UNIVERSIDAD POLITECNICA DE VALENCIA

DEPARTAMENTO DE INGENIERIA MECANICA Y DE MATERIALES

ELEMENTOS FINITOS

(E.T.S.I.I.V)

FORMULACION DE ELEMENTOS FINITOS
SOLUCION DE LAS ECUACIONES

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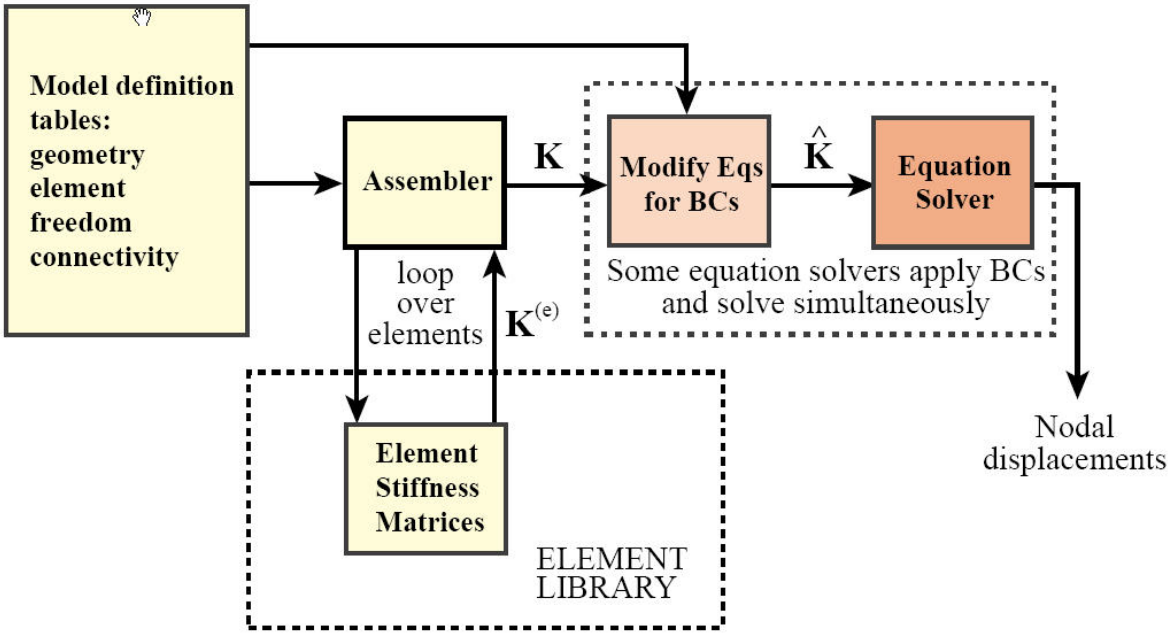
Valencia, 2005

001 PROBLEMA MATEMATICA A RESOLVER CURSO 2004-5

$$\mathbf{K}\mathbf{u} = \mathbf{f}$$

001 VISUALIZACION DEL PROCESO GENERAL DE ANALISIS POR EF CURSO 2004-5



001 RECURSOS COMPUTACIONALES NECESARIOS SOLUCION ECUACIONES – MATRIZ COMPLETA CURSO 2004-5

Storage and Solution Times for a Fully Stored Stiffness Matrix

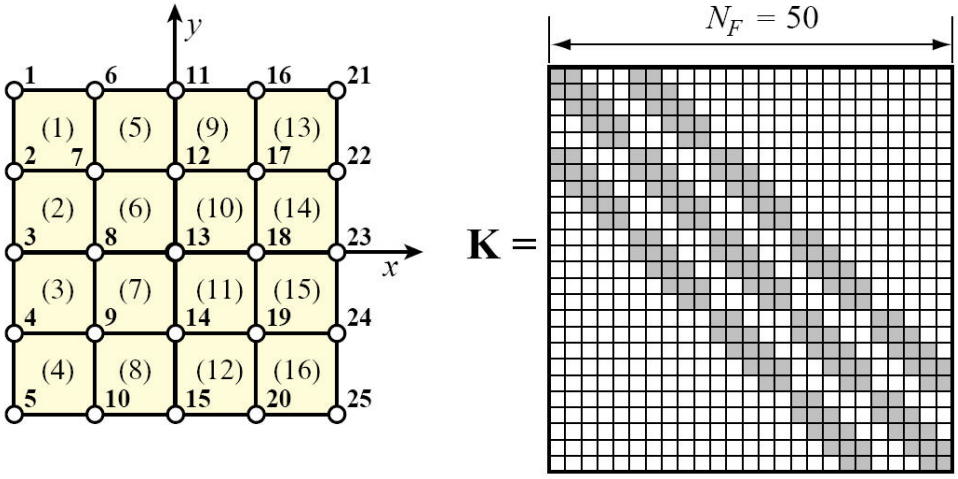
Matrix order N	Storage (double prec)	Factor op. units	Factor time workstation/PC	Factor time supercomputer
10^4	800 MB	$10^{12}/6$	3 hrs	2 min
10^5	80 GB	$10^{15}/6$	4 mos	30 hrs
10^6	8 TB	$10^{18}/6$	300 yrs	3 yrs

time numbers last adjusted in 1998
to get current times divide by 10-20

As regards memory needs, a full square matrix stored without taking advantage of symmetry, requires storage for N^2 entries. If each entry is an 8-byte, double precision floating-point number, the required storage is $8N^2$ bytes. Thus, a matrix of order $N = 10^4$ would require 8×10^8 bytes or 800 MegaBytes (MB) for storage.

For large N the solution of (26.1) is dominated by the factorization of **K**, an operation discussed in §26.2. This operation requires approximately $N^3/6$ floating point operation units. [A floating-point operation unit is conventionally defined as a (multiply,add) pair plus associated indexing and data movement operations.] Now a fast workstation can typically do 10^7 of these operations per second, whereas a supercomputer may be able to sustain 10^9 or more.

001 LA MATRIZ DE RIGIDEZ GLOBAL ES UNA MATRIZ "EN BANDA" CURSO 2004-5



001 RECURSOS COMPUTACIONALES NECESARIOS CURSO 2004-5
SOLUCION ECUACIONES - MATRIZ BANDA

Storage and Solution Times for a Skyline Stiffness Matrix Assuming $B = \sqrt{N}$

Matrix order N	Storage (double prec)	Factor op. units	Factor time workstation/PC	Factor time supercomputer
10^4	8 MB	$10^8/2$	5 sec	0.05 sec
10^5	240 MB	$10^{10}/2$	8 min	5 sec
10^6	8000 MB	$10^{12}/2$	15 hrs	8 min

time numbers last adjusted in 1998
to get current times divide by 10-20

If a skymatrix of order N can be stored in S memory locations, the ratio $B = S/N$ is called the *mean bandwidth*. If the entries are, as usual, 8-byte double-precision floating-point numbers, the storage requirement is $8NB$ bytes. The factorization of a skymatrix requires approximately $\frac{1}{2}NB^2$ floating-point operation units. In two-dimensional problems B is of the order of $\sqrt{N}$.

001 CONTENIDOS A TRATAR EN ESTA LECCION CURSO 2004-5

How the Master Stiffness Equations are Stored in a commonly used "skyline" sparse format

How to Mark BC on the Master Stiffness Eqs (if you write your own solver)

The Basic Solution Steps

(Implementation Details will be Skipped since Built-in Mathematica Solver will be used for Demo Programs)

001 **ALMACENAMIENTO TIPO SKYLINE - EJEMPLO** **CURSO 2004-5**

MATRIZ DE RIGIDEZ

$$\mathbf{K} = \begin{bmatrix} 11 & & 13 & & 16 \\ & 22 & 0 & 24 & 0 \\ & & 33 & 34 & 0 \\ & & & 44 & 46 \\ & & & & 55 & 56 \\ & symm & & & & 66 \end{bmatrix}$$

VECTOR SKYLINE

s= { 11,22,13,0,33,24,34,44,55,16,0,0,46,56,66}

VECTOR LOCALIZACION TERMINOS DIAGONAL

p= { 0,1,2,5,8,9,15}

VECTOR COMPLETO

S= { p, s}

S={ { 0,1,2,5,8,9,15} ,{11,22,13,0,33,24,34,44,55,16,0,0,46,56,66} }

001 **MARCADO CONDICIONES CONTORNO EN DESPLAZAMIENTOS** **CURSO 2004-5**

Equations for which the displacement component is known or prescribed are identified by a ***negative*** diagonal location value. For example, if u_3 and u_5 are prese4cribed displacement components in the sample system,

p : [0, 1, 2, −5, 8, −9, 15]

001 **PASOS BASICOS PROCESO OBTENCION DESPLAZAMIENTOS** CURSO 2004-5

FACTORIZACION

$$\mathbf{K} = \mathbf{LDU} = \mathbf{LDL}^T = \mathbf{U}^T\mathbf{DU}$$

where **L** is a unit lower triangular matrix, **D** is a nonsingular diagonal matrix, and **U** and **L** are the transpose of each other. The original matrix is overwritten by the entries of $\mathbf{D}^{-1}$ and **U**.

`SymmSkyMatrixFactor`

SOLUCION

$$\textit{Forward reduction}: \quad \mathbf{Lz} = \mathbf{f},$$

$$\textit{Diagonal scaling}: \quad \mathbf{Dy} = \mathbf{z},$$

$$\textit{Back substitution}: \quad \mathbf{Uu} = \mathbf{y},$$

`SymmSkyMatrixVectorSolve`