ELEMENTO CUADRILATERO REGULAR - 9 NODOS

Implementación en Mathematica

v.2018

1. DATOS INICIALES

INICIO

In[1]:=

```
Off[General::"spell1"]
Off[General::"spell"]
```

In[3]:=

```
SetDirectory[NotebookDirectory[]]
```

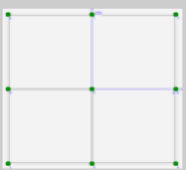
Out[3]=

```
C:\#0-Modulos-M30x_MeF-10\#M308-m6-a5a-swm\11-I-cuadrilatero-i
```

DEFINICION ELEMENTO CUADRILATERO REGULAR DE 9 NODOS

DEFINICION GRAFICA

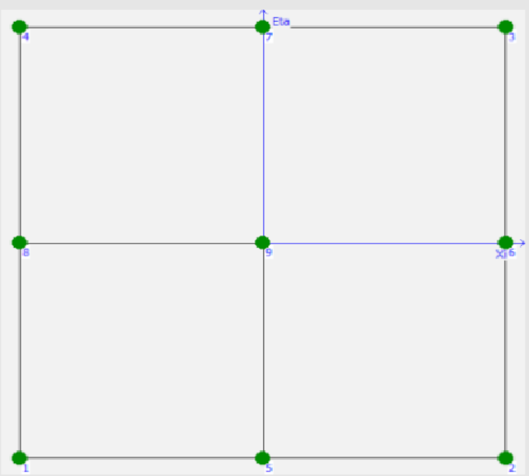
In[4]:=

CuaR9 =  ;

In[5]:=

```
CuaR9r = Show[CuaR9, ImageSize → 250]
```

Out[5]=



COORDENADAS NATURALES NODOS

In[6]:=

```
Cn = {{-1, -1}, {1, -1}, {1, 1}, {-1, 1}, {0, -1}, {1, 0}, {0, 1}, {-1, 0}, {0, 0}};
```

In[7]:=

```
NNodos = Dimensions[Cn][[1]]
```

Out[7]=

```
9
```

IMAGEN DEL ELEMENTO - COMPROBACION

FUNCION REPRESENTACION GRAFICA ELEMENTOS Y NODOS

In[8]:=
```
ElementPlot[b_List, options___] := Module[{asa, color, nr, circles, lines},
    asa = Select[{options}, ((! SameQ[#[[1]], NodeColor]) && (! SameQ[#[[1]], NodeSize])) &];
    {color, nr} = {NodeColor, NodeSize} /. {options} /.
      {NodeColor → GrayLevel[0], NodeSize → PointSize[0.06]};
    circles = Map[Point[#] &, Partition[Flatten[b], 2]];
    lines = Line[Append[b[[1]], First[b[[1]]]]];
    Show[Graphics[{nr, color, circles}], Graphics[lines], Evaluate[Sequence[##] &@@ asa]]];
```

DEFINICION VECTOR DE NODOS

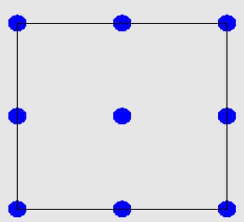
In[9]:=
```
ptsexteriores = {Cn[[1]], Cn[[5]], Cn[[2]], Cn[[6]], Cn[[3]], Cn[[7]], Cn[[4]], Cn[[8]]};
```

In[10]:=
```
ptsinteriores = {Cn[[9]]};
```

IMAGEN DE COMPROBACION

In[11]:=
```
Imagen = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → 1,
  PlotRange → {{-1.5, 1.5}, {-1.5, 1.5}}, ImageSize → 150, Frame → False, NodeColor → RGBColor[0, 0, 1]]
```

Out[11]=



3. FUNCIONES DE FORMA - METODO PRODUCTO DE CURVAS

CURVAS A CONSIDERAR

In[12]:=
```
Cu = Table[0, {i, 6}];
```

LADOS

In[13]:=
```
Cu[[1]] = (η + 1); Cu[[2]] = (ξ - 1); Cu[[3]] = (η - 1); Cu[[4]] = (ξ + 1);
```

MEDIANAS

In[14]:=
```
Cu[[5]] = η; Cu[[6]] = ξ;
```

DEFINICION PRODUCTOS DE CURVAS EN CADA NODO

In[15]:=
```
Nc = Table[0, {i, NNodos}];
```

Tipo 1 - ESQUINA

In[16]:=
```
Nc[[4]] = Cu[[1]] * Cu[[2]] * Cu[[5]] * Cu[[6]]
```
Out[16]= $\eta\,(1+\eta)\,(-1+\xi)\,\xi$

In[17]:=
```
Nc[[3]] = Cu[[1]] * Cu[[4]] * Cu[[5]] * Cu[[6]]
```
Out[17]= $\eta\,(1+\eta)\,\xi\,(1+\xi)$

In[18]:=
```
Nc[[2]] = Cu[[3]] * Cu[[4]] * Cu[[5]] * Cu[[6]]
```
Out[18]= $(-1+\eta)\,\eta\,\xi\,(1+\xi)$

In[19]:=
```
Nc[[1]] = Cu[[2]] * Cu[[3]] * Cu[[5]] * Cu[[6]]
```
Out[19]= $(-1+\eta)\,\eta\,(-1+\xi)\,\xi$

Tipo 2 - LADOS

In[20]:=
```
Nc[[5]] = Cu[[2]] * Cu[[3]] * Cu[[4]] * Cu[[5]]
```
Out[20]= $(-1+\eta)\,\eta\,(-1+\xi)\,(1+\xi)$

In[21]:=
```
Nc[[6]] = Cu[[1]] * Cu[[3]] * Cu[[4]] * Cu[[6]]
```
Out[21]= $(-1+\eta)\,(1+\eta)\,\xi\,(1+\xi)$

In[22]:=
```
Nc[[7]] = Cu[[1]] * Cu[[2]] * Cu[[4]] * Cu[[5]]
```
Out[22]= $\eta\,(1+\eta)\,(-1+\xi)\,(1+\xi)$

In[23]:=
```
Nc[[8]] = Cu[[1]] * Cu[[2]] * Cu[[3]] * Cu[[6]]
```
Out[23]= $(-1+\eta)\,(1+\eta)\,(-1+\xi)\,\xi$

Tipo 3 - INTERIOR

In[24]:=
```
Nc[[9]] = Cu[[1]] * Cu[[2]] * Cu[[3]] * Cu[[4]]
```
Out[24]= $(-1+\eta)\,(1+\eta)\,(-1+\xi)\,(1+\xi)$

OBTENCION FUNCIONES DE FORMA

In[25]:=
```
Clear[Nf]
```

In[26]:=
```
Nfp = Table[0, {i, NNodos}];
```

In[27]:=
```
Nf = Table[0, {i, NNodos}];
```

In[28]:=

```
Do[
  Nfp[[i]] = a * Nc[[i]];
  eq = 1 == Nfp[[i]] /. {ξ -> Cn[[i, 1]], η -> Cn[[i, 2]]};
  as = a /. Solve[eq, a][[1]]; Print["Nodo ", i];
  Nf[[i]] = Simplify[Nfp[[i]] /. {a → as}],
  {i, NNodos}
 ];
```

Nodo 1

Nodo 2

Nodo 3

Nodo 4

Nodo 5

Nodo 6

Nodo 7

Nodo 8

Nodo 9

In[29]:=

```
MatrixForm[Nf]
```

Out[29]//MatrixForm=

$$\begin{pmatrix} \frac{1}{4}(-1+\eta)\,\eta\,(-1+\xi)\,\xi \\ \frac{1}{4}(-1+\eta)\,\eta\,\xi\,(1+\xi) \\ \frac{1}{4}\eta\,(1+\eta)\,\xi\,(1+\xi) \\ \frac{1}{4}\eta\,(1+\eta)\,(-1+\xi)\,\xi \\ -\frac{1}{2}(-1+\eta)\,\eta\,(-1+\xi)\,(1+\xi) \\ -\frac{1}{2}(-1+\eta)\,(1+\eta)\,\xi\,(1+\xi) \\ -\frac{1}{2}\eta\,(1+\eta)\,(-1+\xi)\,(1+\xi) \\ -\frac{1}{2}(-1+\eta)\,(1+\eta)\,(-1+\xi)\,\xi \\ (-1+\eta)\,(1+\eta)\,(-1+\xi)\,(1+\xi) \end{pmatrix}$$

COMPROBACION SUMA UNIDAD

In[30]:=

$$\text{Suma} = \sum_{i=1}^{\text{NNodos}} \text{Nf}[[i]]$$

Out[30]=

$$\frac{1}{4}(-1+\eta)\,\eta\,(-1+\xi)\,\xi-\frac{1}{2}(-1+\eta)\,(1+\eta)\,(-1+\xi)\,\xi+\frac{1}{4}\eta\,(1+\eta)\,(-1+\xi)\,\xi-\frac{1}{2}(-1+\eta)\,\eta\,(-1+\xi)\,(1+\xi)+(-1+\eta)\,(1+\eta)\,(-1+\xi)\,(1+\xi)-\frac{1}{2}\eta\,(1+\eta)\,(-1+\xi)\,(1+\xi)+\frac{1}{4}(-1+\eta)\,\eta\,\xi\,(1+\xi)-\frac{1}{2}(-1+\eta)\,(1+\eta)\,\xi\,(1+\xi)+\frac{1}{4}\eta\,(1+\eta)\,\xi\,(1+\xi)$$

In[31]:=

```
Simplify[%]
```

Out[31]=

1

OK.

▪ REPRESENTACION GRAFICA

▫ Función Representación Gráfica Funciones de Forma

▫ Representación Gráfica Funciones Forma Elemento.

In[33]:=
```
Ng = Table[0, {i, NNodos}];
```

In[34]:=
```
xyc1 = {0, 0, 0}; xyc2 = {3, 0, 0}; xyc3 = {3, 3, 0};
xyc4 = {0, 3, 0}; xyquad = N[{xyc1, xyc2, xyc3, xyc4, xyc1}];
```

Control de Cuadricula

In[35]:=
```
Nsub = 10;
```

In[36]:=
```
Do[
  fi[ξ_, η_] = Nf[[i]];
  Ng[[i]] = PlotQuadrilateralShapeFunction[xyquad, fi, Nsub, 1 / 2],
  {i, NNodos}
 ];
```

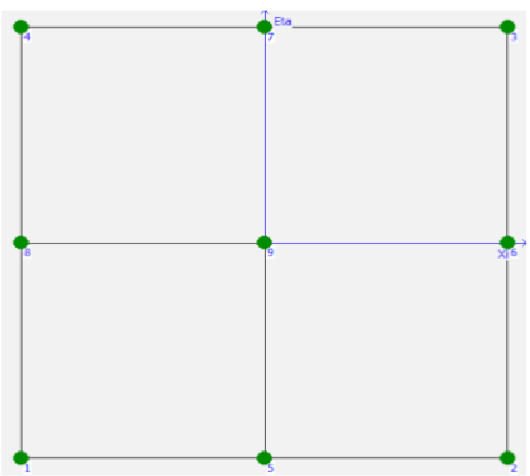
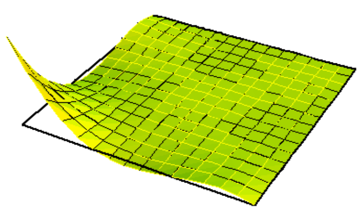
4. RESULTADOS INTERACTIVOS -

In[37]:=
```
Manipulate[{CuaR9r, Ng[[n]], Nf[[n]]}, {n, 1, Dimensions[Nf][[1]], 1}, {n, Range[Dimensions[Nf][[1]]]},
 FrameLabel → {"FUNCION DE FORMA EN NODO n - CUADRILATERO REGULAR 9 NODOS"}, SaveDefinitions → True]
```

Out[37]=

n

n 1

{ , ,

$$\frac{1}{256}(-1+\eta)(-1+3\eta)(1+3\eta)(-1+\xi)(-1+3\xi)(1+3\xi)\}$$

FUNCION DE FORMA EN NODO n – CUADRILATERO REGULAR 9 NODOS

5. DERIVADAS FUNCIONES DE FORMA Y JACOBIANO

■ FUNCIONES DE FORMA

In[38]:=
```
Nf
```

Out[38]=

$$\left\{\frac{1}{4}(-1+\eta)\,\eta\,(-1+\xi)\,\xi,\ \frac{1}{4}(-1+\eta)\,\eta\,\xi\,(1+\xi),\ \frac{1}{4}\eta\,(1+\eta)\,\xi\,(1+\xi),\ \frac{1}{4}\eta\,(1+\eta)\,(-1+\xi)\,\xi,\ -\frac{1}{2}(-1+\eta)\,\eta\,(-1+\xi)\,(1+\xi),\ -\frac{1}{2}(-1+\eta)\,(1+\eta)\,\xi\,(1+\xi),\ -\frac{1}{2}\eta\,(1+\eta)\,(-1+\xi)\,(1+\xi),\ -\frac{1}{2}(-1+\eta)\,(1+\eta)\,(-1+\xi)\,\xi,\ (-1+\eta)\,(1+\eta)\,(-1+\xi)\,(1+\xi)\right\}$$

■ DERIVADAS FUNCIONES DE FORMA / COORDENADAS NATURALES

▫ INICIALIZACIONES

In[39]:=
```
Clear[DNfξ, DNfη]
```

In[40]:=
```
DNfξ = Table[0, {i, NNodos}];
```

In[41]:=
```
DNfη = Table[0, {i, NNodos}];
```

▫ PROCESO DE CALCULO

In[42]:=
```
Do[
  Nfe = Expand[Nf[[i]]];
  DNfξ[[i]] = Simplify[D[Nf[[i]], ξ]];
  DNfη[[i]] = Simplify[D[Nf[[i]], η]];
  Clear[Nfe],
  {i, NNodos}
 ];
```

▫ RESULTADOS

In[43]:=
```
DNfξ
```

Out[43]=

$$\left\{\frac{1}{4}(-1+\eta)\,\eta\,(-1+2\,\xi),\ \frac{1}{4}(-1+\eta)\,\eta\,(1+2\,\xi),\ \frac{1}{4}\eta\,(1+\eta)\,(1+2\,\xi),\ \frac{1}{4}\eta\,(1+\eta)\,(-1+2\,\xi),\ -(-1+\eta)\,\eta\,\xi,\ -\frac{1}{2}\left(-1+\eta^2\right)(1+2\,\xi),\ -\eta\,(1+\eta)\,\xi,\ -\frac{1}{2}\left(-1+\eta^2\right)(-1+2\,\xi),\ 2\left(-1+\eta^2\right)\xi\right\}$$

In[44]:=
```
DNfη
```

Out[44]=

$$\left\{\frac{1}{4}(-1+2\,\eta)\,(-1+\xi)\,\xi,\ \frac{1}{4}(-1+2\,\eta)\,\xi\,(1+\xi),\ \frac{1}{4}(1+2\,\eta)\,\xi\,(1+\xi),\ \frac{1}{4}(1+2\,\eta)\,(-1+\xi)\,\xi,\ -\frac{1}{2}(-1+2\,\eta)\left(-1+\xi^2\right),\ -\eta\,\xi\,(1+\xi),\ -\frac{1}{2}(1+2\,\eta)\left(-1+\xi^2\right),\ -\eta\,(-1+\xi)\,\xi,\ 2\,\eta\left(-1+\xi^2\right)\right\}$$

■ MODULO CALCULO FUNCIONES DE FORMA Y JACOBIANO

▫ MODULO GENERICO A COMPLETAR

MODULO GENERICO A COMPLETAR: XX = No. Nodos

In[45]:=
```
(*QuadXXIsoPShapeFunDer[ncoor_,qcoor_]:=
  Module[{Nf,dNx,dNy,dNξ,dNη,J11,J12,J21,J22,Jdet,ξ,η,x,y},{ξ,η}=qcoor;
   Nf=(*Nf*);

   dNξ=(*dNξ*);

   dNη=(*dNη*);

   x=Table[ncoor[[i,1]],{i,XX}];y=Table[ncoor[[i,2]],{i,XX}];
   J11=dNξ.x;J21=dNξ.y;J12=dNη.x;J22=dNη.y;
   Jdet=Simplify[J11*J22-J12*J21];
   dNx=(J22*dNξ-J21*dNη)/Jdet;dNx=Simplify[dNx];
   dNy=(-J12*dNξ+J11*dNη)/Jdet;dNy=Simplify[dNy];
   Return[{Nf,dNx,dNy,Jdet}]];*)
```

MODULO COMPLETADO -

In[46]:=
```
Quad9IsoPShapeFunDer[ncoor_, qcoor_] :=
  Module[{Nf, dNx, dNy, dNξ, dNη, J11, J12, J21, J22, Jdet, ξ, η, x, y}, {ξ, η} = qcoor;
   Nf = {-1/4 (-1 + η) (-1 + ξ) (η + ξ + η ξ), -1/4 (-1 + η) (1 + ξ) (η (-1 + ξ) + ξ),
     1/4 (η + η^2 + ξ + η ξ + ξ^2 - η^2 ξ^2), -1/4 (1 + η) (-1 + ξ) (η - ξ + η ξ), (-1 + η) (1 + η) (-1 + ξ) (1 + ξ)};

   dNξ = {1/4 (-1 + η) η (-1 + 2 ξ), 1/4 (-1 + η) η (1 + 2 ξ), 1/4 η (1 + η) (1 + 2 ξ), 1/4 η (1 + η) (-1 + 2 ξ),
     -(-1 + η) η ξ, -1/2 (-1 + η^2) (1 + 2 ξ), -η (1 + η) ξ, -1/2 (-1 + η^2) (-1 + 2 ξ), 2 (-1 + η^2) ξ};

   dNη = {1/4 (-1 + 2 η) (-1 + ξ) ξ, 1/4 (-1 + 2 η) ξ (1 + ξ), 1/4 (1 + 2 η) ξ (1 + ξ), 1/4 (1 + 2 η) (-1 + ξ) ξ,
     -1/2 (-1 + 2 η) (-1 + ξ^2), -η ξ (1 + ξ), -1/2 (1 + 2 η) (-1 + ξ^2), -η (-1 + ξ) ξ, 2 η (-1 + ξ^2)};

   x = Table[ncoor[[i, 1]], {i, 9}]; y = Table[ncoor[[i, 2]], {i, 9}];
   J11 = dNξ.x; J21 = dNξ.y; J12 = dNη.x; J22 = dNη.y;
   Jdet = Simplify[J11 * J22 - J12 * J21];
   dNx = (J22 * dNξ - J21 * dNη) / Jdet; dNx = Simplify[dNx];
   dNy = (-J12 * dNξ + J11 * dNη) / Jdet; dNy = Simplify[dNy];
   Return[{Nf, dNx, dNy, Jdet}]];
```

6. MATRIZ DE RIGIDEZ - INTEGRACION NUMERICA

MODULO DE CALCULO DE LA MATRIZ DE RIGIDEZ

MODULO GENERICO A COMPLETAR

MODULO GENERICO: XX = No. Nodos, YY = Grados Libertad

```
(*QuadXXIsoPMembraneStiffness[ncoor_,mprop_,fprop_,options_]:=Module[{i,j,k,p=7,numer=False,
    Emat,th=1,h,qcoor,c,w,Nf,dNx,dNy,Jdet,B,Ke=Table[0,{YY},{YY}]},Emat=mprop[[1]];
   If[Length[options]==2,{numer,p}=options,{numer}=options];
   If[Length[fprop]>0,th=fprop[[1]]];
   If[p<1||p>4,Print["p out of range"];Return[Null]];
   For[k=1,k≤p*p,k++,
    {qcoor,w}=QuadGaussRuleInfo[{p,numer},k];
    {Nf,dNx,dNy,Jdet}=QuadXXIsoPShapeFunDer[ncoor,qcoor];
    If[Length[th]==0,h=th,h=th.Nf];c=w*Jdet*h;
    B={Flatten[Table[{dNx[[i]],0},{i,XX}]],
      Flatten[Table[{0,dNy[[i]]},{i,XX}]],Flatten[Table[{dNy[[i]],dNx[[i]]},{i,XX}]]};
    Ke+=Simplify[c*Transpose[B].(Emat.B)];];
   Return[Simplify[Ke]];*)
```

MODULO COMPLETADO -

In[80]:=

```
Quad9IsoPMembraneStiffness[ncoor_, mprop_, fprop_, options_] :=
  Module[{i, j, k, p = 7, numer = False, Emat, th = 1, h, qcoor, c, w,
    Nf, dNx, dNy, Jdet, B, Ke = Table[0, {18}, {18}]}, Emat = mprop[[1]];
   If[Length[options] == 2, {numer, p} = options, {numer} = options];
   If[Length[fprop] > 0, th = fprop[[1]]];
   If[p < 1 || p > 12, Print["p out of range"]; Return[Null]];
   For[k = 1, k ≤ p * p, k++,
    {qcoor, w} = QuadGaussRuleInfo[{p, numer}, k];
    {Nf, dNx, dNy, Jdet} = Quad9IsoPShapeFunDer[ncoor, qcoor];
    If[Length[th] == 0, h = th, h = th.Nf]; c = w * Jdet * h;
    B = {Flatten[Table[{dNx[[i]], 0}, {i, 9}]],
      Flatten[Table[{0, dNy[[i]]}, {i, 9}]], Flatten[Table[{dNy[[i]], dNx[[i]]}, {i, 9}]]};
    Ke += Simplify[c * Transpose[B].(Emat.B)];];
   Return[Simplify[Ke]];
```

MODULO REGLAS DE CUADRATURA DE GAUSS

OPCION 1: DEFINICION EN MATHEMATICA

In[81]:=

```
<< NumericalDifferentialEquationAnalysis`;
```

In[82]:=

```
? GaussianQuadratureWeights
```

GaussianQuadratureWeights[n, a, b, prec] gives a list of the pairs {abscissa, weight} to prec digits precision for the elementary n–point Gaussian quadrature formula for quadrature on the interval a to b. The argument prec is optional. >>

In[83]:=

```
QuadGaussRuleInfo[{rule_, numer_}, point_] := Module[{xi, eta, p1, p2, i1, i2, w1, w2, k, info = Null},
   If[Length[rule] == 2, {p1, p2} = rule, p1 = p2 = rule];
   If[Length[point] == 2, {i1, i2} = point, k = point; i2 = Floor[(k - 1) / p1] + 1; i1 = k - p1 * (i2 - 1)];
   {xi, w1} = GaussianQuadratureWeights[p1, -1, 1][[i1]];
   {eta, w2} = GaussianQuadratureWeights[p2, -1, 1][[i2]];
   info = {{xi, eta}, w1 * w2};
   If[numer, Return[N[info]], Return[Simplify[info]]];
  ];
```

In[84]:=

```
QuadGaussRuleInfo[{25, False}, 1]
```

Out[84]=

```
{{-0.995557, -0.995557}, 0.000129819}
```

OPCION 2: DEFINICION DE CARLOS FELIPPA

In[85]:=
```
QuadGaussRuleInfo[{rule_, numer_}, point_] := Module[{xi, eta, p1, p2, i1, i2, w1, w2, k, info = Null},
   If[Length[rule] == 2, {p1, p2} = rule, p1 = p2 = rule];
   If[Length[point] == 2, {i1, i2} = point, k = point; i2 = Floor[(k - 1) / p1] + 1; i1 = k - p1 * (i2 - 1)];
   {xi, w1} = LineGaussRuleInfo[{p1, numer}, i1];
   {eta, w2} = LineGaussRuleInfo[{p2, numer}, i2];
   info = {{xi, eta}, w1 * w2};
   If[numer, Return[N[info]], Return[Simplify[info]]];];
```

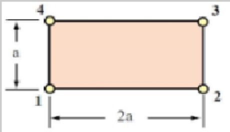
In[86]:=
```
LineGaussRuleInfo[{rule_, numer_}, point_] :=
  Module[{g2 = {-1, 1} / Sqrt[3], w3 = {5 / 9, 8 / 9, 5 / 9}, g3 = {-Sqrt[3 / 5], 0, Sqrt[3 / 5]},
    w4 = {(1 / 2) - Sqrt[5 / 6] / 6, (1 / 2) + Sqrt[5 / 6] / 6, (1 / 2) + Sqrt[5 / 6] / 6, (1 / 2) - Sqrt[5 / 6] / 6},
    g4 = {-Sqrt[(3 + 2 * Sqrt[6 / 5]) / 7], -Sqrt[(3 - 2 * Sqrt[6 / 5]) / 7],
      Sqrt[(3 - 2 * Sqrt[6 / 5]) / 7], Sqrt[(3 + 2 * Sqrt[6 / 5]) / 7]}, g5 = {-Sqrt[5 + 2 * Sqrt[10 / 7]],
       -Sqrt[5 - 2 * Sqrt[10 / 7]], 0, Sqrt[5 - 2 * Sqrt[10 / 7]], Sqrt[5 + 2 * Sqrt[10 / 7]]} / 3,
    w5 = {322 - 13 * Sqrt[70], 322 + 13 * Sqrt[70], 512, 322 + 13 * Sqrt[70], 322 - 13 * Sqrt[70]} / 900,
    i = point, p = rule, info = {Null, 0}},
   If[p == 1, info = {0, 2}];
   If[p == 2, info = {g2[[i]], 1}];
   If[p == 3, info = {g3[[i]], w3[[i]]}];
   If[p == 4, info = {g4[[i]], w4[[i]]}];
   If[p == 5, info = {g5[[i]], w5[[i]]}];
   If[numer, Return[N[info]], Return[Simplify[info]]];
  ];
```

In[87]:=
```
QuadGaussRuleInfo[{2, False}, 1]
```

Out[87]= $\left\{\left\{-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right\}, 1\right\}$

7. TEST DEL RECTANGULO

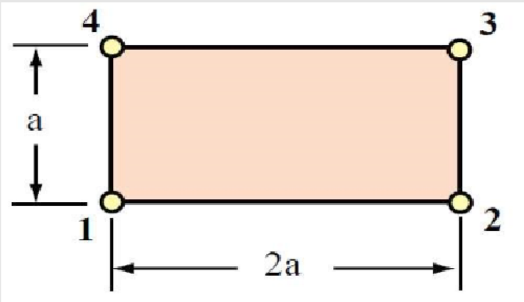
DEFINICION DE LA GEOMETRIA

In[57]:=
```
RectanguloT = 
```


;

In[58]:=
```
RectanguloTr = Show[RectanguloT, ImageSize → 250]
```

Out[58]=



DEFINCION COORDENADAS NODOS ELEMENTO - ANSYS CLASSIC

IMAGEN DEL MODELO EN ANSYS

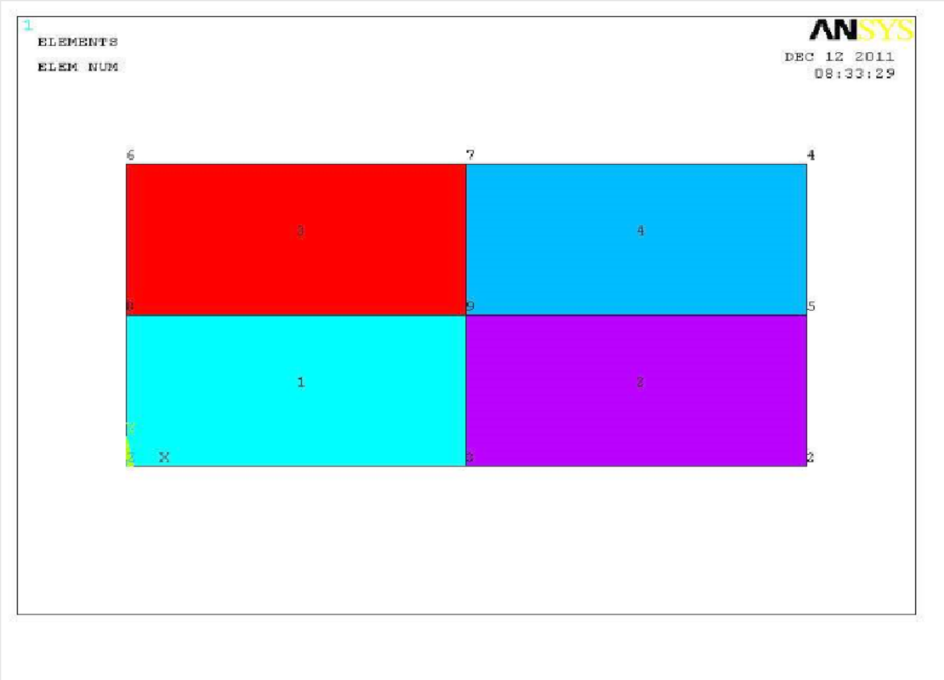
In[59]:=

RectanguloA =  ;

In[60]:=

```
RectanguloAr = Show[RectanguloA, ImageSize → 450]
```

Out[60]=



DATOS COORDENADAS NODALES EN ANSYS

In[61]:=

```
Nodes = {{0.`, 0.`}, {1.`, 0.`}, {0.5`, 0.`}, {1.`, 0.5`},
   {1.`, 0.25`}, {0.`, 0.5`}, {0.5`, 0.5`}, {0.`, 0.25`}, {0.5`, 0.25`}};
```

In[62]:=

```
Dimensions[Nodes][[1]]
```

Out[62]=

```
9
```

ORDENACION DATOS DE ANSYS

In[63]:=

```
CuaT5r
```

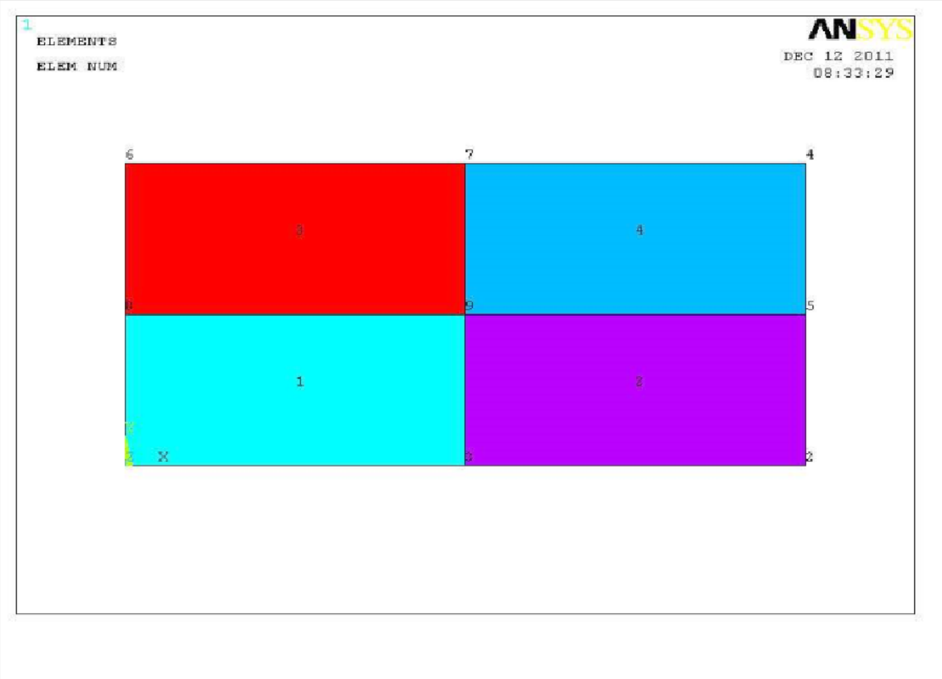
Out[63]=

```
CuaT5r
```

In[64]:=

```
RectanguloAr
```

Out[64]=



In[65]:=

```
ncoor = {Nodes[[1]], Nodes[[2]], Nodes[[4]],
  Nodes[[6]], Nodes[[3]], Nodes[[5]], Nodes[[7]], Nodes[[8]], Nodes[[9]]}
```

Out[65]=

```
{{0., 0.}, {1., 0.}, {1., 0.5}, {0., 0.5}, {0.5, 0.}, {1., 0.25}, {0.5, 0.5}, {0., 0.25}, {0.5, 0.25}}
```

DATOS COORDENADAS NODALES - SEGUN TEMA 23

In[66]:=

```
a = 1 / 2;
```

In[67]:=

```
ncoor = {{0, 0}, {2 * a, 0}, {2 * a, a}, {0, a}, {a, 0}, {2 * a, a / 2}, {a, a}, {0, a / 2}, {a, a / 2}};
```

DEFINICION DEL MATERIAL

In[68]:=

```
ClearAll[Em, nu, a, b, e, h, p, num]; h = 1;
```

DATOS SEGUN ENUNCIADO

In[69]:=

```
Em = 96 * 39 * 11 * 55 * 7; nu = 1 / 3;  (*isotropic material*)
```

In[70]:=

```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

In[71]:=

```
Emat // MatrixForm
```

Out[71]//MatrixForm=

$$\begin{pmatrix} 17\,837\,820 & 5\,945\,940 & 0 \\ 5\,945\,940 & 17\,837\,820 & 0 \\ 0 & 0 & 5\,945\,940 \end{pmatrix}$$

VERIFICACION DE LA MATRIZ DE RIGIDEZ

NUMERO DE PUNTOS DE GAUSS MINIMO PARA CONSEGUIR SUFICIENCIA DE RANGO

In[72]:=
```
NF = NNodos * 2.;
```

In[73]:=
$$NG = \frac{NF - 3}{3}$$

Out[73]=
```
5.
```

Se necesitan como mínimo 5 Puntos -- Regla 3 x 3 minima

BUCLE GENERICO A COMPLETAR

BUCLE GENERICO: XX = No. Nodos, ZZ = GRADOS DE LIBERTAD - 3

```
(*For [p=1, p≤5, p++,
 Ke=QuadXXIsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
 Print["Gauss integration rule: ",p," x ",p];
 Print["Ke=",Chop[Ke]//MatrixForm];
 Valores=Chop[Eigenvalues[N[Ke]],10^-7];
 If[Valores[[ZZ]]≠0,Break[],Print["Valores proprios matriz Ke=",Valores]]
];
Print["Valores proprios matriz Ke=",Valores];
Print["tenemos la suficiencia de rango para p=",p]*)
```

DESARROLLO DE LA MATRIZ DE RIGIDEZ -

In[105]:=
```
? Chop
```

Chop[*expr*] replaces approximate real numbers in *expr* that are close to zero by the exact integer 0. >>

In[114]:=
```
For [p = 1, p ≤ 5, p++,
   Ke = Quad9IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {False, p}];
   Print["Gauss integration rule: ", p, " x ", p];
   Print["Ke=", Chop[Ke] // MatrixForm];
   Valores = Chop[Eigenvalues[N[Ke]], 10^-7];
   If[Valores[[15]] ≠ 0, Break[], Print["Valores proprios matriz Ke=", Valores]]
  ];
Print["Valores proprios matriz Ke=", Valores];
Print["tenemos la suficiencia de rango para p=", p]
```

```
Gauss integration rule: 1 x 1
```

Ke=

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	11 891 880	0	0	−5 945 940	−11 891 880	0	0	5 94
0	0	0	0	0	0	0	0	0	35 675 640	−5 945 940	0	0	−35 675 640	5 945 940	0
0	0	0	0	0	0	0	0	0	−5 945 940	8 918 910	0	0	5 945 940	−8 918 910	0
0	0	0	0	0	0	0	0	−5 945 940	0	0	2 972 970	5 945 940	0	0	−2 97
0	0	0	0	0	0	0	0	−11 891 880	0	0	5 945 940	11 891 880	0	0	−5 94
0	0	0	0	0	0	0	0	0	−35 675 640	5 945 940	0	0	35 675 640	−5 945 940	0
0	0	0	0	0	0	0	0	0	5 945 940	−8 918 910	0	0	−5 945 940	8 918 910	0
0	0	0	0	0	0	0	0	5 945 940	0	0	−2 972 970	−5 945 940	0	0	2 972
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Valores proprios matriz Ke=

$\{7.38749\times 10^{7}, 2.97297\times 10^{7}, 1.53142\times 10^{7}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$

Gauss integration rule: 2 x 2

Ke=

5 395 390	2 972 970	−1 211 210	0	−385 385	−330 330	−715 715	0	440 440	
2 972 970	10 020 010	0	−4 514 510	−330 330	−715 715	0	935 935	0	8 3
−1 211 210	0	5 395 390	−2 972 970	−715 715	0	−385 385	330 330	440 440	
0	−4 514 510	−2 972 970	10 020 010	0	935 935	330 330	−715 715	0	8 3
−385 385	−330 330	−715 715	0	5 395 390	2 972 970	−1 211 210	0	1 761 760	1 3
−330 330	−715 715	0	935 935	2 972 970	10 020 010	0	−4 514 510	1 321 320	1 7
−715 715	0	−385 385	330 330	−1 211 210	0	5 395 390	−2 972 970	1 761 760	−1
0	935 935	330 330	−715 715	0	−4 514 510	−2 972 970	10 020 010	−1 321 320	1 7
440 440	0	440 440	0	1 761 760	1 321 320	1 761 760	−1 321 320	17 617 600	
0	8 368 360	0	8 368 360	1 321 320	1 761 760	−1 321 320	1 761 760	0	38
2 092 090	1 321 320	−1 211 210	0	−1 211 210	0	2 092 090	−1 321 320	−6 166 160	−5
1 321 320	5 395 390	0	−9 799 790	0	−9 799 790	−1 321 320	5 395 390	−5 285 280	−11
1 761 760	1 321 320	1 761 760	−1 321 320	440 440	0	440 440	0	−880 880	
1 321 320	1 761 760	−1 321 320	1 761 760	0	8 368 360	0	8 368 360	0	4 4
−1 211 210	0	2 092 090	−1 321 320	2 092 090	1 321 320	−1 211 210	0	−6 166 160	5 2
0	−9 799 790	−1 321 320	5 395 390	1 321 320	5 395 390	0	−9 799 790	5 285 280	−11
−6 166 160	−5 285 280	−6 166 160	5 285 280	−6 166 160	−5 285 280	−6 166 160	5 285 280	−8 808 800	
−5 285 280	−11 451 440	5 285 280	−11 451 440	−5 285 280	−11 451 440	5 285 280	−11 451 440	0	−40

Valores proprios matriz Ke=

$\{1.44762\times 10^{8}, 5.92079\times 10^{7}, 5.18636\times 10^{7}, 4.15006\times 10^{7}, 2.69107\times 10^{7}, 1.78764\times 10^{7}, 1.67116\times 10^{7},$
$1.49247\times 10^{7}, 1.00074\times 10^{7}, 5.74907\times 10^{6}, 5.03739\times 10^{6}, 1.84505\times 10^{6}, 0, 0, 0, 0, 0, 0\}$

Gauss integration rule: 3 x 3

Ke=

6 474 468	2 972 970	−528 528	0	−231 231	−330 330	−165 165	0	−1 321 320	
2 972 970	12 024 012	0	−2 642 640	−330 330	−429 429	0	1 354 353	0	4
−528 528	0	6 474 468	−2 972 970	−165 165	0	−231 231	330 330	−1 321 320	
0	−2 642 640	−2 972 970	12 024 012	0	1 354 353	330 330	−429 429	0	4
−231 231	−330 330	−165 165	0	6 474 468	2 972 970	−528 528	0	1 057 056	1
−330 330	−429 429	0	1 354 353	2 972 970	12 024 012	0	−2 642 640	1 321 320	1
−165 165	0	−231 231	330 330	−528 528	0	6 474 468	−2 972 970	1 057 056	−
0	1 354 353	330 330	−429 429	0	−2 642 640	−2 972 970	12 024 012	−1 321 320	1
−1 321 320	0	−1 321 320	0	1 057 056	1 321 320	1 057 056	−1 321 320	21 141 120	
0	4 492 488	0	4 492 488	1 321 320	1 057 056	−1 321 320	1 057 056	0	4
1 255 254	1 321 320	−2 840 838	0	−2 840 838	0	1 255 254	−1 321 320	−3 699 696	−
1 321 320	3 237 234	0	−12 222 210	0	−12 222 210	−1 321 320	3 237 234	−5 285 280	−
1 057 056	1 321 320	1 057 056	−1 321 320	−1 321 320	0	−1 321 320	0	528 528	
1 321 320	1 057 056	−1 321 320	1 057 056	0	4 492 488	0	4 492 488	0	5
−2 840 838	0	1 255 254	−1 321 320	1 255 254	1 321 320	−2 840 838	0	−3 699 696	5
0	−12 222 210	−1 321 320	3 237 234	1 321 320	3 237 234	0	−12 222 210	5 285 280	−
−3 699 696	−5 285 280	−3 699 696	5 285 280	−3 699 696	−5 285 280	−3 699 696	5 285 280	−13 741 728	
−5 285 280	−6 870 864	5 285 280	−6 870 864	−5 285 280	−6 870 864	5 285 280	−6 870 864	0	−4

```
Valores proprios matriz Ke={1.61935×10^8, 7.14971×10^7, 5.28465×10^7,
  4.58743×10^7, 2.78483×10^7, 2.71851×10^7, 1.95926×10^7, 1.8804×10^7, 1.21446×10^7,
  1.13707×10^7, 7.49744×10^6, 7.36709×10^6, 6.63769×10^6, 3.47217×10^6, 1.60241×10^6, 0, 0, 0}

tenemos la suficiencia de rango para p=3
```

In[118]:=

```
For [p = 4, p ≤ 5, p++,
  Ke = Quad9IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
  Print["Gauss integration rule: ", p, " x ", p];
  Print["Ke=", Chop[Ke] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]], 10^-7];
  Print["Valores proprios matriz Ke=", Valores];
  Print["tenemos la suficiencia de rango para p=", p]
 ];
```

Gauss integration rule: 4 x 4

$$Ke=\begin{pmatrix}
6.47447\times10^{6} & 2.97297\times10^{6} & -528528. & 1.35515\times10^{-10} & -231231. & -330330. & -1 \\
2.97297\times10^{6} & 1.2024\times10^{7} & 1.35515\times10^{-10} & -2.64264\times10^{6} & -330330. & -429429. & \\
-528528. & 0 & 6.47447\times10^{6} & -2.97297\times10^{6} & -165165. & -1.45519\times10^{-10} & -2 \\
0 & -2.64264\times10^{6} & -2.97297\times10^{6} & 1.2024\times10^{7} & -1.45519\times10^{-10} & 1.35435\times10^{6} & 3 \\
-231231. & -330330. & -165165. & 0 & 6.47447\times10^{6} & 2.97297\times10^{6} & -5 \\
-330330. & -429429. & 0 & 1.35435\times10^{6} & 2.97297\times10^{6} & 1.2024\times10^{7} & \\
-165165. & 0 & -231231. & 330330. & -528528. & 0 & 6.4 \\
0 & 1.35435\times10^{6} & 330330. & -429429. & 0 & -2.64264\times10^{6} & -2.9 \\
-1.32132\times10^{6} & 0 & -1.32132\times10^{6} & 6.93035\times10^{-10} & 1.05706\times10^{6} & 1.32132\times10^{6} & 1.0 \\
0 & 4.49249\times10^{6} & 6.93035\times10^{-10} & 4.49249\times10^{6} & 1.32132\times10^{6} & 1.05706\times10^{6} & -1.3 \\
1.25525\times10^{6} & 1.32132\times10^{6} & -2.84084\times10^{6} & -2.91038\times10^{-10} & -2.84084\times10^{6} & -1.16415\times10^{-10} & 1.2 \\
1.32132\times10^{6} & 3.23723\times10^{6} & -2.91038\times10^{-10} & -1.22222\times10^{7} & -1.16415\times10^{-10} & -1.22222\times10^{7} & -1.3 \\
1.05706\times10^{6} & 1.32132\times10^{6} & 1.05706\times10^{6} & -1.32132\times10^{6} & -1.32132\times10^{6} & -5.82077\times10^{-10} & -1.3 \\
1.32132\times10^{6} & 1.05706\times10^{6} & -1.32132\times10^{6} & 1.05706\times10^{6} & -5.82077\times10^{-10} & 4.49249\times10^{6} & -1.4 \\
-2.84084\times10^{6} & -7.01675\times10^{-10} & 1.25525\times10^{6} & -1.32132\times10^{6} & 1.25525\times10^{6} & 1.32132\times10^{6} & -2.8 \\
-7.01675\times10^{-10} & -1.22222\times10^{7} & -1.32132\times10^{6} & 3.23723\times10^{6} & 1.32132\times10^{6} & 3.23723\times10^{6} & 9.38 \\
-3.6997\times10^{6} & -5.28528\times10^{6} & -3.6997\times10^{6} & 5.28528\times10^{6} & -3.6997\times10^{6} & -5.28528\times10^{6} & -3. \\
-5.28528\times10^{6} & -6.87086\times10^{6} & 5.28528\times10^{6} & -6.87086\times10^{6} & -5.28528\times10^{6} & -6.87086\times10^{6} & 5.2
\end{pmatrix}$$

Valores proprios matriz Ke= $\{1.61935\times10^{8}, 7.14971\times10^{7}, 5.28465\times10^{7}, 4.58743\times10^{7}, 2.78483\times10^{7}, 2.71851\times10^{7}, 1.95926\times10^{7}, 1.8804\times10^{7}, 1.21446\times10^{7}, 1.13707\times10^{7}, 7.49744\times10^{6}, 7.36709\times10^{6}, 6.63769\times10^{6}, 3.47217\times10^{6}, 1.60241\times10^{6}, 0, 0, 0\}$

tenemos la suficiencia de rango para p=4

Gauss integration rule: 5 x 5

$$Ke=\begin{pmatrix}
6.47447\times10^{6} & 2.97297\times10^{6} & -528528. & 0 & -231231. & -330330. & -1651 \\
2.97297\times10^{6} & 1.2024\times10^{7} & 0 & -2.64264\times10^{6} & -330330. & -429429. & 0 \\
-528528. & 0 & 6.47447\times10^{6} & -2.97297\times10^{6} & -165165. & 0 & -2312 \\
0 & -2.64264\times10^{6} & -2.97297\times10^{6} & 1.2024\times10^{7} & 0 & 1.35435\times10^{6} & 33033 \\
-231231. & -330330. & -165165. & 0 & 6.47447\times10^{6} & 2.97297\times10^{6} & -5285 \\
-330330. & -429429. & 0 & 1.35435\times10^{6} & 2.97297\times10^{6} & 1.2024\times10^{7} & -1.74623 \\
-165165. & 0 & -231231. & 330330. & -528528. & -2.03727\times10^{-10} & 6.47447 \\
0 & 1.35435\times10^{6} & 330330. & -429429. & -2.03727\times10^{-10} & -2.64264\times10^{6} & -2.9729 \\
-1.32132\times10^{6} & 1.00044\times10^{-10} & -1.32132\times10^{6} & 0 & 1.05706\times10^{6} & 1.32132\times10^{6} & 1.05706 \\
1.00044\times10^{-10} & 4.49249\times10^{6} & 0 & 4.49249\times10^{6} & 1.32132\times10^{6} & 1.05706\times10^{6} & -1.3213 \\
1.25525\times10^{6} & 1.32132\times10^{6} & -2.84084\times10^{6} & -1.13505\times10^{-9} & -2.84084\times10^{6} & 6.98492\times10^{-10} & 1.25525 \\
1.32132\times10^{6} & 3.23723\times10^{6} & -1.13505\times10^{-9} & -1.22222\times10^{7} & 6.98492\times10^{-10} & -1.22222\times10^{7} & -1.3213 \\
1.05706\times10^{6} & 1.32132\times10^{6} & 1.05706\times10^{6} & -1.32132\times10^{6} & -1.32132\times10^{6} & 1.16415\times10^{-9} & -1.3213 \\
1.32132\times10^{6} & 1.05706\times10^{6} & -1.32132\times10^{6} & 1.05706\times10^{6} & 1.16415\times10^{-9} & 4.49249\times10^{6} & -8.44011 \\
-2.84084\times10^{6} & 9.34961\times10^{-10} & 1.25525\times10^{6} & -1.32132\times10^{6} & 1.25525\times10^{6} & 1.32132\times10^{6} & -2.8408 \\
9.34961\times10^{-10} & -1.22222\times10^{7} & -1.32132\times10^{6} & 3.23723\times10^{6} & 1.32132\times10^{6} & 3.23723\times10^{6} & 0 \\
-3.6997\times10^{6} & -5.28528\times10^{6} & -3.6997\times10^{6} & 5.28528\times10^{6} & -3.6997\times10^{6} & -5.28528\times10^{6} & -3.6997 \\
-5.28528\times10^{6} & -6.87086\times10^{6} & 5.28528\times10^{6} & -6.87086\times10^{6} & -5.28528\times10^{6} & -6.87086\times10^{6} & 5.28528
\end{pmatrix}$$

Valores proprios matriz Ke= $\{1.61935\times10^{8}, 7.14971\times10^{7}, 5.28465\times10^{7}, 4.58743\times10^{7}, 2.78483\times10^{7}, 2.71851\times10^{7}, 1.95926\times10^{7}, 1.8804\times10^{7}, 1.21446\times10^{7}, 1.13707\times10^{7}, 7.49744\times10^{6}, 7.36709\times10^{6}, 6.63769\times10^{6}, 3.47217\times10^{6}, 1.60241\times10^{6}, 0, 0, 0\}$

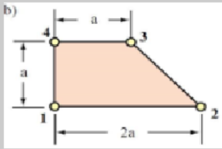
tenemos la suficiencia de rango para p=5

8. TEST DEL TRAPECIO

DEFINICION DE LA GEOMETRIA

In[119]:=

```
TrapecioT = 
```

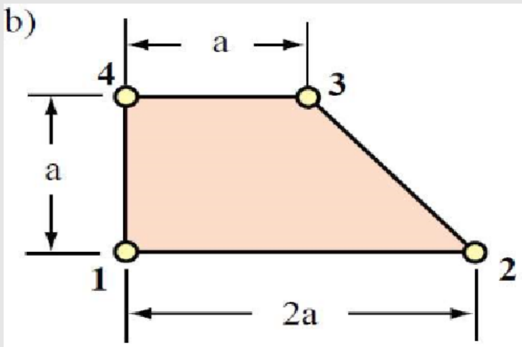



```
;
```

In[120]:=

```
TrapecioTr = Show[TrapecioT, ImageSize → 250]
```

Out[120]=

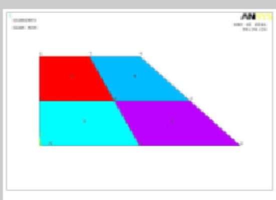


DEFINCION COORDENADAS NODOS ELEMENTO - ANSYS CLASSIC

IMAGEN DEL MODELO EN ANSYS

In[121]:=

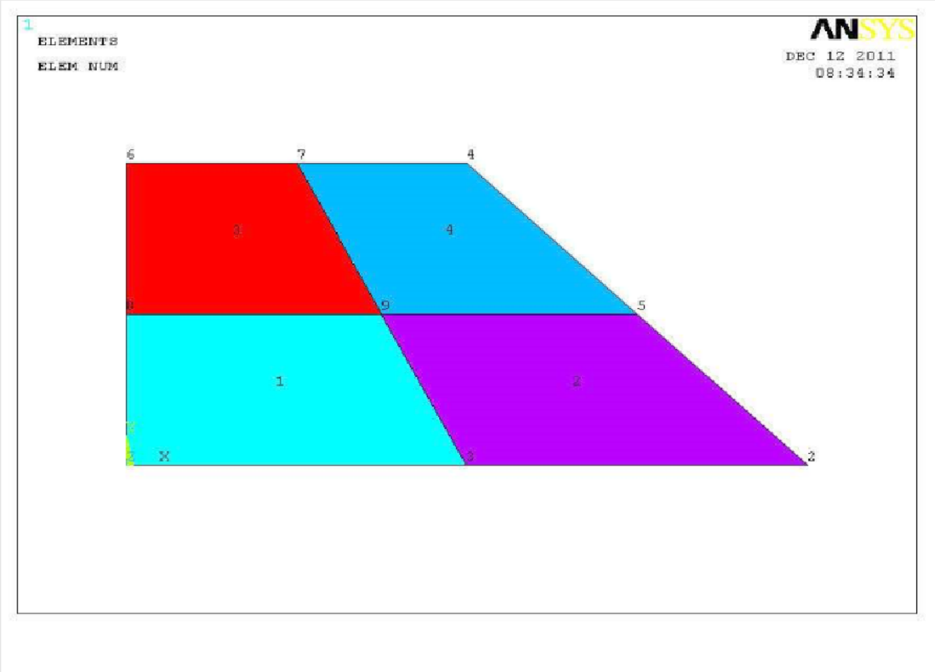
```
TrapecioA = 
```



```
;
```

In[122]:=
```
TrapecioAr = Show[TrapecioA, ImageSize → 450]
```

Out[122]=



▫ DATOS COORDENADAS NODALES EN ANSYS

In[123]:=
```
Nodes = {{0.`, 0.`}, {1.`, 0.`}, {0.5`, 0.`}, {0.5`, 0.5`},
   {0.75`, 0.25`}, {0.`, 0.5`}, {0.25`, 0.5`}, {0.`, 0.25`}, {(0.25` + 0.5`) / 2, 0.25`}};
```

In[124]:=
```
Dimensions[Nodes][[1]]
```

Out[124]=
```
9
```

▫ ORDENACION DATOS DE ANSYS

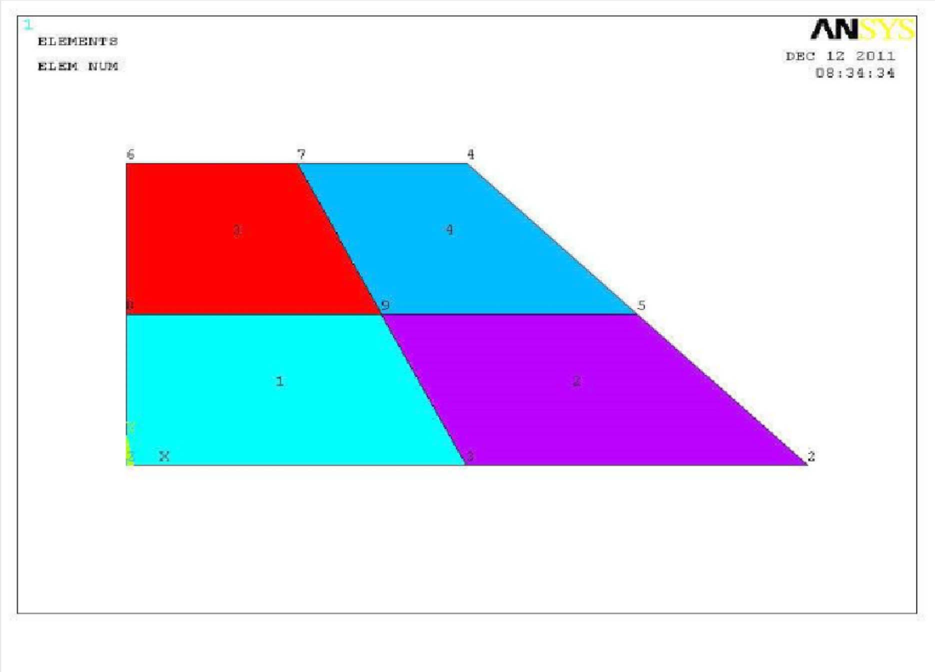
In[125]:=
```
CuaR4r
```

Out[125]=
```
CuaR4r
```

In[126]:=

```
TrapecioAr
```

Out[126]=



In[137]:=

```
ncoor = {Nodes[[1]], Nodes[[2]], Nodes[[4]],
   Nodes[[6]], Nodes[[3]], Nodes[[5]], Nodes[[7]], Nodes[[8]], Nodes[[9]]}
```

Out[137]=

```
{{0., 0.}, {1., 0.}, {0.5, 0.5}, {0., 0.5},
 {0.5, 0.}, {0.75, 0.25}, {0.25, 0.5}, {0., 0.25}, {0.375, 0.25}}
```

DEFINICION DEL MATERIAL

In[138]:=

```
ClearAll[Em, nu, a, b, e, h, p, num]; h = 1;
```

DATOS SEGUN ENUNCIADO

In[139]:=

```
Em = 96 * 39 * 11 * 55 * 7; nu = 1 / 3;  (*isotropic material*)
```

In[140]:=

```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

In[141]:=

```
Emat // MatrixForm
```

Out[141]//MatrixForm=

$$\begin{pmatrix} 17\,837\,820 & 5\,945\,940 & 0 \\ 5\,945\,940 & 17\,837\,820 & 0 \\ 0 & 0 & 5\,945\,940 \end{pmatrix}$$

VERIFICACION DE LA MATRIZ DE RIGIDEZ

NUMERO DE PUNTOS DE GAUSS MINIMO PARA CONSEGUIR SUFICIENCIA DE RANGO

In[142]:=

```
NF = NNodos * 2.;
```

In[143]:=

$$NG = \frac{NF - 3}{3}$$

Out[143]=

5.

Se necesitan como mínimo 5 Puntos -- Regla 3 x 32 minima

BUCLE GENERICO A COMPLETAR

BUCLE GENERICO: XX = No. Nodos, ZZ = GRADOS DE LIBERTAD - 3

```
(*For [p=1, p≤5, p++,
 Ke=QuadXXIsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
 Print["Gauss integration rule: ",p," x ",p];
 Print["Ke=",Chop[Ke]//MatrixForm];
 Valores=Chop[Eigenvalues[N[Ke]]];
 If[Valores[[ZZ]]≠0,Break[],Print["Valores proprios matriz Ke=",Valores]]
];
Print["Valores proprios matriz Ke=",Valores];
Print["tenemos la suficiencia de rango para p=",p]*)
```

DESARROLLO DE LA MATRIZ DE RIGIDEZ -

In[144]:=

```
For [p = 1, p ≤ 5, p++,
   Ke = Quad9IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
   Print["Gauss integration rule: ", p, " x ", p];
   Print["Ke=", Chop[Ke] // MatrixForm];
   Valores = Chop[Eigenvalues[N[Ke]], 10^-7];
   If[Valores[[15]] ≠ 0, Break[], Print["Valores proprios matriz Ke=", Valores]]
  ];
Print["Valores proprios matriz Ke=", Valores];
Print["tenemos la suficiencia de rango para p=", p]
```

Gauss integration rule: 1 x 1

Ke=

0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	8.91891×10^{6}	0	-2.97297×10^{6}	-5.94594×10^{6}	-8.91891×10^{6}	0
0	0	0	0	0	0	0	0	0	2.67567×10^{7}	-5.94594×10^{6}	-8.91891×10^{6}	0	$-2.67567\times$
0	0	0	0	0	0	0	0	-2.97297×10^{6}	-5.94594×10^{6}	1.28829×10^{7}	3.96396×10^{6}	2.97297×10^{6}	$5.94594\times$
0	0	0	0	0	0	0	0	-5.94594×10^{6}	-8.91891×10^{6}	3.96396×10^{6}	6.93693×10^{6}	5.94594×10^{6}	$8.91891\times$
0	0	0	0	0	0	0	0	-8.91891×10^{6}	0	2.97297×10^{6}	5.94594×10^{6}	8.91891×10^{6}	0
0	0	0	0	0	0	0	0	0	-2.67567×10^{7}	5.94594×10^{6}	8.91891×10^{6}	0	$2.67567\times$
0	0	0	0	0	0	0	0	2.97297×10^{6}	5.94594×10^{6}	-1.28829×10^{7}	-3.96396×10^{6}	-2.97297×10^{6}	$-5.94594\times$
0	0	0	0	0	0	0	0	5.94594×10^{6}	8.91891×10^{6}	-3.96396×10^{6}	-6.93693×10^{6}	-5.94594×10^{6}	$-8.91891\times$
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

Valores proprios matriz Ke=

$\{6.61373\times10^{7}, 2.77477\times10^{7}, 1.71058\times10^{7}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$

Gauss integration rule: 2 x 2

$$Ke=\begin{pmatrix}
6.12381\times10^{6} & 3.35412\times10^{6} & -787710. & 127050. & -584430. & -406560. & -1.29591\times10^{6} \\
3.35412\times10^{6} & 1.12566\times10^{7} & 127050. & -3.37953\times10^{6} & -406560. & -1.14345\times10^{6} & -228690. \\
-787710. & 127050. & 3.65904\times10^{6} & -1.57542\times10^{6} & -1.60083\times10^{6} & -838530. & -254100. \\
127050. & -3.37953\times10^{6} & -1.57542\times10^{6} & 3.86232\times10^{6} & -838530. & -533610. & 254100. \\
-584430. & -406560. & -1.60083\times10^{6} & -838530. & 8.69022\times10^{6} & 4.92954\times10^{6} & -508200. \\
-406560. & -1.14345\times10^{6} & -838530. & -533610. & 4.92954\times10^{6} & 1.61099\times10^{7} & 177870. \\
-1.29591\times10^{6} & -228690. & -254100. & 254100. & -508200. & 177870. & 5.00577\times10^{6} \\
-228690. & 381150. & 254100. & -152460. & 177870. & -2.94756\times10^{6} & -2.43936\times10^{6} \\
-711480. & -508200. & -1.21968\times10^{6} & -1.5246\times10^{6} & 3.50658\times10^{6} & 2.23608\times10^{6} & 1.88034\times10^{6} \\
-508200. & 5.99676\times10^{6} & -1.5246\times10^{6} & 4.47216\times10^{6} & 2.23608\times10^{6} & 5.64102\times10^{6} & -1.0164\times10^{6} \\
2.541\times10^{6} & 1.42296\times10^{6} & -508200. & 1.11804\times10^{6} & 2.94756\times10^{6} & 2.23608\times10^{6} & 1.11804\times10^{6} \\
1.42296\times10^{6} & 6.80988\times10^{6} & 1.11804\times10^{6} & -7.21644\times10^{6} & 2.23608\times10^{6} & -2.541\times10^{6} & -1.11804\times10^{6} \\
2.541\times10^{6} & 1.62624\times10^{6} & 1.5246\times10^{6} & -406560. & -2.89674\times10^{6} & -2.13444\times10^{6} & -2.18526\times10^{6} \\
1.62624\times10^{6} & 2.74428\times10^{6} & -406560. & -304920. & -2.13444\times10^{6} & 2.69346\times10^{6} & -711480. \\
-914760. & 304920. & 1.21968\times10^{6} & -1.21968\times10^{6} & 2.43936\times10^{6} & 1.5246\times10^{6} & 2.13444\times10^{6} \\
304920. & -8.43612\times10^{6} & -1.21968\times10^{6} & 2.84592\times10^{6} & 1.5246\times10^{6} & 5.69184\times10^{6} & 609840. \\
-6.91152\times10^{6} & -5.69184\times10^{6} & -2.0328\times10^{6} & 4.0656\times10^{6} & -1.19935\times10^{7} & -7.72464\times10^{6} & -5.89512\times10^{6} \\
-5.69184\times10^{6} & -1.42296\times10^{7} & 4.0656\times10^{6} & 406560. & -7.72464\times10^{6} & -2.29706\times10^{7} & 4.47216\times10^{6}
\end{pmatrix}$$

Valores proprios matriz Ke=
$\{1.20394\times10^{8}, 7.26579\times10^{7}, 5.70785\times10^{7}, 3.16655\times10^{7}, 2.90261\times10^{7}, 2.21045\times10^{7}, 1.64904\times10^{7}, 1.1803\times10^{7}, 9.07415\times10^{6}, 7.89\times10^{6}, 4.11185\times10^{6}, 1.90283\times10^{6}, 0, 0, 0, 0, 0, 0\}$

Gauss integration rule: 3 x 3

$$Ke=\begin{pmatrix}
6.75761\times10^{6} & 3.39768\times10^{6} & -415272. & 141570. & -434148. & -377520. & -622908. \\
3.39768\times10^{6} & 1.23449\times10^{7} & 141570. & -2.37838\times10^{6} & -377520. & -924924. & -141570. \\
-415272. & 141570. & 4.35092\times10^{6} & -1.4157\times10^{6} & -811668. & -519090. & -103818. \\
141570. & -2.37838\times10^{6} & -1.4157\times10^{6} & 5.12483\times10^{6} & -519090. & 207636. & 283140. \\
-434148. & -377520. & -811668. & -519090. & 1.08631\times10^{7} & 5.56842\times10^{6} & 386958. \\
-377520. & -924924. & -519090. & 207636. & 5.56842\times10^{6} & 1.93762\times10^{7} & 235950. \\
-622908. & -141570. & -103818. & 283140. & 386958. & 235950. & 6.94637\times10^{6} \\
-141570. & 773916. & 283140. & 66066. & 235950. & -726726. & -2.26512\times10^{6} \\
-1.71772\times10^{6} & -566280. & -2.284\times10^{6} & -1.69884\times10^{6} & 2.56714\times10^{6} & 1.8876\times10^{6} & 1.05706\times10^{6} \\
-566280. & 3.90733\times10^{6} & -1.69884\times10^{6} & 2.20849\times10^{6} & 1.8876\times10^{6} & 4.68125\times10^{6} & -1.13256\times10^{6} \\
1.8121\times10^{6} & 1.38424\times10^{6} & -2.08894\times10^{6} & 692120. & 578864. & 1.38424\times10^{6} & 453024. \\
1.38424\times10^{6} & 4.93293\times10^{6} & 692120. & -9.79035\times10^{6} & 1.38424\times10^{6} & -5.31045\times10^{6} & -1.19548\times10^{6} \\
1.71772\times10^{6} & 1.51008\times10^{6} & 585156. & -755040. & -5.96482\times10^{6} & -2.8314\times10^{6} & -5.02102\times10^{6} \\
1.51008\times10^{6} & 2.13299\times10^{6} & -755040. & -1.26469\times10^{6} & -2.8314\times10^{6} & -2.79365\times10^{6} & -943800. \\
-2.34062\times10^{6} & 188760. & 490776. & -1.2584\times10^{6} & 1.77434\times10^{6} & 1.44716\times10^{6} & 75504. \\
188760. & -1.05454\times10^{7} & -1.2584\times10^{6} & 968968. & 1.44716\times10^{6} & 4.31631\times10^{6} & 377520. \\
-4.75675\times10^{6} & -5.53696\times10^{6} & 276848. & 4.53024\times10^{6} & -8.95981\times10^{6} & -6.79536\times10^{6} & -3.17117\times10^{6} \\
-5.53696\times10^{6} & -1.02434\times10^{7} & 4.53024\times10^{6} & 4.85742\times10^{6} & -6.79536\times10^{6} & -1.88257\times10^{7} & 4.78192\times10^{6}
\end{pmatrix}$$

Valores proprios matriz Ke= $\{1.38791\times10^{8}, 8.20819\times10^{7}, 6.23979\times10^{7}, 3.71169\times10^{7}, 3.02355\times10^{7}, 2.54861\times10^{7}, 2.1303\times10^{7}, 1.93668\times10^{7}, 1.43981\times10^{7}, 9.62784\times10^{6}, 8.73506\times10^{6}, 7.54787\times10^{6}, 4.69658\times10^{6}, 3.89752\times10^{6}, 1.20118\times10^{6}, 0, 0, 0\}$

tenemos la suficiencia de rango para p=3

In[147]:=

```
For [p = 4, p ≤ 5, p++,
  Ke = Quad9IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
  Print["Gauss integration rule: ", p, " x ", p];
  Print["Ke=", Chop[Ke] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]], 10^-7];
  Print["Valores proprios matriz Ke=", Valores];
  Print["tenemos la suficiencia de rango para p=", p]
 ];
```

Gauss integration rule: 4 x 4

$$Ke=\begin{pmatrix}
6.76713\times10^{6} & 3.399\times10^{6} & -413684. & 142011. & -430973. & -376638. & -603856. \\
3.399\times10^{6} & 1.23488\times10^{7} & 142011. & -2.37714\times10^{6} & -376638. & -922454. & -138924. \\
-413684. & 142011. & 4.36221\times10^{6} & -1.41085\times10^{6} & -789087. & -509387. & -100643. \\
142011. & -2.37714\times10^{6} & -1.41085\times10^{6} & 5.13401\times10^{6} & -509387. & 225983. & 284022. \\
-430973. & -376638. & -789087. & -509387. & 1.09083\times10^{7} & 5.58783\times10^{6} & 393309. \\
-376638. & -922454. & -509387. & 225983. & 5.58783\times10^{6} & 1.94129\times10^{7} & 237714. \\
-603856. & -138924. & -100643. & 284022. & 393309. & 237714. & 6.98447\times10^{6} \\
-138924. & 781678. & 284022. & 68535.8 & 237714. & -721786. & -2.25983\times10^{6} \\
-1.72883\times10^{6} & -568044. & -2.29687\times10^{6} & -1.70413\times10^{6} & 2.54138\times10^{6} & 1.87702\times10^{6} & 1.03483\times10^{6} \\
-568044. & 3.90222\times10^{6} & -1.70413\times10^{6} & 2.19808\times10^{6} & 1.87702\times10^{6} & 4.66043\times10^{6} & -1.13609\times10^{6} \\
1.80786\times10^{6} & 1.38306\times10^{6} & -2.11905\times10^{6} & 679183. & 518649. & 1.35837\times10^{6} & 444556. \\
1.38306\times10^{6} & 4.92963\times10^{6} & 679183. & -9.81481\times10^{6} & 1.35837\times10^{6} & -5.35937\times10^{6} & -1.19783\times10^{6} \\
1.69549\times10^{6} & 1.50655\times10^{6} & 559400. & -765625. & -6.01633\times10^{6} & -2.85257\times10^{6} & -5.06547\times10^{6} \\
1.50655\times10^{6} & 2.12276\times10^{6} & -765625. & -1.28551\times10^{6} & -2.85257\times10^{6} & -2.83528\times10^{6} & -950856. \\
-2.36603\times10^{6} & 185232. & 486542. & -1.25958\times10^{6} & 1.76588\times10^{6} & 1.44481\times10^{6} & 24697.6 \\
185232. & -1.05557\times10^{7} & -1.25958\times10^{6} & 965675. & 1.44481\times10^{6} & 4.30973\times10^{6} & 370464. \\
-4.72711\times10^{6} & -5.53226\times10^{6} & 311189. & 4.54435\times10^{6} & -8.89113\times10^{6} & -6.76713\times10^{6} & -3.11189\times10^{6} \\
-5.53226\times10^{6} & -1.02297\times10^{7} & 4.54435\times10^{6} & 4.88518\times10^{6} & -6.76713\times10^{6} & -1.87702\times10^{7} & 4.79133\times10^{6}
\end{pmatrix}$$

Valores proprios matriz Ke= $\{1.38952\times10^{8}, 8.22628\times10^{7}, 6.24702\times10^{7}, 3.71838\times10^{7}, 3.02571\times10^{7}, 2.55282\times10^{7}, 2.13724\times10^{7}, 1.94618\times10^{7}, 1.44611\times10^{7}, 9.65925\times10^{6}, 8.75718\times10^{6}, 7.58293\times10^{6}, 4.70868\times10^{6}, 3.91142\times10^{6}, 1.20325\times10^{6}, 0, 0, 0\}$

tenemos la suficiencia de rango para p=4

Gauss integration rule: 5 x 5

$$Ke=\begin{pmatrix}
6.76742\times10^{6} & 3.39904\times10^{6} & -413637. & 142024. & -430878. & -376612. & -603285. \\
3.39904\times10^{6} & 1.23489\times10^{7} & 142024. & -2.3771\times10^{6} & -376612. & -922380. & -138845. \\
-413637. & 142024. & 4.36255\times10^{6} & -1.4107\times10^{6} & -788411. & -509097. & -100548. \\
142024. & -2.3771\times10^{6} & -1.4107\times10^{6} & 5.13428\times10^{6} & -509097. & 226532. & 284048. \\
-430878. & -376612. & -788411. & -509097. & 1.09097\times10^{7} & 5.58841\times10^{6} & 393499. \\
-376612. & -922380. & -509097. & 226532. & 5.58841\times10^{6} & 1.9414\times10^{7} & 237767. \\
-603285. & -138845. & -100548. & 284048. & 393499. & 237767. & 6.98561\times10^{6} \\
-138845. & 781911. & 284048. & 68609.7 & 237767. & -721639. & -2.25967\times10^{6} \\
-1.72916\times10^{6} & -568097. & -2.29726\times10^{6} & -1.70429\times10^{6} & 2.54061\times10^{6} & 1.8767\times10^{6} & 1.03416\times10^{6} \\
-568097. & 3.90206\times10^{6} & -1.70429\times10^{6} & 2.19777\times10^{6} & 1.8767\times10^{6} & 4.65981\times10^{6} & -1.13619\times10^{6} \\
1.80774\times10^{6} & 1.38303\times10^{6} & -2.11995\times10^{6} & 678796. & 516846. & 1.35759\times10^{6} & 444303. \\
1.38303\times10^{6} & 4.92954\times10^{6} & 678796. & -9.81555\times10^{6} & 1.35759\times10^{6} & -5.36084\times10^{6} & -1.1979\times10^{6} \\
1.69482\times10^{6} & 1.50645\times10^{6} & 558629. & -765942. & -6.01787\times10^{6} & -2.8532\times10^{6} & -5.0668\times10^{6} \\
1.50645\times10^{6} & 2.12245\times10^{6} & -765942. & -1.28613\times10^{6} & -2.8532\times10^{6} & -2.83653\times10^{6} & -951068. \\
-2.36679\times10^{6} & 185126. & 486415. & -1.25961\times10^{6} & 1.76562\times10^{6} & 1.44474\times10^{6} & 23176.1 \\
185126. & -1.05561\times10^{7} & -1.25961\times10^{6} & 965576. & 1.44474\times10^{6} & 4.30953\times10^{6} & 370252. \\
-4.72623\times10^{6} & -5.53211\times10^{6} & 312218. & 4.54478\times10^{6} & -8.88907\times10^{6} & -6.76629\times10^{6} & -3.11012\times10^{6} \\
-5.53211\times10^{6} & -1.02293\times10^{7} & 4.54478\times10^{6} & 4.88601\times10^{6} & -6.76629\times10^{6} & -1.87685\times10^{7} & 4.79161\times10^{6}
\end{pmatrix}$$

Valores proprios matriz Ke= $\{1.38957\times10^{8}, 8.22683\times10^{7}, 6.24724\times10^{7}, 3.71858\times10^{7}, 3.02578\times10^{7}, 2.55295\times10^{7}, 2.13745\times10^{7}, 1.94646\times10^{7}, 1.4463\times10^{7}, 9.6602\times10^{6}, 8.75783\times10^{6}, 7.58395\times10^{6}, 4.70905\times10^{6}, 3.91183\times10^{6}, 1.20332\times10^{6}, 0, 0, 0\}$

tenemos la suficiencia de rango para p=5