ncoor = Cnc;
```

```
ptsexteriores = {Cnc[[1]], Cnc[[4]], Cnc[[2]], Cnc[[3]]};
```

```
ptsinteriores = {};
```

```
Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
  Axes → True, PlotRange → {{-12, 12}, {-2, 20}}, NodeColor → RGBColor[0, 0, 1]]
```



■ **DEFINICION DEL MATERIAL**

```
ClearAll[Em, nu, h];
```

```
h = 1; Em = 288; nu = 1 / 3;
```

```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

```
Print["Emat=", Emat // MatrixForm]
```

Emat=
324	108	0
108	324	0
0	0	108

VERIFICACION DE LA MATRIZ DE RIGIDEZ -

NUMERO DE PUNTOS DE GAUSS MINIMO PARA CONSEGUIR SUFICIENCIA DE RANGO -

```
NF = NNodos * 2.;
```

$$NG = \frac{NF - 3}{3}$$

1.66667

Se necesitan como mínimo 2 Puntos -- Regla 3 minima

BUCLE GENERICO A COMPLETAR -

BUCLE GENERICO: XX = No. Nodos, ZZ = GRADOS DE LIBERTAD - 3

```
(*For [i=1, i≤7,i++,p={1,-3,3,6,-6,7,12}[[i]];
  Ke=TrigXXIsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
  Print["Gauss integration rule: ",p];
  Print["Ke=",Chop[Simplify[Ke]]//MatrixForm];
  Valores=Chop[Eigenvalues[N[Ke]]];
  If[Valores[[ZZ]]!=0,Print["Valores propios matriz Ke=",Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=",p];Break[],
   Print["Valores proprios matriz Ke=",Valores];Print["NO tenemos la suficiencia de rango para p=",p]]
 ];*)
```

DESARROLLO DE LA MATRIZ DE RIGIDEZ -

```
For [i = 1, i ≤ 7, i++, p = {1, -3, 3, 6, -6, 7, 12}[[i]];
  Ke = Trig4TIsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
  Print["Gauss integration rule: ", p];
  Print["Ke=", Chop[Simplify[Ke]] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]]];
  If[Valores[[5]] != 0, Print["Valores propios matriz Ke=", Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=", p](*;Break[]*), Print[
    "Valores proprios matriz Ke=", Valores]; Print["NO tenemos la suficiencia de rango para p=", p]]
 ];
```

Gauss integration rule: 1

Ke=
142.028	-18.	-138.564	0	10.3923	-54.	-13.8564	72.
-18.	51.9615	0	-41.5692	-54.	31.1769	72.	-41.5692
-138.564	0	142.028	18.	10.3923	54.	-13.8564	-72.
0	-41.5692	18.	51.9615	54.	31.1769	-72.	-41.5692
10.3923	-54.	10.3923	54.	62.3538	0	-83.1384	0
-54.	31.1769	54.	31.1769	0	187.061	0	-249.415
-13.8564	72.	-13.8564	-72.	-83.1384	0	110.851	0
72.	-41.5692	-72.	-41.5692	0	-249.415	0	332.554

Valores proprios matriz Ke={584.398, 270.2, 226.202, 0, 0, 0, 0, 0}

NO tenemos la suficiencia de rango para p=1

Gauss integration rule: -3

$$Ke=\begin{pmatrix} 239.023 & -18. & -41.5692 & 0 & 10.3923 & -54. & -207.846 & 72. \\ -18. & 93.5307 & 0 & 0 & -54. & 31.1769 & 72. & -124.708 \\ -41.5692 & 0 & 239.023 & 18. & 10.3923 & 54. & -207.846 & -72. \\ 0 & 0 & 18. & 93.5307 & 54. & 31.1769 & -72. & -124.708 \\ 10.3923 & -54. & 10.3923 & 54. & 62.3538 & 0 & -83.1384 & 0 \\ -54. & 31.1769 & 54. & 31.1769 & 0 & 187.061 & 0 & -249.415 \\ -207.846 & 72. & -207.846 & -72. & -83.1384 & 0 & 498.831 & 0 \\ 72. & -124.708 & -72. & -124.708 & 0 & -249.415 & 0 & 498.831 \end{pmatrix}$$

Valores propios matriz Ke={724.768, 706.677, 249.415, 145.492, 85.8316, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=-3

Gauss integration rule: 3

$$Ke=\begin{pmatrix} 239.023 & -18. & -41.5692 & 0 & 10.3923 & -54. & -207.846 & 72. \\ -18. & 93.5307 & 0 & 0 & -54. & 31.1769 & 72. & -124.708 \\ -41.5692 & 0 & 239.023 & 18. & 10.3923 & 54. & -207.846 & -72. \\ 0 & 0 & 18. & 93.5307 & 54. & 31.1769 & -72. & -124.708 \\ 10.3923 & -54. & 10.3923 & 54. & 62.3538 & 0 & -83.1384 & 0 \\ -54. & 31.1769 & 54. & 31.1769 & 0 & 187.061 & 0 & -249.415 \\ -207.846 & 72. & -207.846 & -72. & -83.1384 & 0 & 498.831 & 0 \\ 72. & -124.708 & -72. & -124.708 & 0 & -249.415 & 0 & 498.831 \end{pmatrix}$$

Valores propios matriz Ke={724.768, 706.677, 249.415, 145.492, 85.8316, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=3

Gauss integration rule: 6

$$Ke=\begin{pmatrix} 239.023 & -18. & -41.5692 & 0 & 10.3923 & -54. & -207.846 & 72. \\ -18. & 93.5307 & 0 & 0 & -54. & 31.1769 & 72. & -124.708 \\ -41.5692 & 0 & 239.023 & 18. & 10.3923 & 54. & -207.846 & -72. \\ 0 & 0 & 18. & 93.5307 & 54. & 31.1769 & -72. & -124.708 \\ 10.3923 & -54. & 10.3923 & 54. & 62.3538 & 0 & -83.1384 & 0 \\ -54. & 31.1769 & 54. & 31.1769 & 0 & 187.061 & 0 & -249.415 \\ -207.846 & 72. & -207.846 & -72. & -83.1384 & 0 & 498.831 & 0 \\ 72. & -124.708 & -72. & -124.708 & 0 & -249.415 & 0 & 498.831 \end{pmatrix}$$

Valores propios matriz Ke={724.768, 706.677, 249.415, 145.492, 85.8316, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=6

Gauss integration rule: -6

$$Ke=\begin{pmatrix} 239.023 & -18. & -41.5692 & 0 & 10.3923 & -54. & -207.846 & 72. \\ -18. & 93.5307 & 0 & 0 & -54. & 31.1769 & 72. & -124.708 \\ -41.5692 & 0 & 239.023 & 18. & 10.3923 & 54. & -207.846 & -72. \\ 0 & 0 & 18. & 93.5307 & 54. & 31.1769 & -72. & -124.708 \\ 10.3923 & -54. & 10.3923 & 54. & 62.3538 & 0 & -83.1384 & 0 \\ -54. & 31.1769 & 54. & 31.1769 & 0 & 187.061 & 0 & -249.415 \\ -207.846 & 72. & -207.846 & -72. & -83.1384 & 0 & 498.831 & 0 \\ 72. & -124.708 & -72. & -124.708 & 0 & -249.415 & 0 & 498.831 \end{pmatrix}$$

Valores propios matriz Ke={724.768, 706.677, 249.415, 145.492, 85.8316, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=-6

Gauss integration rule: 7

$$Ke=\begin{pmatrix} 239.023 & -18. & -41.5692 & 0 & 10.3923 & -54. & -207.846 & 72. \\ -18. & 93.5307 & 0 & 0 & -54. & 31.1769 & 72. & -124.708 \\ -41.5692 & 0 & 239.023 & 18. & 10.3923 & 54. & -207.846 & -72. \\ 0 & 0 & 18. & 93.5307 & 54. & 31.1769 & -72. & -124.708 \\ 10.3923 & -54. & 10.3923 & 54. & 62.3538 & 0 & -83.1384 & 0 \\ -54. & 31.1769 & 54. & 31.1769 & 0 & 187.061 & 0 & -249.415 \\ -207.846 & 72. & -207.846 & -72. & -83.1384 & 0 & 498.831 & 0 \\ 72. & -124.708 & -72. & -124.708 & 0 & -249.415 & 0 & 498.831 \end{pmatrix}$$

Valores propios matriz Ke={724.768, 706.677, 249.415, 145.492, 85.8316, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=7

Gauss integration rule: 12

Ke=

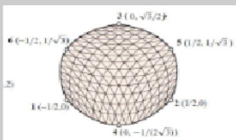
239.023	-18.	-41.5692	0	10.3923	-54.	-207.846	72.
-18.	93.5307	0	0	-54.	31.1769	72.	-124.708
-41.5692	0	239.023	18.	10.3923	54.	-207.846	-72.
0	0	18.	93.5307	54.	31.1769	-72.	-124.708
10.3923	-54.	10.3923	54.	62.3538	0	-83.1384	0
-54.	31.1769	54.	31.1769	0	187.061	0	-249.415
-207.846	72.	-207.846	-72.	-83.1384	0	498.831	0
72.	-124.708	-72.	-124.708	0	-249.415	0	498.831

Valores propios matriz Ke={724.768, 706.677, 249.415, 145.492, 85.8316, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=12

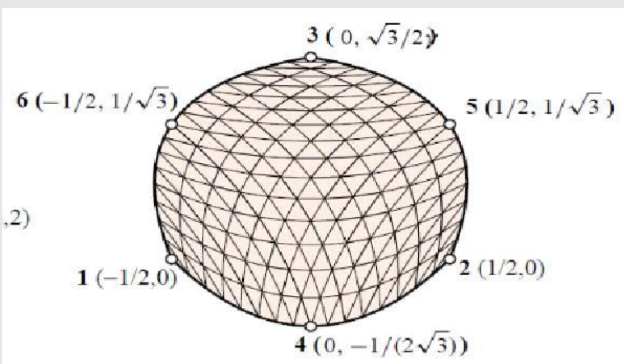
8. TEST DEL ELEMENTO DE LADOS CURVOS.

■ DEFINICION DE LA GEOMETRIA

```
LadosRectosTesT = 
```


```
;
```

```
LadosRectosTesTr = Show[LadosRectosTesT, ImageSize → 300]
```



■ DEFINICION DE LOS NODOS ELEMENTO -

□ ELEMENTO BASE REAL

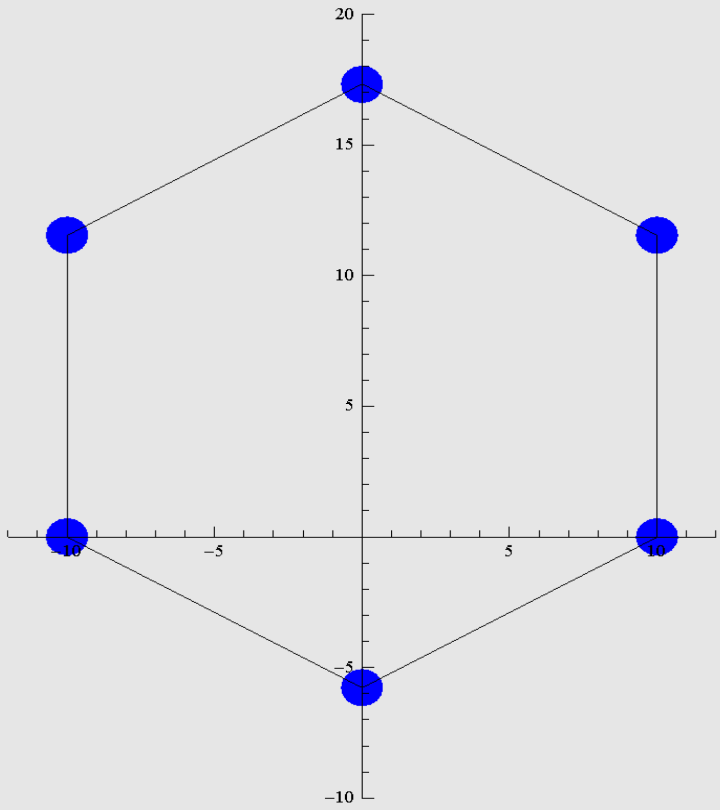
```
Cng = Table[{0, 0}, {i, 6}];
```

```
Cng[[1]] = {-10, 0}; Cng[[2]] = {10, 0}; Cng[[3]] = {0, 10*Sqrt[3]}; Cng[[4]] = {0, -10*Sqrt[3]*1/3};
Cng[[5]] = {10, 10*Sqrt[3]*2/3}; Cng[[6]] = {-10, 10*Sqrt[3]*2/3};
```

```
ptsexteriores = {Cng[[1]], Cng[[4]], Cnc[[2]], Cng[[5]], Cng[[3]], Cng[[6]]};
```

```
ptsinteriores = {};
```

```
Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
  Axes → True, PlotRange → {{-12, 12}, {-10, 20}}, NodeColor → RGBColor[0, 0, 1]]
```



- **TRANSFORMACION DE COORDENADAS TRIANGULARES A CARTESIANAS**

```
Nf6 = {ξ1 * (2 * ξ1 - 1), ξ2 * (2 * ξ2 - 1), ξ3 * (2 * ξ3 - 1), 4 * ξ1 * ξ2, 4 * ξ2 * ξ3, 4 * ξ3 * ξ1};
```

```
TtC[ξ1_, ξ2_, ξ3_] =
  (         1                  1                  1                  1                  1                  1
   Cng[[1]][[1]]  Cng[[2]][[1]]  Cng[[3]][[1]]  Cng[[4]][[1]]  Cng[[5]][[1]]  Cng[[6]][[1]]
   Cng[[1]][[2]]  Cng[[2]][[2]]  Cng[[3]][[2]]  Cng[[4]][[2]]  Cng[[5]][[2]]  Cng[[6]][[2]] ).Nf6;
```

- **COORDENADAS TRIANGULARES DEL ELEMENTO CONSIDERADO**

```
Cnt
```

$$\left\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}, \left\{\frac{1}{2}, \frac{1}{2}, 0\right\}\right\}$$

- **COORDENADAS CARTESIANAS NODOS ELEMENTO REAL CONSIDERADO Y COMPROBACION GRAFICA - #**

```
Cnc = Table[{0, 0}, {i, NNodos}];
```

```
Do[
  Cnc[[i]] = {TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[2]],
    TtC[Cnt[[i]][[1]], Cnt[[i]][[2]], Cnt[[i]][[3]]][[3]]},
  {i, NNodos}
 ];
```

```
Cnc
```

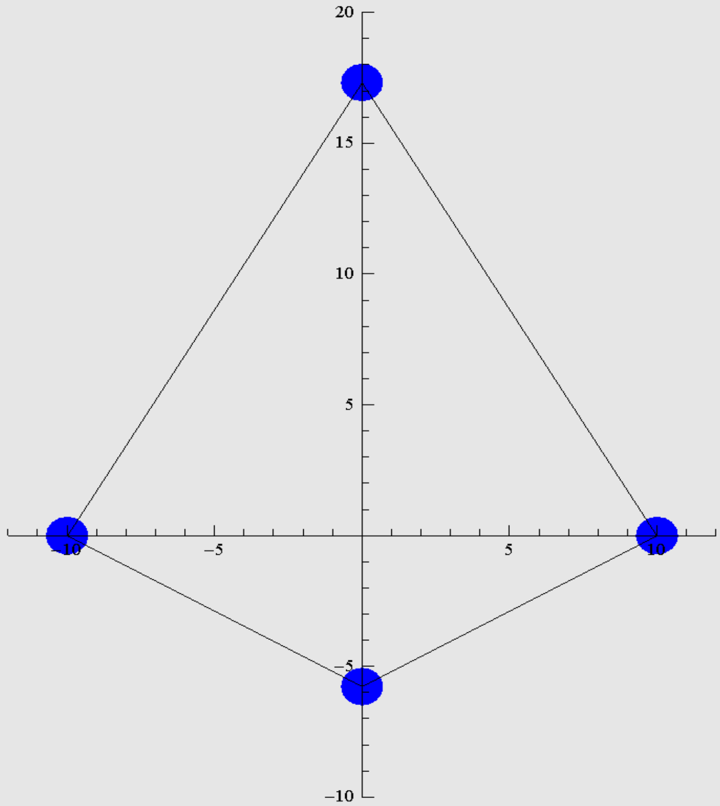
$$\left\{\{-10, 0\}, \{10, 0\}, \left\{0, 10\sqrt{3}\right\}, \left\{0, -\frac{10}{\sqrt{3}}\right\}\right\}$$

```
ncoor = Cnc;
```

```
ptsexteriores = {Cnc[[1]], Cnc[[4]], Cnc[[2]], Cnc[[3]]};
```

```
ptsinteriores = {};
```

```
Elemento = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → Automatic,
  Axes → True, PlotRange → {{-12, 12}, {-10, 20}}, NodeColor → RGBColor[0, 0, 1]]
```



■ **DEFINICION DEL MATERIAL**

```
ClearAll[Em, nu, h];
```

```
h = 1; Em = 7*72; nu = 0; h = 1;
```

```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

■ **VERIFICACION DE LA MATRIZ DE RIGIDEZ - #**

□ **NUMERO DE PUNTOS DE GAUSS MINIMO PARA CONSEGUIR SUFICIENCIA DE RANGO - #**

```
NF = NNodos*2.;
```

$$NG = \frac{NF - 3}{3}$$

1.66667

Se necesitan como mínimo 2 Puntos -- Regla 3 minima

BUCLE GENERICO A COMPLETAR -

BUCLE GENERICO: XX = No. Nodos, ZZ = GRADOS DE LIBERTAD - 3

```
(*For [i=1, i≤7,i++,p={1,-3,3,6,-6,7,12}[[i]];
  Ke=TrigXXIsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
  Print["Gauss integration rule: ",p];
  Print["Ke=",Chop[Simplify[Ke]]//MatrixForm];
  Valores=Chop[Eigenvalues[N[Ke]]];
  If[Valores[[ZZ]]!=0,Print["Valores propios matriz Ke=",Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=",p];Break[],
   Print["Valores proprios matriz Ke=",Valores];Print["NO tenemos la suficiencia de rango para p=",p]]
 ];*)
```

DESARROLLO DE LA MATRIZ DE RIGIDEZ -

```
For [i = 1, i ≤ 7, i++, p = {1, -3, 3, 6, -6, 7, 12}[[i]];
  Ke = Trig4TIsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
  Print["Gauss integration rule: ", p];
  Print["Ke=", Chop[Simplify[Ke]] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]]];
  If[Valores[[5]] != 0, Print["Valores propios matriz Ke=", Valores];
   Print["TENEMOS LA SUFICIENCIA DE RANGO PARA p=", p](*;Break[]*), Print[
    "Valores proprios matriz Ke=", Valores]; Print["NO tenemos la suficiencia de rango para p=", p]]
 ];
```

Gauss integration rule: 1

$$Ke=\begin{pmatrix} 318.031 & -21. & -312.435 & 21. & 16.7876 & 0 & -22.3834 & 0 \\ -21. & 163.212 & -21. & -152.021 & -126. & 33.5751 & 168. & -44.7669 \\ -312.435 & -21. & 318.031 & 21. & 16.7876 & 0 & -22.3834 & 0 \\ 21. & -152.021 & 21. & 163.212 & 126. & 33.5751 & -168. & -44.7669 \\ 16.7876 & -126. & 16.7876 & 126. & 100.725 & 0 & -134.301 & 0 \\ 0 & 33.5751 & 0 & 33.5751 & 0 & 201.451 & 0 & -268.601 \\ -22.3834 & 168. & -22.3834 & -168. & -134.301 & 0 & 179.067 & 0 \\ 0 & -44.7669 & 0 & -44.7669 & 0 & -268.601 & 0 & 358.135 \end{pmatrix}$$

Valores proprios matriz Ke={630.466, 600.622, 570.777, 0, 0, 0, 0, 0}

NO tenemos la suficiencia de rango para p=1

Gauss integration rule: -3

$$Ke=\begin{pmatrix} 516.498 & -21. & -113.969 & 21. & -82.4456 & 0 & -320.083 & 0 \\ -21. & 269.161 & -21. & -46.0726 & -126. & -19.399 & 168. & -203.689 \\ -113.969 & -21. & 516.498 & 21. & -82.4456 & 0 & -320.083 & 0 \\ 21. & -46.0726 & 21. & 269.161 & 126. & -19.399 & -168. & -203.689 \\ -82.4456 & -126. & -82.4456 & 126. & 150.342 & 0 & 14.5492 & 0 \\ 0 & -19.399 & 0 & -19.399 & 0 & 227.938 & 0 & -189.14 \\ -320.083 & 168. & -320.083 & -168. & 14.5492 & 0 & 625.617 & 0 \\ 0 & -203.689 & 0 & -203.689 & 0 & -189.14 & 0 & 596.518 \end{pmatrix}$$

Valores propios matriz Ke={1044.48, 795.704, 630.466, 449.243, 251.84, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=-3

Gauss integration rule: 3

$$Ke=\begin{pmatrix} 489.541 & -21. & -140.926 & 21. & -68.9671 & 0 & -279.647 & 0 \\ -21. & 256.816 & -21. & -58.4173 & -126. & -13.2266 & 168. & -185.172 \\ -140.926 & -21. & 489.541 & 21. & -68.9671 & 0 & -279.647 & 0 \\ 21. & -58.4173 & 21. & 256.816 & 126. & -13.2266 & -168. & -185.172 \\ -68.9671 & -126. & -68.9671 & 126. & 143.603 & 0 & -5.66853 & 0 \\ 0 & -13.2266 & 0 & -13.2266 & 0 & 224.852 & 0 & -198.399 \\ -279.647 & 168. & -279.647 & -168. & -5.66853 & 0 & 564.963 & 0 \\ 0 & -185.172 & 0 & -185.172 & 0 & -198.399 & 0 & 568.743 \end{pmatrix}$$

Valores propios matriz Ke={941.92, 758.62, 630.466, 430.494, 233.373, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=3

Gauss integration rule: 6

$$Ke=\begin{pmatrix} 492.853 & -21. & -137.613 & 21. & -70.6235 & 0 & -284.617 & 0 \\ -21. & 258.463 & -21. & -56.7704 & -126. & -14.05 & 168. & -187.642 \\ -137.613 & -21. & 492.853 & 21. & -70.6235 & 0 & -284.617 & 0 \\ 21. & -56.7704 & 21. & 258.463 & 126. & -14.05 & -168. & -187.642 \\ -70.6235 & -126. & -70.6235 & 126. & 144.431 & 0 & -3.18394 & 0 \\ 0 & -14.05 & 0 & -14.05 & 0 & 225.263 & 0 & -197.163 \\ -284.617 & 168. & -284.617 & -168. & -3.18394 & 0 & 572.417 & 0 \\ 0 & -187.642 & 0 & -187.642 & 0 & -197.163 & 0 & 572.448 \end{pmatrix}$$

Valores propios matriz Ke={954.126, 763.417, 630.466, 433.196, 235.987, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=6

Gauss integration rule: -6

$$Ke=\begin{pmatrix} 492.236 & -21. & -138.23 & 21. & -70.315 & 0 & -283.691 & 0 \\ -21. & 258.05 & -21. & -57.1829 & -126. & -13.8438 & 168. & -187.024 \\ -138.23 & -21. & 492.236 & 21. & -70.315 & 0 & -283.691 & 0 \\ 21. & -57.1829 & 21. & 258.05 & 126. & -13.8438 & -168. & -187.024 \\ -70.315 & -126. & -70.315 & 126. & 144.277 & 0 & -3.64675 & 0 \\ 0 & -13.8438 & 0 & -13.8438 & 0 & 225.16 & 0 & -197.473 \\ -283.691 & 168. & -283.691 & -168. & -3.64675 & 0 & 571.029 & 0 \\ 0 & -187.024 & 0 & -187.024 & 0 & -197.473 & 0 & 571.52 \end{pmatrix}$$

Valores propios matriz Ke={951.843, 762.211, 630.466, 432.702, 235.337, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=-6

Gauss integration rule: 7

$$Ke=\begin{pmatrix} 492.865 & -21. & -137.601 & 21. & -70.6294 & 0 & -284.634 & 0 \\ -21. & 258.464 & -21. & -56.7688 & -126. & -14.0508 & 168. & -187.645 \\ -137.601 & -21. & 492.865 & 21. & -70.6294 & 0 & -284.634 & 0 \\ 21. & -56.7688 & 21. & 258.464 & 126. & -14.0508 & -168. & -187.645 \\ -70.6294 & -126. & -70.6294 & 126. & 144.434 & 0 & -3.17515 & 0 \\ 0 & -14.0508 & 0 & -14.0508 & 0 & 225.264 & 0 & -197.162 \\ -284.634 & 168. & -284.634 & -168. & -3.17515 & 0 & 572.444 & 0 \\ 0 & -187.645 & 0 & -187.645 & 0 & -197.162 & 0 & 572.452 \end{pmatrix}$$

Valores propios matriz Ke={954.169, 763.422, 630.466, 433.205, 235.989, 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=7

Gauss integration rule: 12

$$Ke=\begin{pmatrix} 492.875 & -21. & -137.591 & 21. & -70.6344 & 0 & -284.649 & 0 \\ -21. & 258.471 & -21. & -56.7623 & -126. & -14.0541 & 168. & -187.655 \\ -137.591 & -21. & 492.875 & 21. & -70.6344 & 0 & -284.649 & 0 \\ 21. & -56.7623 & 21. & 258.471 & 126. & -14.0541 & -168. & -187.655 \\ -70.6344 & -126. & -70.6344 & 126. & 144.436 & 0 & -3.16762 & 0 \\ 0 & -14.0541 & 0 & -14.0541 & 0 & 225.265 & 0 & -197.157 \\ -284.649 & 168. & -284.649 & -168. & -3.16762 & 0 & 572.466 & 0 \\ 0 & -187.655 & 0 & -187.655 & 0 & -197.157 & 0 & 572.466 \end{pmatrix}$$

Valores propios matriz Ke={954.207, 763.441, 630.466, 433.213, 236., 0, 0, 0}

TENEMOS LA SUFICIENCIA DE RANGO PARA p=12