ELEMENTO CUADRILATERO TRANSICION - 5 NODOS

Implementación en Mathematica

v.2018

1. DATOS INICIALES

■ INICIO

In[1]:=

```
Off[General::"spell1"]
Off[General::"spell"]
```

In[3]:=

```
SetDirectory[NotebookDirectory[]]
```

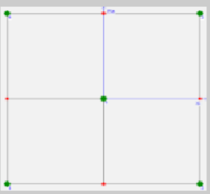
Out[3]=

```
C:\#0-Modulos-M30x_MeF-10\#M308-m6-a5a-swm\11-I-cuadrilatero-i
```

■ DEFINICION ELEMENTO CUADRILATERO DE TRANSICION DE 5 NODOS

□ DEFINICION GRAFICA

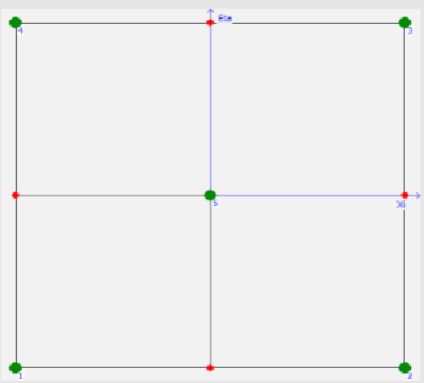
In[4]:=

CuaT5 =  ;

In[5]:=

```
CuaT5r = Show[CuaT5, ImageSize → 200]
```

Out[5]=



□ COORDENADAS NATURALES NODOS

In[6]:=

```
Cn = {{-1, -1}, {1, -1}, {1, 1}, {-1, 1}, {0, 0}};
```

In[7]:=

```
NNodos = Dimensions[Cn][[1]]
```

Out[7]=

```
5
```

IMAGEN DEL ELEMENTO - COMPROBACION

FUNCION REPRESENTACION GRAFICA ELEMENTOS Y NODOS

In[8]:=

```
ElementPlot[b_List, options___] := Module[{asa, color, nr, circles, lines},
    asa = Select[{options}, ((! SameQ[#[[1]], NodeColor]) && (! SameQ[#[[1]], NodeSize])) &];
    {color, nr} = {NodeColor, NodeSize} /. {options} /.
      {NodeColor → GrayLevel[0], NodeSize → PointSize[0.06]};
    circles = Map[Point[#] &, Partition[Flatten[b], 2]];
    lines = Line[Append[b[[1]], First[b[[1]]]]];
    Show[Graphics[{nr, color, circles}], Graphics[lines], Evaluate[Sequence[##] & @@ asa]]];
```

DEFINICION VECTOR DE NODOS

In[9]:=

```
ptsexteriores = {Cn[[1]], Cn[[2]], Cn[[3]], Cn[[4]]};
```

In[10]:=

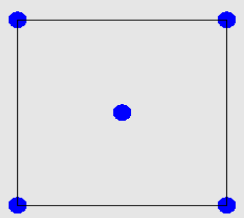
```
ptsinteriores = {Cn[[5]]};
```

IMAGEN DE COMPROBACION

In[11]:=

```
Imagen = ElementPlot[{ptsexteriores, ptsinteriores}, AspectRatio → 1,
   PlotRange → {{-1.5, 1.5}, {-1.5, 1.5}}, ImageSize → 150, Frame → False, NodeColor → RGBColor[0, 0, 1]]
```

Out[11]=

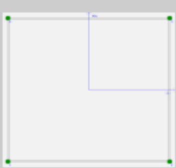


ELEMENTOS COMPLETOS NECESARIOS

REGULAR DE 4 NODOS

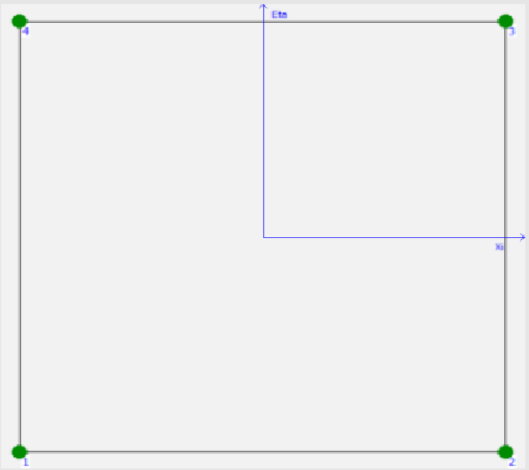
In[12]:=

```
CuaR4 =
```

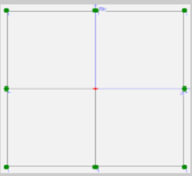


```
;
```

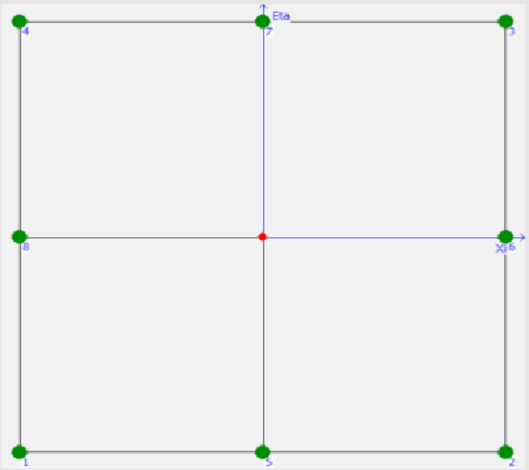
In[13]:=
```
CuaR4r = Show[CuaR4, ImageSize → 250]
```

Out[13]=


SERENDIPITO DE 8 NODOS - 2 DIVISIONES POR LADO

In[14]:=
```
CuaS8 = 
```

```
;
```

In[15]:=
```
CuaS8r = Show[CuaS8, ImageSize → 250]
```

Out[15]=


3. FUNCIONES DE FORMA - METODO PRODUCTO DE CURVAS

Curvas a Considerar

LADOS - CUADRILATERO REGULAR DE 4 NODOS

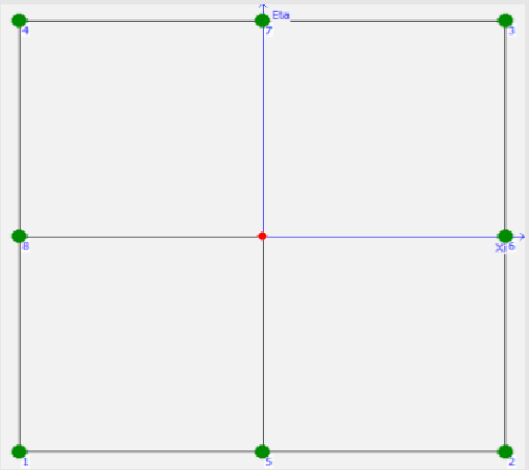
In[16]:=
```
CuCR04N = Table[0, {i, 4}];
```

In[17]:=
```
CuCR04N[[1]] = (η + 1); CuCR04N[[2]] = (ξ - 1); CuCR04N[[3]] = (η - 1); CuCR04N[[4]] = (ξ + 1);
```

LADOS Y MEDIANAS - CUADRILATERO SERENDIPITO DE 8 NODOS

In[18]:=
```
Show[CuaS8, ImageSize → 250]
```

Out[18]=



In[19]:=
```
CuCS8N = Table[0, {i, 6 }];
```

In[20]:=
```
CuCS8N[[1]] = (η + 1);
CuCS8N[[2]] = (ξ - 1);
CuCS8N[[3]] = (η - 1);
CuCS8N[[4]] = (ξ + 1);
```

In[24]:=
```
CuCS8N[[5]] = (ξ);
```

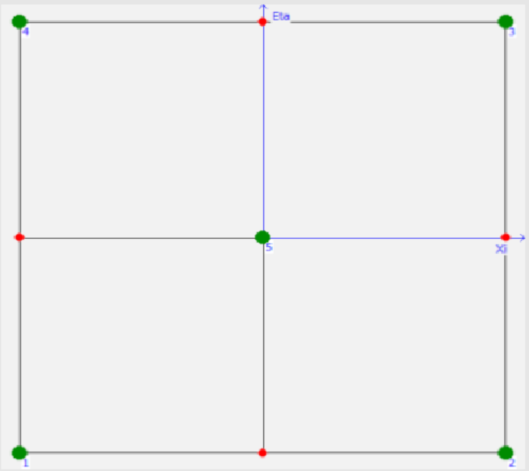
In[25]:=
```
CuCS8N[[6]] = (η);
```

Definición Productos de Curvas en cada Nodo - # - NODOS NO ESQUINA

In[26]:=
```
Nc = Table[{0, 0}, {i, NNodos}];
```

In[27]:=
```
Show[CuaT5, ImageSize → 250]
```

Out[27]=



Tipo 1 - INTERIOR

In[28]:=
```
Nc[[5]] = CuCS8N[[1]] * CuCS8N[[2]] * CuCS8N[[3]] * CuCS8N[[4]];
```

Obtención Funciones de Forma - NODOS NO ESQUINA

In[29]:=
```
Clear[Nf]
```

In[30]:=
```
Nfp = Table[0, {i, NNodos}];
```

In[31]:=
```
Nf = Table[0, {i, NNodos}];
```

In[32]:=
```
Do[
  Nfp[[i]] = a * Nc[[i]];
  eq = 1 == Nfp[[i]] /. {ξ -> Cn[[i, 1]], η -> Cn[[i, 2]]};
  as = a /. Solve[eq, a][[1]]; Print["Nodo ", i];
  Nf[[i]] = Simplify[Nfp[[i]] /. {a → as}],
  {i, 5, NNodos}
 ];
```

Nodo 5

In[33]:=
```
MatrixForm[Nf]
```

Out[33]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ (-1+\eta)\ (1+\eta)\ (-1+\xi)\ (1+\xi) \end{pmatrix}$$

Obtención Funciones de Forma - NODOS ESQUINA

Utilizamos las Funciones de Forma del Cuadrilatero de 4 Nodos.

In[34]:=

$$\text{NfCR4} = \left\{\frac{1}{4}(-1+\eta)(-1+\xi),\ -\frac{1}{4}(-1+\eta)(1+\xi),\ \frac{1}{4}(1+\eta)(1+\xi),\ -\frac{1}{4}(1+\eta)(-1+\xi)\right\};$$

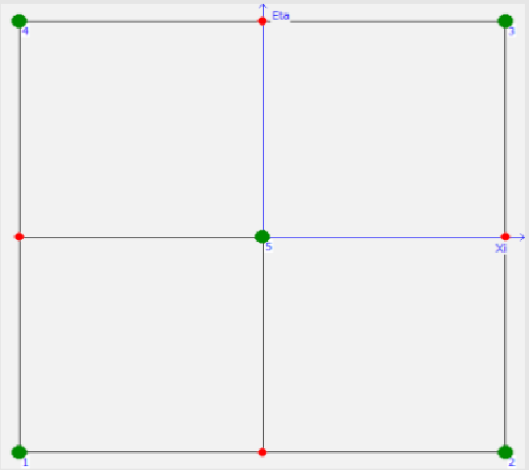
NODO 1 - Desarrollo -

In[35]:=
```
Clear[a5];
```

In[36]:=
```
Nf[[1]] = NfCR4[[1]] + a5 * Nf[[5]];
```

In[37]:=
```
Show[CuaT5, ImageSize → 250]
```

Out[37]=



In[38]:=
```
eq = 0 == Nf[[1]] /. {ξ -> Cn[[5]][[1]], η -> Cn[[5]][[2]]}
a5s = a5 /. Solve[eq, a5][[1]]
```

Out[38]= $0 == \frac{1}{4} + a5$

Out[39]= $-\frac{1}{4}$

In[40]:=
```
Nf[[1]] = Simplify[Nf[[1]] /. {a5 → a5s}];
```

In[41]:=
```
Nf[[1]]
```

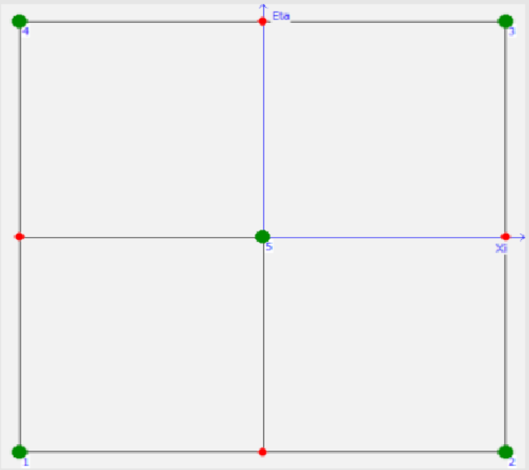
Out[41]= $-\frac{1}{4}\ (-1+\eta)\ (-1+\xi)\ (\eta+\xi+\eta\,\xi)$

NODO 2 - Desarrollo -

In[42]:=
```
Clear[a5];
```

In[43]:=
```
Nf[[2]] = NfCR4[[2]] + a5 * Nf[[5]];
```

In[44]:=
```
Show[CuaT5, ImageSize → 250]
```
Out[44]=



In[45]:=
```
eq = 0 == Nf[[2]] /. {ξ -> Cn[[5]][[1]], η -> Cn[[5]][[2]]}
a5s = a5 /. Solve[eq, a5][[1]]
```
Out[45]=

$$0 == \frac{1}{4} + a5$$

Out[46]=

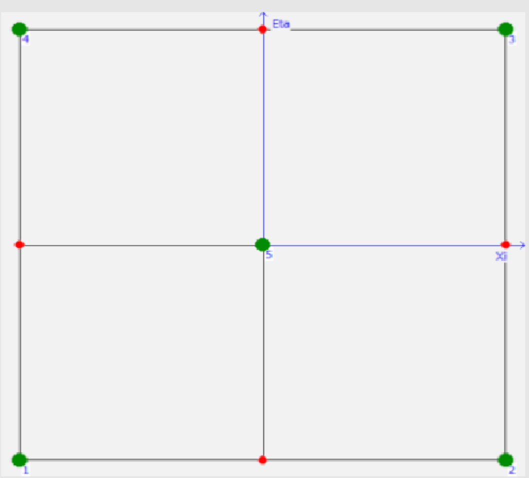
$$-\frac{1}{4}$$

In[47]:=
```
Nf[[2]] = Simplify[Nf[[2]] /. {a5 → a5s}]
```
Out[47]=

$$-\frac{1}{4}\,(-1+\eta)\,(1+\xi)\,(\eta\,(-1+\xi)+\xi)$$

NODO 3 - Desarrollo -

In[48]:=
```
Clear[a5];
```
In[49]:=
```
Nf[[3]] = NfCR4[[3]] + a5 * Nf[[5]];
```
In[50]:=
```
Show[CuaT5, ImageSize → 250]
```
Out[50]=



In[51]:=
```
eq = 0 == Nf[[3]] /. {ξ -> Cn[[5]][[1]], η -> Cn[[5]][[2]]}
a5s = a5 /. Solve[eq, a5][[1]]
```

Out[51]= $0 == \frac{1}{4} + a5$

Out[52]= $-\frac{1}{4}$

In[53]:=
```
Nf[[3]] = Simplify[Nf[[3]] /. {a5 → a5s}]
```

Out[53]= $\frac{1}{4}\left(\eta + \eta^2 + \xi + \eta\,\xi + \xi^2 - \eta^2\,\xi^2\right)$

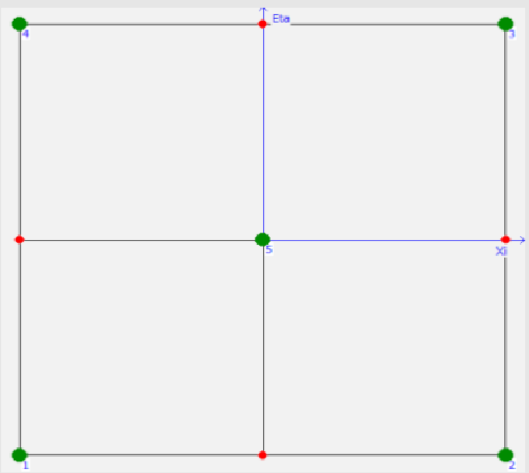
NODO 4 - Desarrollo -

In[54]:=
```
Clear[a5];
```

In[55]:=
```
Nf[[4]] = NfCR4[[4]] + a5 * Nf[[5]];
```

In[56]:=
```
Show[CuaT5, ImageSize → 250]
```

Out[56]=



In[57]:=
```
eq = 0 == Nf[[4]] /. {ξ -> Cn[[5]][[1]], η -> Cn[[5]][[2]]}
a5s = a5 /. Solve[eq, a5][[1]]
```

Out[57]= $0 == \frac{1}{4} + a5$

Out[58]= $-\frac{1}{4}$

In[59]:=
```
Nf[[4]] = Simplify[Nf[[4]] /. {a5 → a5s}]
```

Out[59]= $-\frac{1}{4}(1 + \eta)\,(-1 + \xi)\,(\eta - \xi + \eta\,\xi)$

Funciones de Forma de todos los Nodos.

In[60]:=
```
MatrixForm[Nf]
```

Out[60]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{4}(-1+\eta)(-1+\xi)(\eta+\xi+\eta\,\xi) \\ -\frac{1}{4}(-1+\eta)(1+\xi)(\eta(-1+\xi)+\xi) \\ \frac{1}{4}\left(\eta+\eta^2+\xi+\eta\,\xi+\xi^2-\eta^2\,\xi^2\right) \\ -\frac{1}{4}(1+\eta)(-1+\xi)(\eta-\xi+\eta\,\xi) \\ (-1+\eta)(1+\eta)(-1+\xi)(1+\xi) \end{pmatrix}$$

Comprobación Suma Unidad -

In[61]:=

$$\text{Suma} = \sum_{i=1}^{\text{NNodos}} \text{Nf}[[i]]$$

Out[61]=

$$(-1+\eta)(1+\eta)(-1+\xi)(1+\xi) - \frac{1}{4}(-1+\eta)(1+\xi)(\eta(-1+\xi)+\xi) - \frac{1}{4}(1+\eta)(-1+\xi)(\eta-\xi+\eta\,\xi) - \frac{1}{4}(-1+\eta)(-1+\xi)(\eta+\xi+\eta\,\xi) + \frac{1}{4}\left(\eta+\eta^2+\xi+\eta\,\xi+\xi^2-\eta^2\,\xi^2\right)$$

In[62]:=
```
Simplify[%]
```

Out[62]=

1

OK.

Proceso para comprobar Valor Funciones de Forman en Nodos - en caso de Error

In[63]:=
```
Do[
  Print["NODO ", j];
  Do[
   Print[i, "   ", Simplify[Nf[[j]] /. {ξ -> Cn[[i, 1]], η -> Cn[[i, 2]]}]],
   {i, NNodos}
    ],
  {j, NNodos}
 ];
```

```
NODO 1
1  1
2  0
3  0
4  0
5  0
NODO 2
1  0
2  1
3  0
4  0
5  0
NODO 3
1  0
2  0
3  1
4  0
5  0
NODO 4
1  0
2  0
3  0
4  1
5  0
NODO 5
1  0
2  0
3  0
4  0
5  1
```

Representación Gráfica.

Función Representación Gráfica Funciones de Forma

In[64]:=

```
PlotQuadrilateralShapeFunction[xyquad_, f_, Nsub_, aspect_] :=
  Module[{Ne, Nev, line3D = {}, poly3D = {}, xyf1, xyf2, xyf3, i, j, n,
    ixi, ieta, xi, eta, x1, x2, x3, x4, y1, y2, y3, y4, z1, z2, z3, z4, xc, yc},
   {{x1, y1, z1}, {x2, y2, z2}, {x3, y3, z3}, {x4, y4, z4}} = Take[xyquad, 4];
   xc = {x1, x2, x3, x4}; yc = {y1, y2, y3, y4};
   Ne[xi_, eta_] := N[{(1 - xi) * (1 - eta), (1 + xi) * (1 - eta), (1 + xi) * (1 + eta), (1 - xi) * (1 + eta)} / 4];
   n = Nsub; Do[Do[ixi = (2 * i - n - 1) / n; ieta = (2 * j - n - 1) / n;
     {xi, eta} = N[{ixi - 1 / n, ieta - 1 / n}]; Nev = Ne[xi, eta];
     xyf1 = {xc.Nev, yc.Nev, f[xi, eta]}; {xi, eta} = N[{ixi + 1 / n, ieta - 1 / n}]; Nev = Ne[xi, eta];
     xyf2 = {xc.Nev, yc.Nev, f[xi, eta]}; {xi, eta} = N[{ixi + 1 / n, ieta + 1 / n}]; Nev = Ne[xi, eta];
     xyf3 = {xc.Nev, yc.Nev, f[xi, eta]}; {xi, eta} = N[{ixi - 1 / n, ieta + 1 / n}]; Nev = Ne[xi, eta];
     xyf4 = {xc.Nev, yc.Nev, f[xi, eta]}; AppendTo[poly3D, Polygon[{xyf1, xyf2, xyf3, xyf4}]];
     AppendTo[line3D, Line[{xyf1, xyf2, xyf3, xyf4, xyf1}]], {i, 1, Nsub}], {j, 1, Nsub}];
   Show[Graphics3D[RGBColor[1, 1, 0]], Graphics3D[poly3D], Graphics3D[Thickness[.002]],
    Graphics3D[line3D], Graphics3D[RGBColor[0, 0, 0]], Graphics3D[Thickness[.005]],
    Graphics3D[Line[xyquad]], PlotRange → All, BoxRatios → {1, 1, aspect}, Boxed → False]];
```

Representación Gráfica Funciones Forma Elemento.

In[65]:=

```
Ng = Table[0, {i, NNodos}];
```

In[66]:=

```
xyc1 = {0, 0, 0}; xyc2 = {3, 0, 0}; xyc3 = {3, 3, 0};
xyc4 = {0, 3, 0}; xyquad = N[{xyc1, xyc2, xyc3, xyc4, xyc1}];
```

Control de Cuadricula

In[67]:=

```
Nsub = 20;
```

In[68]:=

```
Do[
  fi[ξ_, η_] = Nf[[i]];
  Ng[[i]] = PlotQuadrilateralShapeFunction[xyquad, fi, Nsub, 1 / 2],
  {i, NNodos}
 ];
```

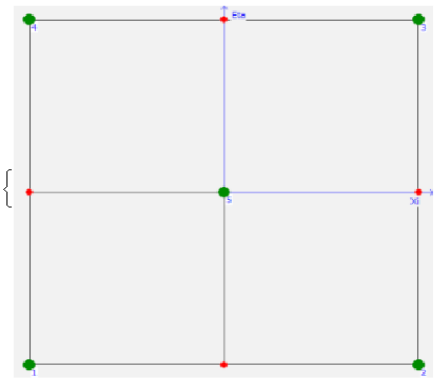
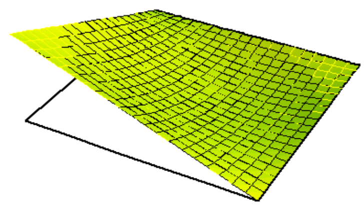
4. RESULTADOS INTERACTIVOS

In[69]:=
```
Manipulate[{CuaT5r, Ng[[n]], Nf[[n]]}, {n, 1, Dimensions[Nf][[1]], 1}, {n, Range[Dimensions[Nf][[1]]]},
 FrameLabel → {"FUNCION DE FORMA EN NODO n - CUADRILATERO TRANSICION 5 NODOS"},
 SaveDefinitions → True]
```

Out[69]=

n

n 1 2 3 4 5

{ , , $-\frac{1}{4}(-1+\eta)(-1+\xi)(\eta+\xi+\eta\,\xi)$ }

FUNCION DE FORMA EN NODO n – CUADRILATERO TRANSICION 5 NODOS

5. DERIVADAS FUNCIONES DE FORMA Y JACOBIANO

FUNCIONES DE FORMA

In[70]:=
```
Nf
```

Out[70]=

$$\left\{-\frac{1}{4}(-1+\eta)(-1+\xi)(\eta+\xi+\eta\,\xi),\ -\frac{1}{4}(-1+\eta)(1+\xi)(\eta(-1+\xi)+\xi),\ \frac{1}{4}\left(\eta+\eta^2+\xi+\eta\,\xi+\xi^2-\eta^2\,\xi^2\right),\ -\frac{1}{4}(1+\eta)(-1+\xi)(\eta-\xi+\eta\,\xi),\ (-1+\eta)(1+\eta)(-1+\xi)(1+\xi)\right\}$$

DERIVADAS FUNCIONES DE FORMA / COORDENADAS NATURALES

iNICIALIZACIONES

In[71]:=
```
Clear[DNfξ, DNfη]
```

In[72]:=
```
DNfξ = Table[0, {i, NNodos}];
```

In[73]:=
```
DNfη = Table[0, {i, NNodos}];
```

PROCESO DE CALCULO

In[74]:=
```
Do[
  Nfe = Expand[Nf[[i]]];
  DNfξ[[i]] = Simplify[D[Nf[[i]], ξ]];
  DNfη[[i]] = Simplify[D[Nf[[i]], η]];
  Clear[Nfe],
  {i, NNodos}
 ];
```

RESULTADOS

In[75]:=
```
DNfξ
```

Out[75]=

$$\left\{\frac{1}{4}\left(-1+\eta+2\,\xi-2\,\eta^2\,\xi\right),\ -\frac{1}{4}\,(-1+\eta)\,(1+2\,(1+\eta)\,\xi),\ \frac{1}{4}\left(1+\eta+2\,\xi-2\,\eta^2\,\xi\right),\ -\frac{1}{4}\,(1+\eta)\,(1+2\,(-1+\eta)\,\xi),\ 2\left(-1+\eta^2\right)\xi\right\}$$

In[76]:=
```
DNfη
```

Out[76]=

$$\left\{\frac{1}{4}\left(-1+\xi-2\,\eta\left(-1+\xi^2\right)\right),\ -\frac{1}{4}\,(1+2\,\eta\,(-1+\xi))\,(1+\xi),\ \frac{1}{4}\left(1+\xi-2\,\eta\left(-1+\xi^2\right)\right),\ -\frac{1}{4}\,(-1+\xi)\,(1+2\,\eta\,(1+\xi)),\ 2\,\eta\left(-1+\xi^2\right)\right\}$$

MODULO CALCULO FUNCIONES DE FORMA Y JACOBIANO

MODULO GENERICO A COMPLETAR

MODULO GENERICO A COMPLETAR: XX = No. Nodos

In[77]:=
```
(*QuadXXIsoPShapeFunDer[ncoor_,qcoor_]:=
  Module[{Nf,dNx,dNy,dNξ,dNη,J11,J12,J21,J22,Jdet,ξ,η,x,y},{ξ,η}=qcoor;
   Nf=(*Nf*);

   dNξ=(*dNξ*);

   dNη=(*dNη*);

   x=Table[ncoor[[i,1]],{i,XX}];y=Table[ncoor[[i,2]],{i,XX}];
   J11=dNξ.x;J21=dNξ.y;J12=dNη.x;J22=dNη.y;
   Jdet=Simplify[J11*J22-J12*J21];
   dNx=(J22*dNξ-J21*dNη)/Jdet;dNx=Simplify[dNx];
   dNy=(-J12*dNξ+J11*dNη)/Jdet;dNy=Simplify[dNy];
   Return[{Nf,dNx,dNy,Jdet}]];*)
```

▫ MODULO COMPLETADO -

In[78]:=

```
Quad5IsoPShapeFunDer[ncoor_, qcoor_] :=
  Module[{Nf, dNx, dNy, dNξ, dNη, J11, J12, J21, J22, Jdet, ξ, η, x, y}, {ξ, η} = qcoor;
   Nf = {-1/4 (-1 + η) (-1 + ξ) (η + ξ + η ξ), -1/4 (-1 + η) (1 + ξ) (η (-1 + ξ) + ξ),
     1/4 (η + η^2 + ξ + η ξ + ξ^2 - η^2 ξ^2), -1/4 (1 + η) (-1 + ξ) (η - ξ + η ξ), (-1 + η) (1 + η) (-1 + ξ) (1 + ξ)};

   dNξ = {1/4 (-1 + η + 2 ξ - 2 η^2 ξ), -1/4 (-1 + η) (1 + 2 (1 + η) ξ),
     1/4 (1 + η + 2 ξ - 2 η^2 ξ), -1/4 (1 + η) (1 + 2 (-1 + η) ξ), 2 (-1 + η^2) ξ};

   dNη = {1/4 (-1 + ξ - 2 η (-1 + ξ^2)), -1/4 (1 + 2 η (-1 + ξ)) (1 + ξ),
     1/4 (1 + ξ - 2 η (-1 + ξ^2)), -1/4 (-1 + ξ) (1 + 2 η (1 + ξ)), 2 η (-1 + ξ^2)};

   x = Table[ncoor[[i, 1]], {i, 5}]; y = Table[ncoor[[i, 2]], {i, 5}];
   J11 = dNξ.x; J21 = dNξ.y; J12 = dNη.x; J22 = dNη.y;
   Jdet = Simplify[J11 * J22 - J12 * J21];
   dNx = (J22 * dNξ - J21 * dNη) / Jdet; dNx = Simplify[dNx];
   dNy = (-J12 * dNξ + J11 * dNη) / Jdet; dNy = Simplify[dNy];
   Return[{Nf, dNx, dNy, Jdet}]];
```

▫ USO Y DESARROLLO

In[79]:=

```
{Nf, dNx, dNy, Jdet} = Quad5IsoPShapeFunDer[{{x1, y1}, {x2, y2}, {x3, y3}, {x4, y4}, {x5, y5}}, {ξ, η}];
```

In[80]:=

```
Jdet
```

Out[80]=

$$\frac{1}{16}\Big(-\big(y1\,(-1+\eta)\,(-1+2\,(1+\eta)\,\xi)+y2\,(-1+\eta)\,(1+2\,(1+\eta)\,\xi)+(1+\eta)\,(-8\,y5\,(-1+\eta)\,\xi+y3\,(-1+2\,(-1+\eta)\,\xi)+y4\,(1+2\,(-1+\eta)\,\xi))\big)\\ \big(-x3-x4-2\,x3\,\eta-2\,x4\,\eta+8\,x5\,\eta-x3\,\xi+x4\,\xi+2\,x3\,\eta\,\xi^2+2\,x4\,\eta\,\xi^2-8\,x5\,\eta\,\xi^2+x1\,(-1+\xi)\,(-1+2\,\eta\,(1+\xi))+x2\,(1+\xi+2\,\eta\,(-1+\xi^2))\big)+\big(x1\,(-1+\eta)\,(-1+2\,(1+\eta)\,\xi)+x2\,(-1+\eta)\,(1+2\,(1+\eta)\,\xi)+(1+\eta)\,(-8\,x5\,(-1+\eta)\,\xi+x3\,(-1+2\,(-1+\eta)\,\xi)+x4\,(1+2\,(-1+\eta)\,\xi))\big)\\ \big(-y3-y4-2\,y3\,\eta-2\,y4\,\eta+8\,y5\,\eta-y3\,\xi+y4\,\xi+2\,y3\,\eta\,\xi^2+2\,y4\,\eta\,\xi^2-8\,y5\,\eta\,\xi^2+y1\,(-1+\xi)\,(-1+2\,\eta\,(1+\xi))+y2\,(1+\xi+2\,\eta\,(-1+\xi^2))\big)\Big)$$

In[81]:=

```
dNy[[1]]
```

Out[81]=

$$\Big(2\,\big(x4\,(-1+\xi)\,(-1+2\,\eta^2+2\,\xi)+(\eta-\xi)\,(x3+2\,x3\,\eta\,\xi+4\,x5\,(-1+\eta+\xi-\eta\,\xi))+x2\,(-1-2\,\eta^2+2\,\xi^2+\eta\,(3-2\,\xi^2))\big)\Big)\Big/\\ \Big(-\big(y1\,(-1+\eta)\,(-1+2\,(1+\eta)\,\xi)+y2\,(-1+\eta)\,(1+2\,(1+\eta)\,\xi)+(1+\eta)\,(-8\,y5\,(-1+\eta)\,\xi+y3\,(-1+2\,(-1+\eta)\,\xi)+y4\,(1+2\,(-1+\eta)\,\xi))\big)\\ \big(-x3-x4-2\,x3\,\eta-2\,x4\,\eta+8\,x5\,\eta-x3\,\xi+x4\,\xi+2\,x3\,\eta\,\xi^2+2\,x4\,\eta\,\xi^2-8\,x5\,\eta\,\xi^2+x1\,(-1+\xi)\,(-1+2\,\eta\,(1+\xi))+x2\,(1+\xi+2\,\eta\,(-1+\xi^2))\big)+\big(x1\,(-1+\eta)\,(-1+2\,(1+\eta)\,\xi)+x2\,(-1+\eta)\,(1+2\,(1+\eta)\,\xi)+(1+\eta)\,(-8\,x5\,(-1+\eta)\,\xi+x3\,(-1+2\,(-1+\eta)\,\xi)+x4\,(1+2\,(-1+\eta)\,\xi))\big)\\ \big(-y3-y4-2\,y3\,\eta-2\,y4\,\eta+8\,y5\,\eta-y3\,\xi+y4\,\xi+2\,y3\,\eta\,\xi^2+2\,y4\,\eta\,\xi^2-8\,y5\,\eta\,\xi^2+y1\,(-1+\xi)\,(-1+2\,\eta\,(1+\xi))+y2\,(1+\xi+2\,\eta\,(-1+\xi^2))\big)\Big)$$

In[82]:=
```
coorn = {x1 → 0, y1 → 0, x2 → 1, y2 → 0, x3 → 1, y3 → 1 / 2, x4 → 0, y4 → 1 / 2, x5 → 1 / 2, y4 → 1 / 4}
```

Out[82]=

$$\left\{x1 \to 0,\ y1 \to 0,\ x2 \to 1,\ y2 \to 0,\ x3 \to 1,\ y3 \to \frac{1}{2},\ x4 \to 0,\ y4 \to \frac{1}{2},\ x5 \to \frac{1}{2},\ y4 \to \frac{1}{4}\right\}$$

In[83]:=
```
dNx /. coorn
```

6. MATRIZ DE RIGIDEZ - INTEGRACION NUMERICA

MODULO DE CALCULO DE LA MATRIZ DE RIGIDEZ

MODULO GENERICO A COMPLETAR

MODULO GENERICO: XX = No. Nodos, YY = Grados Libertad

In[84]:=
```
(*QuadXXIsoPMembraneStiffness[ncoor_,mprop_,fprop_,options_]:=Module[{i,j,k,p=7,numer=False,
    Emat,th=1,h,qcoor,c,w,Nf,dNx,dNy,Jdet,B,Ke=Table[0,{YY},{YY}]},Emat=mprop[[1]];
   If[Length[options]==2,{numer,p}=options,{numer}=options];
   If[Length[fprop]>0,th=fprop[[1]]];
   If[p<1||p>4,Print["p out of range"];Return[Null]];
   For[k=1,k≤p*p,k++,
    {qcoor,w}=QuadGaussRuleInfo[{p,numer},k];
    {Nf,dNx,dNy,Jdet}=QuadXXIsoPShapeFunDer[ncoor,qcoor];
    If[Length[th]==0,h=th,h=th.Nf];c=w*Jdet*h;
    B={Flatten[Table[{dNx[[i]],0},{i,XX}]],
      Flatten[Table[{0,dNy[[i]]},{i,XX}]],Flatten[Table[{dNy[[i]],dNx[[i]]},{i,XX}]]};
    Ke+=Simplify[c*Transpose[B].(Emat.B)];];
   Return[Simplify[Ke]];*)
```

MODULO COMPLETADO -

In[85]:=
```
Quad5IsoPMembraneStiffness[ncoor_, mprop_, fprop_, options_] :=
  Module[{i, j, k, p = 7, numer = False, Emat, th = 1, h, qcoor, c, w,
    Nf, dNx, dNy, Jdet, B, Ke = Table[0, {10}, {10}]}, Emat = mprop[[1]];
   If[Length[options] == 2, {numer, p} = options, {numer} = options];
   If[Length[fprop] > 0, th = fprop[[1]]];
   If[p < 1 || p > 12, Print["p out of range"]; Return[Null]];
   For[k = 1, k ≤ p * p, k++,
    {qcoor, w} = QuadGaussRuleInfo[{p, numer}, k];
    {Nf, dNx, dNy, Jdet} = Quad5IsoPShapeFunDer[ncoor, qcoor];
    If[Length[th] == 0, h = th, h = th.Nf]; c = w * Jdet * h;
    B = {Flatten[Table[{dNx[[i]], 0}, {i, 5}]],
      Flatten[Table[{0, dNy[[i]]}, {i, 5}]], Flatten[Table[{dNy[[i]], dNx[[i]]}, {i, 5}]]};
    Ke += Simplify[c * Transpose[B].(Emat.B)];];
   Return[Simplify[Ke]]];
```

USO Y DESARROLLO

In[86]:=
```
B = {Flatten[Table[{dNx[[i]], 0}, {i, 5}]],
   Flatten[Table[{0, dNy[[i]]}, {i, 5}]], Flatten[Table[{dNy[[i]], dNx[[i]]}, {i, 5}]]};
```

In[87]:=
```
B // MatrixForm
```

Out[87]//MatrixForm=
```
( ────────────────────────────────────────────────────────────────
  -(y1 (-1+η) (-1+2 (1+η) ξ)+y2 (-1+η) (1+2 (1+η) ξ)+(1+η) (-8 y5 (-1+η) ξ+y3 (-1+2 (-1+η) ξ)+y4 (1+2 (-1+η) ξ))) (-x3-x4-2 x3 η-2 x4 η+8 x5 η-x3 ξ+x4 ξ+2

  ────────────────────────────────────────────────────────────────
  -(y1 (-1+η) (-1+2 (1+η) ξ)+y2 (-1+η) (1+2 (1+η) ξ)+(1+η) (-8 y5 (-1+η) ξ+y3 (-1+2 (-1+η) ξ)+y4 (1+2 (-1+η) ξ))) (-x3-x4-2 x3 η-2 x4 η+8 x5 η-x3 ξ+x4 ξ+2
```

In[88]:=
```
FullSimplify[dNx[[1]] /. coorn]
```

Out[88]=
$$\frac{(-1+\eta)\ (1+\eta\ (2+8\ y5\ (-1+\xi)-4\ \xi)+2\ \xi\ (-2-4\ y5\ (-1+\xi)+\xi))}{2\ \left(1+2\ (-1+4\ y5)\ \eta\ \left(-1+\xi^2\right)\right)}$$

In[89]:=
```
FullSimplify[B /. coorn] // MatrixForm
```

Out[89]//MatrixForm=
$$\begin{pmatrix} \frac{(-1+\eta)\ (1+\eta\ (2+8\ y5\ (-1+\xi)-4\ \xi)+2\ \xi\ (-2-4\ y5\ (-1+\xi)+\xi))}{2\ \left(1+2\ (-1+4\ y5)\ \eta\ \left(-1+\xi^2\right)\right)} & 0 & \frac{(-1+\eta)\ (-1+2\ \xi\ (-2-\xi+4\ y5\ (1+\xi))+\eta\ \ldots}{2\ \left(1+2\ (-1+4\ y5)\ \eta\ (-1 \ldots\right.} \\ 0 & \frac{-1+\xi-2\ \eta\ \left(-1+\xi^2\right)}{1+2\ (-1+4\ y5)\ \eta\ \left(-1+\xi^2\right)} & 0 \\ \frac{-1+\xi-2\ \eta\ \left(-1+\xi^2\right)}{1+2\ (-1+4\ y5)\ \eta\ \left(-1+\xi^2\right)} & \frac{(-1+\eta)\ (1+\eta\ (2+8\ y5\ (-1+\xi)-4\ \xi)+2\ \xi\ (-2-4\ y5\ (-1+\xi)+\xi))}{2\ \left(1+2\ (-1+4\ y5)\ \eta\ \left(-1+\xi^2\right)\right)} & -\frac{(1+2\ \eta\ (-1+\xi))\ (1 \ldots}{1+2\ (-1+4\ y5)\ \eta\ (-1 \ldots} \end{pmatrix}$$

In[90]:=
```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

In[91]:=
```
Integrando = Jdet * h * Transpose[B] . (Emat.B) ;
```

In[92]:=
```
matern = {Em → 96, nu → 1 / 3, h → 1}
```

Out[92]=
$$\left\{Em \to 96,\ nu \to \frac{1}{3},\ h \to 1\right\}$$

In[93]:=
```
Simplify[Integrando /. coorn /. matern] // MatrixForm
```

Out[93]//MatrixForm=

A very large output was generated. Here is a sample of it:

(<< 1 >>)

Show Less | Show More | Show Full Output | Set Size Limit...

In[94]:=

```
Simplify[Integrando /. coorn /. matern /. {ξ → -1/√3, η -> -1/√3}] // MatrixForm
```

Out[94]//MatrixForm=

$$\begin{pmatrix}
\frac{798+441\sqrt{3}}{36-16\sqrt{3}+64\sqrt{3}\,y5} & \frac{114+63\sqrt{3}}{9-4\sqrt{3}+16\sqrt{3}\,y5} & \frac{366+171\sqrt{3}-144\left(7+3\sqrt{3}\right)y5}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} & -\frac{3\left(7+3\sqrt{3}\right)(-5+16\,y5)}{18-8\sqrt{3}+32\sqrt{3}\,y5} \\
\frac{114+63\sqrt{3}}{9-4\sqrt{3}+16\sqrt{3}\,y5} & \frac{39\left(38+21\sqrt{3}\right)}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} & -\frac{3\left(7+3\sqrt{3}\right)(-5+16\,y5)}{18-8\sqrt{3}+32\sqrt{3}\,y5} & \frac{9\left(74+49\sqrt{3}\right)-48\left(7+3\sqrt{3}\right)y5}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} \\
\frac{366+171\sqrt{3}-144\left(7+3\sqrt{3}\right)y5}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} & -\frac{3\left(7+3\sqrt{3}\right)(-5+16\,y5)}{18-8\sqrt{3}+32\sqrt{3}\,y5} & \frac{3\left(218-69\sqrt{3}+288\left(-3+\sqrt{3}\right)y5+1536\,y5^2\right)}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} & -\frac{3\left(9-12\sqrt{3}+16\left(1+3\sqrt{3}\right)y5\right)}{9-4\sqrt{3}+16\sqrt{3}\,y5} \\
-\frac{3\left(7+3\sqrt{3}\right)(-5+16\,y5)}{18-8\sqrt{3}+32\sqrt{3}\,y5} & \frac{9\left(74+49\sqrt{3}\right)-48\left(7+3\sqrt{3}\right)y5}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} & -\frac{3\left(9-12\sqrt{3}+16\left(1+3\sqrt{3}\right)y5\right)}{9-4\sqrt{3}+16\sqrt{3}\,y5} & \frac{3\left(222+9\sqrt{3}+96\left(-3+\sqrt{3}\right)y5+512\,y5^2\right)}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} \\
\frac{231}{36-16\sqrt{3}+64\sqrt{3}\,y5} & \frac{33}{9-4\sqrt{3}+16\sqrt{3}\,y5} & \frac{3\left(95-36\sqrt{3}+48\left(-7+3\sqrt{3}\right)y5\right)}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} & \frac{3\left(-7+3\sqrt{3}\right)(-5+16\,y5)}{18-8\sqrt{3}+32\sqrt{3}\,y5} \\
\frac{33}{9-4\sqrt{3}+16\sqrt{3}\,y5} & \frac{429}{36-16\sqrt{3}+64\sqrt{3}\,y5} & \frac{3\left(-7+3\sqrt{3}\right)(-5+16\,y5)}{18-8\sqrt{3}+32\sqrt{3}\,y5} & \frac{3\left(-75+84\sqrt{3}+16\left(-7+3\sqrt{3}\right)y5\right)}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} \\
\frac{3\left(-73-36\sqrt{3}+48\left(7+3\sqrt{3}\right)y5\right)}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} & \frac{3\left(7+3\sqrt{3}\right)(-3+16\,y5)}{18-8\sqrt{3}+32\sqrt{3}\,y5} & -\frac{3\left(187-72\sqrt{3}+96\left(-8+3\sqrt{3}\right)y5+1536\,y5^2\right)}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} & \frac{6\left(7-6\sqrt{3}+24\sqrt{3}\,y5\right)}{9-4\sqrt{3}+16\sqrt{3}\,y5} \\
\frac{3\left(7+3\sqrt{3}\right)(-3+16\,y5)}{18-8\sqrt{3}+32\sqrt{3}\,y5} & \frac{-393-324\sqrt{3}+48\left(7+3\sqrt{3}\right)y5}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} & \frac{6\left(7-6\sqrt{3}+24\sqrt{3}\,y5\right)}{9-4\sqrt{3}+16\sqrt{3}\,y5} & -\frac{3\left(201-24\sqrt{3}+32\left(-8+3\sqrt{3}\right)y5+512\,y\ldots\right.}{4\left(9-4\sqrt{3}+16\sqrt{3}\,y5\right)} \\
-\frac{14\left(9+7\sqrt{3}\right)}{-4+3\sqrt{3}+16\,y5} & -\frac{8\left(9+7\sqrt{3}\right)}{-4+3\sqrt{3}+16\,y5} & \frac{2\left(-9-31\sqrt{3}+96\sqrt{3}\,y5\right)}{-4+3\sqrt{3}+16\,y5} & \frac{24\,(-5+16\,y5)}{9-4\sqrt{3}+16\sqrt{3}\,y5} \\
-\frac{8\left(9+7\sqrt{3}\right)}{-4+3\sqrt{3}+16\,y5} & -\frac{26\left(9+7\sqrt{3}\right)}{-4+3\sqrt{3}+16\,y5} & \frac{24\,(-5+16\,y5)}{9-4\sqrt{3}+16\sqrt{3}\,y5} & \frac{198+42\sqrt{3}-64\sqrt{3}\,y5}{4-3\sqrt{3}-16\,y5}
\end{pmatrix}$$

MODULO REGLAS DE CUADRATURA DE GAUSS

OPCION 1: DEFINICION EN MATHEMATICA

In[95]:=

```
<< NumericalDifferentialEquationAnalysis`;
```

In[96]:=

```
? GaussianQuadratureWeights
```

GaussianQuadratureWeights[n, a, b, prec] gives a list of the pairs {abscissa, weight} to prec digits precision for the elementary n–point Gaussian quadrature formula for quadrature on the interval a to b. The argument prec is optional. >>

In[97]:=

```
QuadGaussRuleInfo[{rule_, numer_}, point_] := Module[{xi, eta, p1, p2, i1, i2, w1, w2, k, info = Null},
    If[Length[rule] == 2, {p1, p2} = rule, p1 = p2 = rule];
    If[Length[point] == 2, {i1, i2} = point, k = point; i2 = Floor[(k - 1) / p1] + 1; i1 = k - p1 * (i2 - 1)];
    {xi, w1} = GaussianQuadratureWeights[p1, -1, 1][[i1]];
    {eta, w2} = GaussianQuadratureWeights[p2, -1, 1][[i2]];
    info = {{xi, eta}, w1 * w2};
    If[numer, Return[N[info]], Return[Simplify[info]]];
  ];
```

In[98]:=

```
QuadGaussRuleInfo[{25, False}, 1]
```

Out[98]=

```
{{-0.995557, -0.995557}, 0.000129819}
```

OPCION 2: DEFINICION DE CARLOS FELIPPA

In[99]:=
```
QuadGaussRuleInfo[{rule_, numer_}, point_] := Module[{xi, eta, p1, p2, i1, i2, w1, w2, k, info = Null},
    If[Length[rule] == 2, {p1, p2} = rule, p1 = p2 = rule];
    If[Length[point] == 2, {i1, i2} = point, k = point; i2 = Floor[(k - 1) / p1] + 1; i1 = k - p1 * (i2 - 1)];
    {xi, w1} = LineGaussRuleInfo[{p1, numer}, i1];
    {eta, w2} = LineGaussRuleInfo[{p2, numer}, i2];
    info = {{xi, eta}, w1 * w2};
    If[numer, Return[N[info]], Return[Simplify[info]]];];
```

In[100]:=
```
LineGaussRuleInfo[{rule_, numer_}, point_] :=
  Module[{g2 = {-1, 1} / Sqrt[3], w3 = {5 / 9, 8 / 9, 5 / 9}, g3 = {-Sqrt[3 / 5], 0, Sqrt[3 / 5]},
    w4 = {(1 / 2) - Sqrt[5 / 6] / 6, (1 / 2) + Sqrt[5 / 6] / 6, (1 / 2) + Sqrt[5 / 6] / 6, (1 / 2) - Sqrt[5 / 6] / 6},
    g4 = {-Sqrt[(3 + 2 * Sqrt[6 / 5]) / 7], -Sqrt[(3 - 2 * Sqrt[6 / 5]) / 7],
      Sqrt[(3 - 2 * Sqrt[6 / 5]) / 7], Sqrt[(3 + 2 * Sqrt[6 / 5]) / 7]}, g5 = {-Sqrt[5 + 2 * Sqrt[10 / 7]],
       -Sqrt[5 - 2 * Sqrt[10 / 7]], 0, Sqrt[5 - 2 * Sqrt[10 / 7]], Sqrt[5 + 2 * Sqrt[10 / 7]]} / 3,
    w5 = {322 - 13 * Sqrt[70], 322 + 13 * Sqrt[70], 512, 322 + 13 * Sqrt[70], 322 - 13 * Sqrt[70]} / 900,
    i = point, p = rule, info = {Null, 0}},
   If[p == 1, info = {0, 2}];
   If[p == 2, info = {g2[[i]], 1}];
   If[p == 3, info = {g3[[i]], w3[[i]]}];
   If[p == 4, info = {g4[[i]], w4[[i]]}];
   If[p == 5, info = {g5[[i]], w5[[i]]}];
   If[numer, Return[N[info]], Return[Simplify[info]]];
  ];
```

In[101]:=
```
QuadGaussRuleInfo[{2, False}, 1]
```

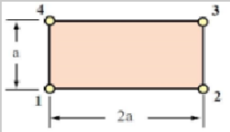
Out[101]=

$$\left\{\left\{-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right\}, 1\right\}$$

7. TEST DEL RECTANGULO

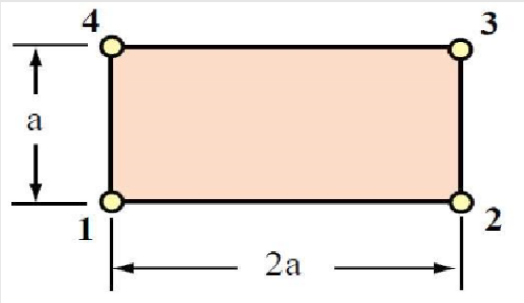
DEFINICION DE LA GEOMETRIA

In[102]:=

RectanguloT =  ;

In[103]:=
```
RectanguloTr = Show[RectanguloT, ImageSize → 250]
```

Out[103]=



■ DEFINCION COORDENADAS NODOS ELEMENTO - ANSYS CLASSIC

▫ IMAGEN DEL MODELO EN ANSYS

In[104]:=

```
RectanguloA = 
```

```
;
```

In[105]:=

```
RectanguloAr = Show[RectanguloA, ImageSize → 450]
```

Out[105]=

▫ DATOS COORDENADAS NODALES EN ANSYS

In[106]:=

```
Nodes = {{0.`, 0.`}, {1.`, 0.`}, {0.5`, 0.`}, {1.`, 0.5`},
   {1.`, 0.25`}, {0.`, 0.5`}, {0.5`, 0.5`}, {0.`, 0.25`}, {0.5`, 0.25`}};
```

In[107]:=

```
Dimensions[Nodes][[1]]
```

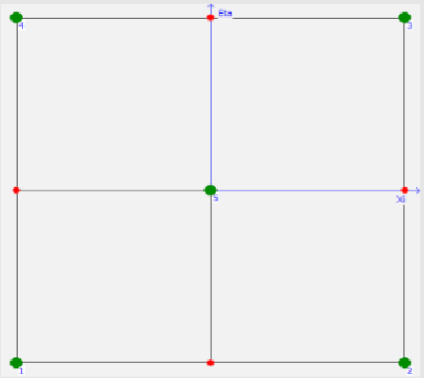
Out[107]=

```
9
```

ORDENACION DATOS DE ANSYS

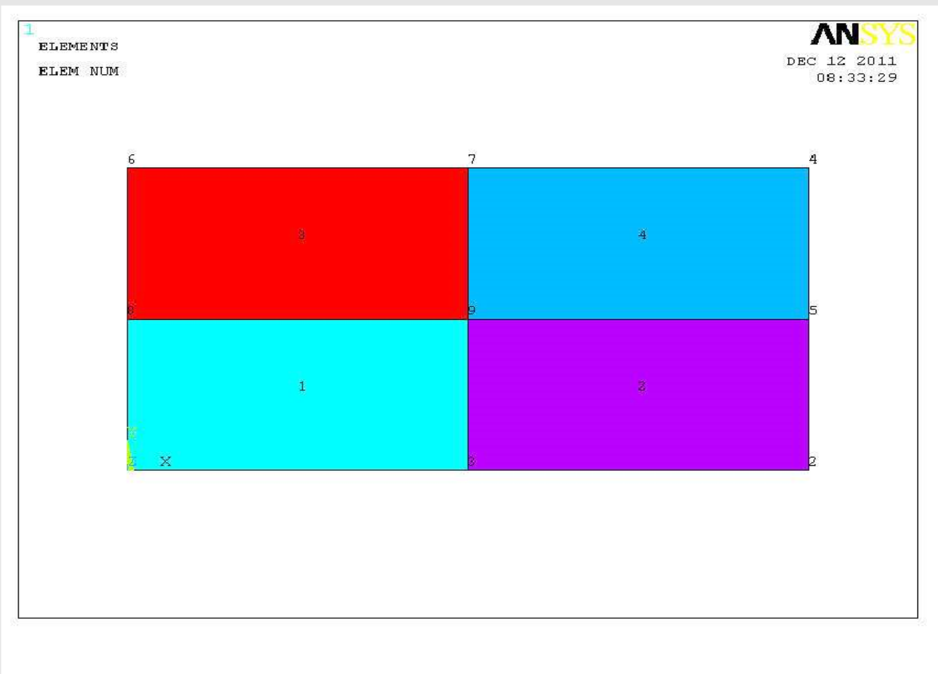
In[108]:=
```
CuaT5r
```

Out[108]=



In[109]:=
```
RectanguloAr
```

Out[109]=



In[110]:=
```
ncoor = {Nodes[[1]], Nodes[[2]], Nodes[[4]], Nodes[[6]], Nodes[[9]]}
```

Out[110]=
```
{{0., 0.}, {1., 0.}, {1., 0.5}, {0., 0.5}, {0.5, 0.25}}
```

DATOS COORDENADAS NODALES - SEGUN TEMA

In[111]:=
```
a = 1 / 2;
```

In[112]:=
```
ncoor = {{0, 0}, {2 * a, 0}, {2 * a, a}, {0, a}, {a, a / 2}};
```

DEFINICION DEL MATERIAL

In[113]:=
```
ClearAll[Em, nu, a, b, e, h, p, num]; h = 1;
```

DATOS SEGUN ENUNCIADO

In[114]:=
```
Em = 96 * 30; nu = 1 / 3;  (*isotropic material*)
```

In[115]:=
```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

In[116]:=
```
Emat // MatrixForm
```

Out[116]//MatrixForm=

$$\begin{pmatrix} 3240 & 1080 & 0 \\ 1080 & 3240 & 0 \\ 0 & 0 & 1080 \end{pmatrix}$$

VERIFICACION DE LA MATRIZ DE RIGIDEZ

NUMERO DE PUNTOS DE GAUSS MINIMO PARA CONSEGUIR SUFICIENCIA DE RANGO

In[117]:=
```
NF = NNodos * 2.;
```

In[118]:=

$$NG = \frac{NF - 3}{3}$$

Out[118]=
```
2.33333
```

Se necesitan como mínimo 3 Puntos -- Regla 2 x 2 minima

BUCLE GENERICO A COMPLETAR

BUCLE GENERICO: XX = No. Nodos, ZZ = GRADOS DE LIBERTAD - 3

In[110]:=
```
(*For [p=1, p≤5, p++,
 Ke=QuadXXIsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
 Print["Gauss integration rule: ",p," x ",p];
 Print["Ke=",Chop[Ke]//MatrixForm];
 Valores=Chop[Eigenvalues[N[Ke]]];
 If[Valores[[ZZ]]≠0,Break[],Print["Valores proprios matriz Ke=",Valores]]
];
Print["Valores proprios matriz Ke=",Valores];
Print["tenemos la suficiencia de rango para p=",p]*)
```

DESARROLLO DE LA MATRIZ DE RIGIDEZ -

In[119]:=
```
For [p = 1, p ≤ 5, p++,
  Ke = Quad5IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {False, p}];
  Print["Gauss integration rule: ", p, " x ", p];
  Print["Ke=", Chop[Ke] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]]];
  If[Valores[[5]] ≠ 0, Break[], Print["Valores proprios matriz Ke=", Valores]]
 ];
Print["Valores proprios matriz Ke=", Valores];
Print["tenemos la suficiencia de rango para p=", p]
```

Gauss integration rule: 1 x 1

Ke=

945	540	135	0	-945	-540	-135	0	0	0
540	1755	0	1485	-540	-1755	0	-1485	0	0
135	0	945	-540	-135	0	-945	540	0	0
0	1485	-540	1755	0	-1485	540	-1755	0	0
-945	-540	-135	0	945	540	135	0	0	0
-540	-1755	0	-1485	540	1755	0	1485	0	0
-135	0	-945	540	135	0	945	-540	0	0
0	-1485	540	-1755	0	1485	-540	1755	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Valores proprios matriz Ke={6709.19, 2700., 1390.81, 0, 0, 0, 0, 0, 0, 0}

Gauss integration rule: 2 x 2

Ke=

1820	1020	380	0	-70	-60	110	0	-2240	-960
1020	3380	0	1940	-60	-130	0	-1030	-960	-4160
380	0	1820	-1020	110	0	-70	60	-2240	960
0	1940	-1020	3380	0	-1030	60	-130	960	-4160
-70	-60	110	0	1820	1020	380	0	-2240	-960
-60	-130	0	-1030	1020	3380	0	1940	-960	-4160
110	0	-70	60	380	0	1820	-1020	-2240	960
0	-1030	60	-130	0	1940	-1020	3380	960	-4160
-2240	-960	-2240	960	-2240	-960	-2240	960	8960	0
-960	-4160	960	-4160	-960	-4160	960	-4160	0	16640

Valores proprios matriz Ke={21033., 11692.7, 6709.19, 2700., 1847.31, 1390.81, 1026.96, 0, 0, 0}

tenemos la suficiencia de rango para p=2

In[123]:=

```
For [p = 3, p ≤ 5, p++,
  Ke = Quad5IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {False, p}];
  Print["Gauss integration rule: ", p, " x ", p];
  Print["Ke=", Chop[Ke] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]]];
  Print["Valores proprios matriz Ke=", Valores];
  Print["tenemos la suficiencia de rango para p=", p]
 ];
```

Gauss integration rule: 3 x 3

$$Ke=\begin{pmatrix} 1932 & 1020 & 492 & 0 & 42 & -60 & 222 & 0 & -2688 & -960 \\ 1020 & 3588 & 0 & 2148 & -60 & 78 & 0 & -822 & -960 & -4992 \\ 492 & 0 & 1932 & -1020 & 222 & 0 & 42 & 60 & -2688 & 960 \\ 0 & 2148 & -1020 & 3588 & 0 & -822 & 60 & 78 & 960 & -4992 \\ 42 & -60 & 222 & 0 & 1932 & 1020 & 492 & 0 & -2688 & -960 \\ -60 & 78 & 0 & -822 & 1020 & 3588 & 0 & 2148 & -960 & -4992 \\ 222 & 0 & 42 & 60 & 492 & 0 & 1932 & -1020 & -2688 & 960 \\ 0 & -822 & 60 & 78 & 0 & 2148 & -1020 & 3588 & 960 & -4992 \\ -2688 & -960 & -2688 & 960 & -2688 & -960 & -2688 & 960 & 10752 & 0 \\ -960 & -4992 & 960 & -4992 & -960 & -4992 & 960 & -4992 & 0 & 19968 \end{pmatrix}$$

Valores proprios matriz Ke={25152.9, 13840.7, 6709.19, 2700., 1939.33, 1390.81, 1067.14, 0, 0, 0}

tenemos la suficiencia de rango para p=3

Gauss integration rule: 4 x 4

$$Ke=\begin{pmatrix} 1932 & 1020 & 492 & 0 & 42 & -60 & 222 & 0 & -2688 & -960 \\ 1020 & 3588 & 0 & 2148 & -60 & 78 & 0 & -822 & -960 & -4992 \\ 492 & 0 & 1932 & -1020 & 222 & 0 & 42 & 60 & -2688 & 960 \\ 0 & 2148 & -1020 & 3588 & 0 & -822 & 60 & 78 & 960 & -4992 \\ 42 & -60 & 222 & 0 & 1932 & 1020 & 492 & 0 & -2688 & -960 \\ -60 & 78 & 0 & -822 & 1020 & 3588 & 0 & 2148 & -960 & -4992 \\ 222 & 0 & 42 & 60 & 492 & 0 & 1932 & -1020 & -2688 & 960 \\ 0 & -822 & 60 & 78 & 0 & 2148 & -1020 & 3588 & 960 & -4992 \\ -2688 & -960 & -2688 & 960 & -2688 & -960 & -2688 & 960 & 10752 & 0 \\ -960 & -4992 & 960 & -4992 & -960 & -4992 & 960 & -4992 & 0 & 19968 \end{pmatrix}$$

Valores proprios matriz Ke={25152.9, 13840.7, 6709.19, 2700., 1939.33, 1390.81, 1067.14, 0, 0, 0}

tenemos la suficiencia de rango para p=4

Gauss integration rule: 5 x 5

$$Ke=\begin{pmatrix} 1932 & 1020 & 492 & 0 & 42 & -60 & 222 & 0 & -2688 & -960 \\ 1020 & 3588 & 0 & 2148 & -60 & 78 & 0 & -822 & -960 & -4992 \\ 492 & 0 & 1932 & -1020 & 222 & 0 & 42 & 60 & -2688 & 960 \\ 0 & 2148 & -1020 & 3588 & 0 & -822 & 60 & 78 & 960 & -4992 \\ 42 & -60 & 222 & 0 & 1932 & 1020 & 492 & 0 & -2688 & -960 \\ -60 & 78 & 0 & -822 & 1020 & 3588 & 0 & 2148 & -960 & -4992 \\ 222 & 0 & 42 & 60 & 492 & 0 & 1932 & -1020 & -2688 & 960 \\ 0 & -822 & 60 & 78 & 0 & 2148 & -1020 & 3588 & 960 & -4992 \\ -2688 & -960 & -2688 & 960 & -2688 & -960 & -2688 & 960 & 10752 & 0 \\ -960 & -4992 & 960 & -4992 & -960 & -4992 & 960 & -4992 & 0 & 19968 \end{pmatrix}$$

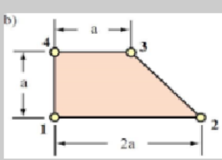
Valores proprios matriz Ke={25152.9, 13840.7, 6709.19, 2700., 1939.33, 1390.81, 1067.14, 0, 0, 0}

tenemos la suficiencia de rango para p=5

8. TEST DEL TRAPECIO

■ DEFINICION DE LA GEOMETRIA

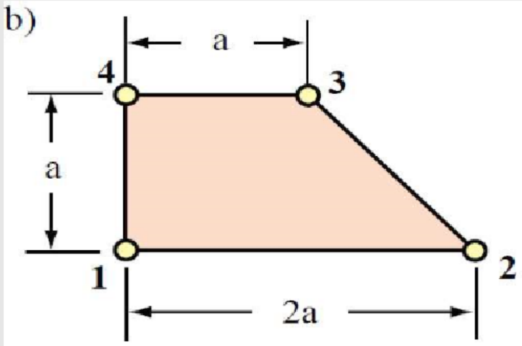
In[124]:=

TrapecioT = 

;

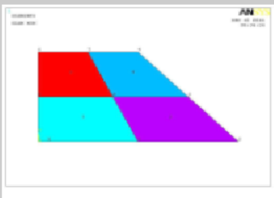
In[125]:=
```
TrapecioTr = Show[TrapecioT, ImageSize → 250]
```

Out[125]=



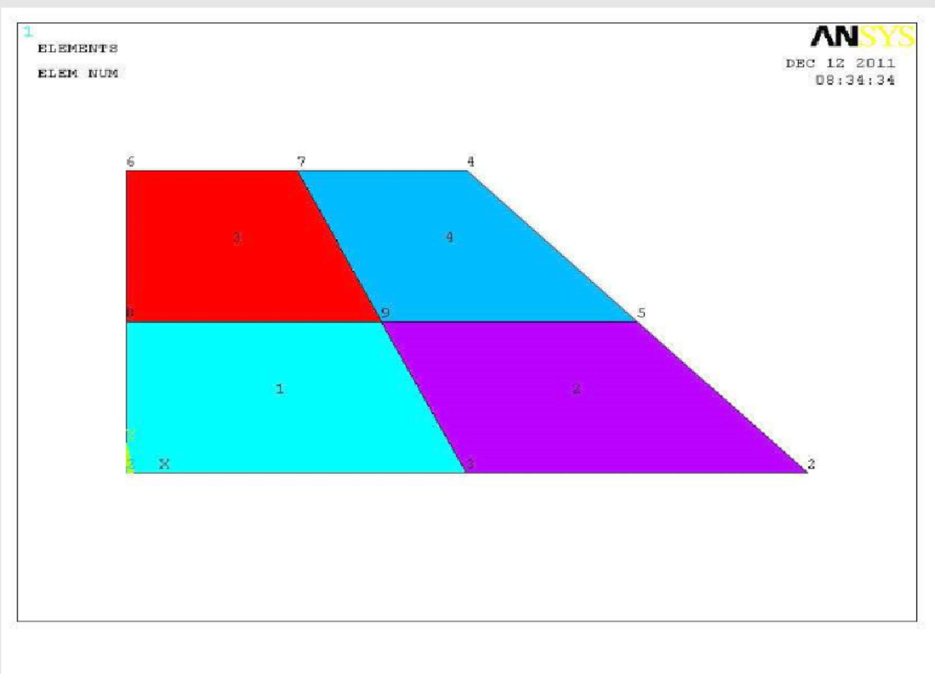
■ DEFINCION COORDENADAS NODOS ELEMENTO - ANSYS CLASSIC

▫ IMAGEN DEL MODELO EN ANSYS

In[126]:=
```
TrapecioA =
```

```
;
```

In[127]:=
```
TrapecioAr = Show[TrapecioA, ImageSize → 450]
```

Out[127]=



▫ DATOS COORDENADAS NODALES EN ANSYS

In[128]:=
```
Nodes = {{0.`, 0.`}, {1.`, 0.`}, {0.5`, 0.`}, {0.5`, 0.5`},
    {0.75`, 0.25`}, {0.`, 0.5`}, {0.25`, 0.5`}, {0.`, 0.25`}, {(0.25` + 0.5`) / 2, 0.25`}};
```

In[129]:=

```
Dimensions[Nodes][[1]]
```

Out[129]=

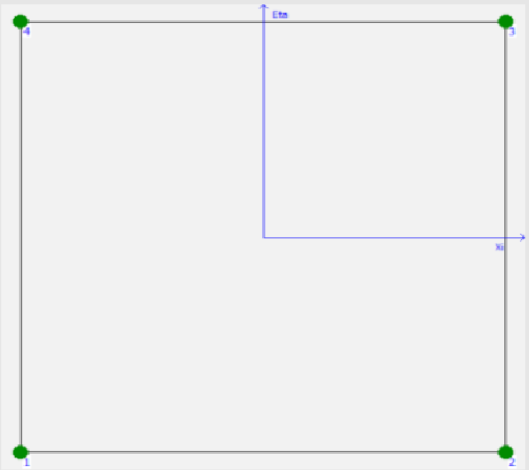
```
9
```

ORDENACION DATOS DE ANSYS

In[130]:=

```
CuaR4r
```

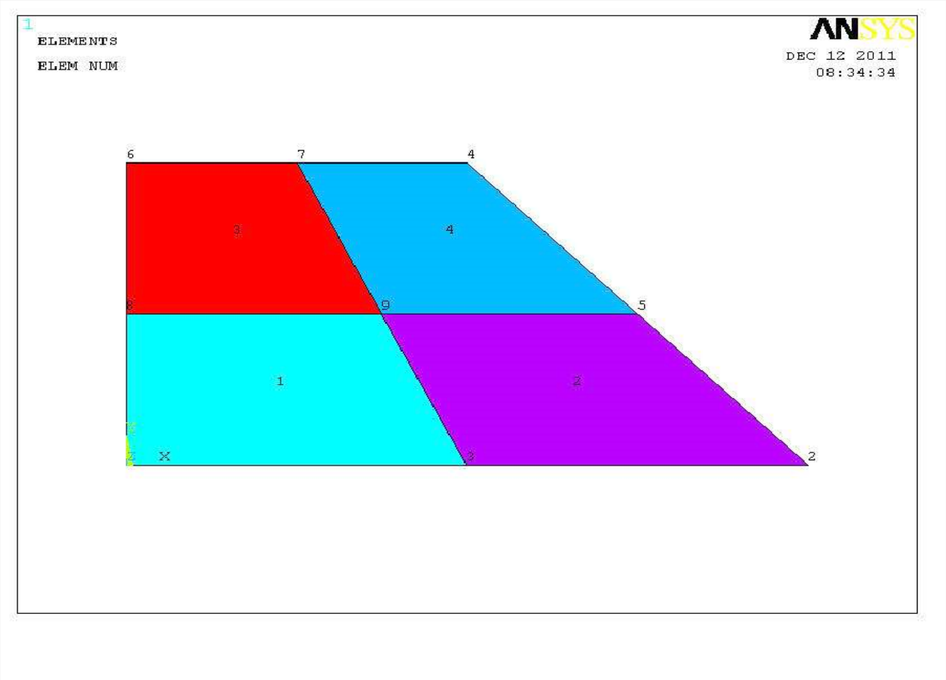
Out[130]=



In[131]:=

```
TrapecioAr
```

Out[131]=



In[132]:=

```
ncoor = {Nodes[[1]], Nodes[[2]], Nodes[[4]], Nodes[[6]], Nodes[[9]]}
```

Out[132]=

```
{{0., 0.}, {1., 0.}, {0.5, 0.5}, {0., 0.5}, {0.375, 0.25}}
```

DEFINICION DEL MATERIAL

In[133]:=

```
ClearAll[Em, nu, a, b, e, h, p, num]; h = 1;
```

In[134]:=
```
Em = 96; nu = 1 / 3; (*isotropic material*)
```

In[135]:=
```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

In[136]:=
```
Emat // MatrixForm
```

Out[136]//MatrixForm=

$$\begin{pmatrix} 108 & 36 & 0 \\ 36 & 108 & 0 \\ 0 & 0 & 36 \end{pmatrix}$$

VERIFICACION DE LA MATRIZ DE RIGIDEZ

NUMERO DE PUNTOS DE GAUSS MINIMO PARA CONSEGUIR SUFICIENCIA DE RANGO

In[137]:=
```
NF = NNodos * 2.;
```

In[138]:=

$$NG = \frac{NF - 3}{3}$$

Out[138]=
```
2.33333
```

Se necesitan como mínimo 3 Puntos -- Regla 2 x 2 minima

BUCLE GENERICO A COMPLETAR

BUCLE GENERICO: XX = No. Nodos, ZZ = GRADOS DE LIBERTAD - 3

In[139]:=
```
(*For [p=1, p≤5, p++,
 Ke=QuadXXIsoPMembraneStiffness[ncoor,{Emat,0,0},{h},{True,p}];
 Print["Gauss integration rule: ",p," x ",p];
 Print["Ke=",Chop[Ke]//MatrixForm];
 Valores=Chop[Eigenvalues[N[Ke]]];
 If[Valores[[ZZ]]≠0,Break[],Print["Valores proprios matriz Ke=",Valores]]
];
Print["Valores proprios matriz Ke=",Valores];
Print["tenemos la suficiencia de rango para p=",p]*)
```

DESARROLLO DE LA MATRIZ DE RIGIDEZ -

In[147]:=
```
For [p = 1, p ≤ 5, p++,
  Ke = Quad5IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
  Print["Gauss integration rule: ", p, " x ", p];
  Print["Ke=", Chop[Ke] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]]];
  If[Valores[[5]] ≠ 0, Break[], Print["Valores proprios matriz Ke=", Valores]]
 ];
Print["Valores proprios matriz Ke=", Valores];
Print["tenemos la suficiencia de rango para p=", p]
```

Gauss integration rule: 1 x 1

Ke=

42.	24.	-6.	-6.	-42.	-24.	6.	6.	0	0
24.	78.	-6.	30.	-24.	-78.	6.	-30.	0	0
-6.	-6.	24.	-12.	6.	6.	-24.	12.	0	0
-6.	30.	-12.	24.	6.	-30.	12.	-24.	0	0
-42.	-24.	6.	6.	42.	24.	-6.	-6.	0	0
-24.	-78.	6.	-30.	24.	78.	-6.	30.	0	0
6.	6.	-24.	12.	-6.	-6.	24.	-12.	0	0
6.	-30.	12.	-24.	-6.	30.	-12.	24.	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Valores proprios matriz Ke={200.216, 84., 51.7841, 0, 0, 0, 0, 0, 0, 0}

Gauss integration rule: 2 x 2

Ke=

72.9231	39.6923	9.23077	-3.23077	-3.23077	-1.84615	13.3846	2.30769	-92.3077	-36.9231
39.6923	133.846	-3.23077	53.5385	-1.84615	-6.	2.30769	-22.6154	-36.9231	-158.769
9.23077	-3.23077	43.8462	-18.4615	18.9231	13.3846	-1.84615	0.923077	-70.1538	7.38462
-3.23077	53.5385	-18.4615	46.6154	13.3846	-6.	0.923077	-1.84615	7.38462	-92.3077
-3.23077	-1.84615	18.9231	13.3846	93.6923	53.5385	-6.	-6.	-103.385	-59.0769
-1.84615	-6.	13.3846	-6.	53.5385	174.	-6.	30.	-59.0769	-192.
13.3846	2.30769	-1.84615	0.923077	-6.	-6.	53.5385	-26.7692	-59.0769	29.5385
2.30769	-22.6154	0.923077	-1.84615	-6.	30.	-26.7692	53.5385	29.5385	-59.0769
-92.3077	-36.9231	-70.1538	7.38462	-103.385	-59.0769	-59.0769	29.5385	324.923	59.0769
-36.9231	-158.769	7.38462	-92.3077	-59.0769	-192.	29.5385	-59.0769	59.0769	502.154

Valores proprios matriz Ke={693.004, 388.658, 200.508, 85.4466, 64.9622, 40.399, 26.0998, 0, 0, 0}

tenemos la suficiencia de rango para p=2

In[150]:=

```
For [p = 3, p ≤ 5, p++,
  Ke = Quad5IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
  Print["Gauss integration rule: ", p, " x ", p];
  Print["Ke=", Chop[Ke] // MatrixForm];
  Valores = Chop[Eigenvalues[N[Ke]]];
  Print["Valores proprios matriz Ke=", Valores];
  Print["tenemos la suficiencia de rango para p=", p]
 ];
```

Gauss integration rule: 3 x 3

$$Ke=\begin{pmatrix}
76.9143 & 40.1905 & 13.1048 & -2.63492 & 0.819048 & -1.39683 & 17.2 & 2.95238 & -108.038 & -39.1111 \\
40.1905 & 140.584 & -2.63492 & 60.3937 & -1.39683 & 0.679365 & 2.95238 & -15.7016 & -39.1111 & -185.956 \\
13.1048 & -2.63492 & 47.9937 & -17.6508 & 22.6603 & 13.873 & 2.4381 & 1.84127 & -86.1968 & 4.57143 \\
-2.63492 & 60.3937 & -17.6508 & 53.8222 & 13.873 & 0.679365 & 1.84127 & 5.53651 & 4.57143 & -120.432 \\
0.819048 & -1.39683 & 22.6603 & 13.873 & 97.8984 & 53.9683 & -2.41905 & -5.49206 & -118.959 & -60.9524 \\
-1.39683 & 0.679365 & 13.873 & 0.679365 & 53.9683 & 180.679 & -5.49206 & 36.6794 & -60.9524 & -218.717 \\
17.2 & 2.95238 & 2.4381 & 1.84127 & -2.41905 & -5.49206 & 58.0571 & -25.7143 & -75.2762 & 26.4127 \\
2.95238 & -15.7016 & 1.84127 & 5.53651 & -5.49206 & 36.6794 & -25.7143 & 61.1556 & 26.4127 & -87.6698 \\
-108.038 & -39.1111 & -86.1968 & 4.57143 & -118.959 & -60.9524 & -75.2762 & 26.4127 & 388.47 & 69.0794 \\
-39.1111 & -185.956 & 4.57143 & -120.432 & -60.9524 & -218.717 & 26.4127 & -87.6698 & 69.0794 & 612.775
\end{pmatrix}$$

Valores proprios matriz Ke={824.337, 465.796, 201.069, 86.1508, 65.6922, 43.88, 31.4249, 0, 0, 0}

tenemos la suficiencia de rango para p=3

Gauss integration rule: 4 x 4

$$Ke=\begin{pmatrix}
76.9794 & 40.2056 & 13.1664 & -2.61682 & 0.885981 & -1.38318 & 17.2598 & 2.97196 & -108.292 & -39.1776 \\
40.2056 & 140.621 & -2.61682 & 60.4336 & -1.38318 & 0.714019 & 2.97196 & -15.6598 & -39.1776 & -186.108 \\
13.1664 & -2.61682 & 48.0636 & -17.6262 & 22.7178 & 13.8879 & 2.51215 & 1.86916 & -86.4598 & 4.48598 \\
-2.61682 & 60.4336 & -17.6262 & 53.8729 & 13.8879 & 0.714019 & 1.86916 & 5.59252 & 4.48598 & -120.613 \\
0.885981 & -1.38318 & 22.7178 & 13.8879 & 97.9701 & 53.9813 & -2.36636 & -5.47664 & -119.207 & -61.0093 \\
-1.38318 & 0.714019 & 13.8879 & 0.714019 & 53.9813 & 180.714 & -5.47664 & 36.714 & -61.0093 & -218.856 \\
17.2598 & 2.97196 & 2.51215 & 1.86916 & -2.36636 & -5.47664 & 58.1383 & -25.6822 & -75.5439 & 26.3178 \\
2.97196 & -15.6598 & 1.86916 & 5.59252 & -5.47664 & 36.714 & -25.6822 & 61.2187 & 26.3178 & -87.8654 \\
-108.292 & -39.1776 & -86.4598 & 4.48598 & -119.207 & -61.0093 & -75.5439 & 26.3178 & 389.503 & 69.3832 \\
-39.1776 & -186.108 & 4.48598 & -120.613 & -61.0093 & -218.856 & 26.3178 & -87.8654 & 69.3832 & 613.443
\end{pmatrix}$$

Valores proprios matriz Ke={825.354, 466.797, 201.076, 86.1624, 65.7088, 43.9427, 31.4822, 0, 0, 0}

tenemos la suficiencia de rango para p=4

Gauss integration rule: 5 x 5

$$Ke=\begin{pmatrix}
76.9814 & 40.2061 & 13.1682 & -2.61628 & 0.887986 & -1.38277 & 17.2616 & 2.97255 & -108.299 & -39.1796 \\
40.2061 & 140.622 & -2.61628 & 60.4348 & -1.38277 & 0.715056 & 2.97255 & -15.6586 & -39.1796 & -186.113 \\
13.1682 & -2.61628 & 48.0656 & -17.6254 & 22.7195 & 13.8883 & 2.51437 & 1.86999 & -86.4677 & 4.48342 \\
-2.61628 & 60.4348 & -17.6254 & 53.8744 & 13.8883 & 0.715056 & 1.86999 & 5.5942 & 4.48342 & -120.619 \\
0.887986 & -1.38277 & 22.7195 & 13.8883 & 97.9722 & 53.9817 & -2.36478 & -5.47617 & -119.215 & -61.0111 \\
-1.38277 & 0.715056 & 13.8883 & 0.715056 & 53.9817 & 180.715 & -5.47617 & 36.7151 & -61.0111 & -218.86 \\
17.2616 & 2.97255 & 2.51437 & 1.86999 & -2.36478 & -5.47617 & 58.1407 & -25.6813 & -75.5519 & 26.3149 \\
2.97255 & -15.6586 & 1.86999 & 5.5942 & -5.47617 & 36.7151 & -25.6813 & 61.2206 & 26.3149 & -87.8713 \\
-108.299 & -39.1796 & -86.4677 & 4.48342 & -119.215 & -61.0111 & -75.5519 & 26.3149 & 389.534 & 69.3923 \\
-39.1796 & -186.113 & 4.48342 & -120.619 & -61.0111 & -218.86 & 26.3149 & -87.8713 & 69.3923 & 613.463
\end{pmatrix}$$

Valores proprios matriz Ke={825.384, 466.827, 201.076, 86.1627, 65.7093, 43.9445, 31.4839, 0, 0, 0}

tenemos la suficiencia de rango para p=5