LECCION 7 - EJERCICIO 8 (18.8) v.2005

INICIO

```
Off[General::"spell1"]
Off[General::"spell"]
```

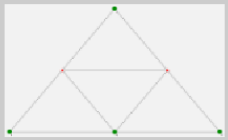
```
SetDirectory[NotebookDirectory[]]
```

```
C:\#0-Modulos-M30x_MeF-10\#M306-m6-a3a-sws\08-Funciones-forma
```

DEFINICION ELEMENTO TRIANGULAR DE TRANSICION DE 4 NODOS

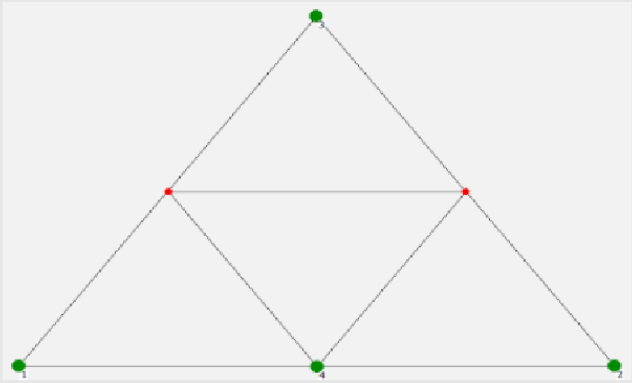
DEFINICION GRAFICA

```
TriT4 =
```



```
;
```

```
TriT4r = Show[TriT4, ImageSize → 300]
```



COORDENADAS TRIANGULARES NODOS

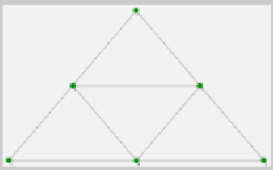
```
Cnt = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}, {1/2, 1/2, 0}};
```

```
NNodos = Dimensions[Cnt][[1]]
```

```
4
```

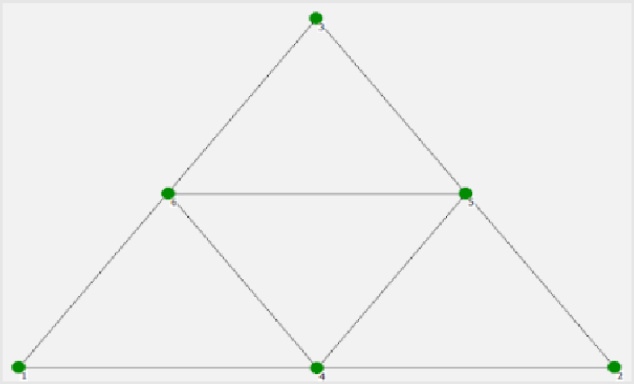
ELEMENTO COMPLETO NECESARIO - 2 DIVISIONES POR LADO

```
TriR6 =
```



```
;
```

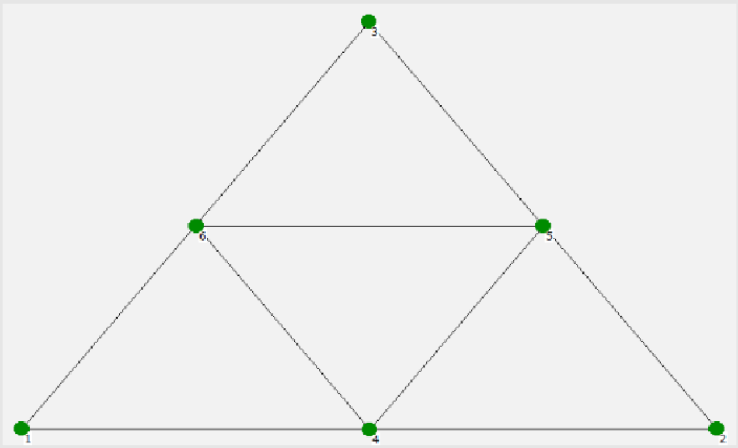
```
Show[TriR6, ImageSize → 300]
```



FUNCIONES DE FORMA - METODO PRODUCTO DE CURVAS

■ CURVAS A CONSIDERAR - TRIANGULO REGULAR DE 6 NODOS

```
Show[TriR6, ImageSize → 350]
```



```
Cu = Table[0, {i, 9}];
```

□ LADOS

```
Cu[[1]] = ξ3; Cu[[2]] = ξ1; Cu[[3]] = ξ2;
```

□ MEDIANAS

```
Cu[[4]] = (ξ1 - 1 / 2);
```

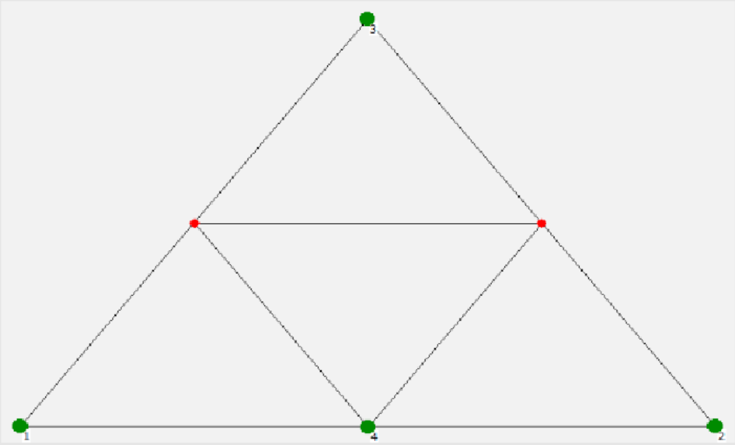
```
Cu[[5]] = (ξ2 - 1 / 2);
```

```
Cu[[6]] = (ξ3 - 1 / 2);
```

■ DEFINICION PRODUCTO DE CURVAS EN CADA NODO - NODOS NO ESQUINA

```
Nc = Table[0, {i, NNodos}];
```

```
Show[TriT4, ImageSize → 350]
```



Tipo 2 - LADOS

```
Nc[[4]] = Cu[[2]] * Cu[[3]];
```

OBTENCION FUNCIONES DE FORMA - NODOS NO ESQUINA

```
Clear[Nf]
```

```
Nfp = Table[0, {i, NNodos}];
```

```
Nf = Table[0, {i, NNodos}];
```

```
Do[
  Nfp[[i]] = a * Nc[[i]];
  eq = 1 == Nfp[[i]] /. {ξ1 -> Cnt[[i, 1]], ξ2 -> Cnt[[i, 2]], ξ3 -> Cnt[[i, 3]]};
  as = a /. Solve[eq, a][[1]]; Print["Nodo ", i];
  Nf[[i]] = Simplify[Nfp[[i]] /. {a → as}],
  {i, 4, NNodos}
 ];
```

Nodo 4

```
MatrixForm[Nf]
```

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 4\,\xi 1\,\xi 2 \end{pmatrix}$$

OBTENCION FUNCIONES DE FORMA - NODOS ESQUINA

Utilizamos las Funciones de Forma del Triangulo de 3 Nodos.

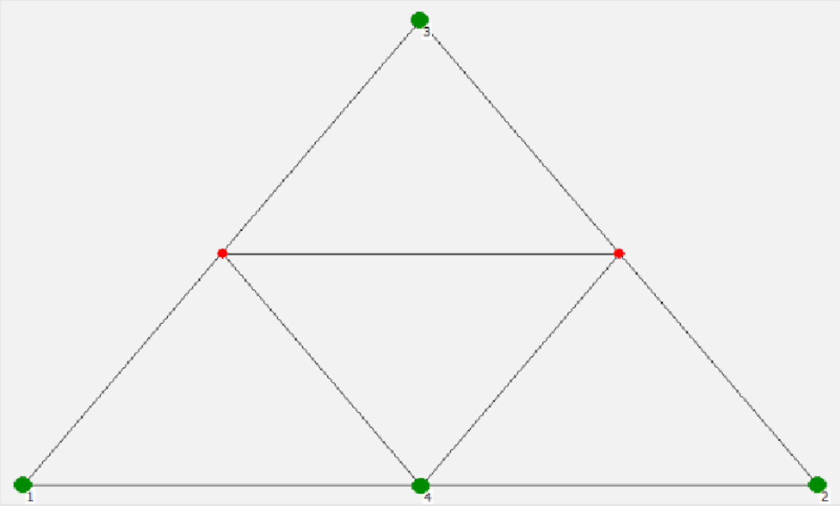
```
Nf3 = {ξ1, ξ2, ξ3};
```

NODO 1

```
Clear[a4]
```

```
Nf[[1]] = Nf3[[1]] + a4 * Nf[[4]];
```

```
Show[TriT4, ImageSize → 400]
```




```
eq = 0 == Nf[[1]] /. {ξ1 -> Cnt[[4]][[1]], ξ2 -> Cnt[[4]][[2]], ξ3 -> Cnt[[4]][[3]]}
a4s = a4 /. Solve[eq, a4][[1]]
```

$0 == \frac{1}{2} + a4$

$-\frac{1}{2}$

```
Nf[[1]] = Simplify[Nf[[1]] /. {a4 → a4s}]
```

$\xi1 - 2\,\xi1\,\xi2$

$\xi1 - 2\,\xi1\,\xi2$

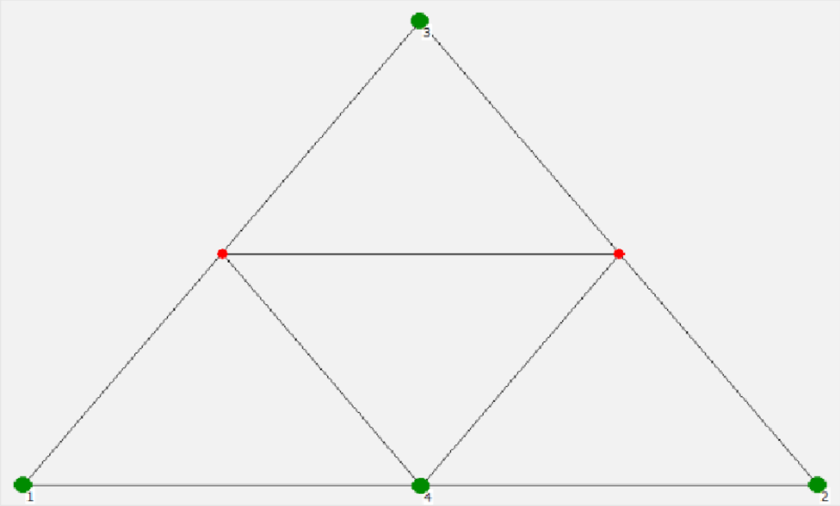
$\xi1 - 2\,\xi1\,\xi2$

NODO 2

```
Clear[a4]
```

```
Nf[[2]] = Nf3[[2]] + a4 * Nf[[4]];
```

```
Show[TriT4, ImageSize → 400]
```




```
eq = 0 == Nf[[2]] /. {ξ1 -> Cnt[[4]][[1]], ξ2 -> Cnt[[4]][[2]], ξ3 -> Cnt[[4]][[3]]}
a4s = a4 /. Solve[eq, a4][[1]]
```

$0 == \frac{1}{2} + a4$

$-\frac{1}{2}$

```
Nf[[2]] = Simplify[Nf[[2]] /. {a4 → a4s}]
```

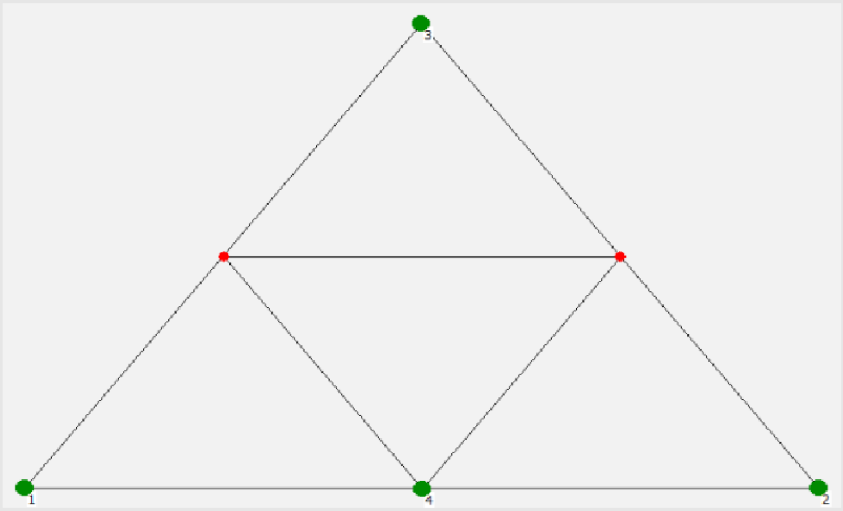
$\xi 2 - 2\,\xi 1\,\xi 2$

NODO 3

```
Clear[a4]
```

```
Nf[[3]] = Nf3[[3]] + a4 * Nf[[4]];
```

```
Show[TriT4, ImageSize → 400]
```




```
eq = 0 == Nf[[3]] /. {ξ1 -> Cnt[[4]][[1]], ξ2 -> Cnt[[4]][[2]], ξ3 -> Cnt[[4]][[3]]}
a4s = a4 /. Solve[eq, a4][[1]]
```

```
0 == a4
```

```
0
```

```
Nf[[3]] = Simplify[Nf[[3]] /. {a4 → a4s}]
```

```
ξ3
```

■ FUNCIONES DE FORMA DE TODOS LOS NODOS

```
MatrixForm[Nf]
```

$$\begin{pmatrix} \xi_1 - 2\,\xi_1\,\xi_2 \\ \xi_2 - 2\,\xi_1\,\xi_2 \\ \xi_3 \\ 4\,\xi_1\,\xi_2 \end{pmatrix}$$

■ COMPROBACION SUMA UNIDAD

```
Suma = Sum[Nf[[i]], {i, 1, NNodos}]
```

$$\sum_{i=1}^{NNodos} Nf[[i]]$$

```
ξ1 + ξ2 + ξ3
```

```
Simplify[Suma /. {ξ1 → 1 - ξ2 - ξ3}]
```

```
1
```

OK - SE CUMPLE LA CONDICION DE COMPLETITUD

Proceso para comprobar Valor Funciones de Forman en Nodos - en caso de Error

```
(*Do[
  Print["NODO ",j];
  Do[
   Print[i,"  ",Simplify[Nf[[j]]/.{ξ1 ->Cnt[[i,1]],ξ2->Cnt[[i,2]],ξ3->Cnt[[i,3]]}]/.{ξ1→1-ξ2-ξ3}],
   {i,NNodos}
     ],
  {j,NNodos}
 ];*)
```

REPRESENTACION GRAFICA

Función Representación Gráfica Funciones de Forma

```
PlotTriangleShapeFunction[xytrig_, f_, Nsub_, aspect_] :=
  Module[{Ni, line3D = {}, poly3D = {}, zc1, zc2, zc3, xyf1, xyf2, xyf3, xc, yc, x1, x2, x3, y1, y2,
    y3, z1, z2, z3, iz1, iz2, iz3, d}, {{x1, y1, z1}, {x2, y2, z2}, {x3, y3, z3}} = Take[xytrig, 3];
   xc = {x1, x2, x3}; yc = {y1, y2, y3}; Ni = Nsub * 3;
   Do[Do[iz3 = Ni - iz1 - iz2; If[iz3 ≤ 0, Continue[]]; d = 0; If[Mod[iz1 + 2, 3] == 0 && Mod[iz2 - 1, 3] == 0,
      d = 1]; If[Mod[iz1 - 2, 3] == 0 && Mod[iz2 + 1, 3] == 0, d = -1]; If[d == 0, Continue[]];
     zc1 = N[{iz1 + d + d, iz2 - d, iz3 - d} / Ni]; zc2 = N[{iz1 - d, iz2 + d + d, iz3 - d} / Ni];
     zc3 = N[{iz1 - d, iz2 - d, iz3 + d + d} / Ni]; xyf1 = {xc.zc1, yc.zc1, f[zc1[[1]], zc1[[2]], zc1[[3]]]};
     xyf2 = {xc.zc2, yc.zc2, f[zc2[[1]], zc2[[2]], zc2[[3]]]}; xyf3 = {xc.zc3, yc.zc3,
       f[zc3[[1]], zc3[[2]], zc3[[3]]]}; AppendTo[poly3D, Polygon[{xyf1, xyf2, xyf3}]];
     AppendTo[line3D, Line[{xyf1, xyf2, xyf3, xyf1}]], {iz2, 1, Ni - iz1}], {iz1, 1, Ni}];
   Show[Graphics3D[RGBColor[1, 0, 0]], Graphics3D[poly3D], Graphics3D[Thickness[.002]],
    Graphics3D[line3D], Graphics3D[RGBColor[0, 0, 0]], Graphics3D[Thickness[.005]],
    Graphics3D[Line[xytrig]], PlotRange → All, BoxRatios → {1, 1, aspect}, Boxed → False]];
```

Representación Gráfica Funciones Forma Elemento.

```
Ng = Table[0, {i, NNodos}];
```

```
xyc1 = {0, 0, 0}; xyc2 = {3, 0, 0}; xyc3 = {Sqrt[3], 3 / 2, 0}; xytrig = N[{xyc1, xyc2, xyc3, xyc1}];
```

Control de Cuadricula

```
Nsub = 20;
```

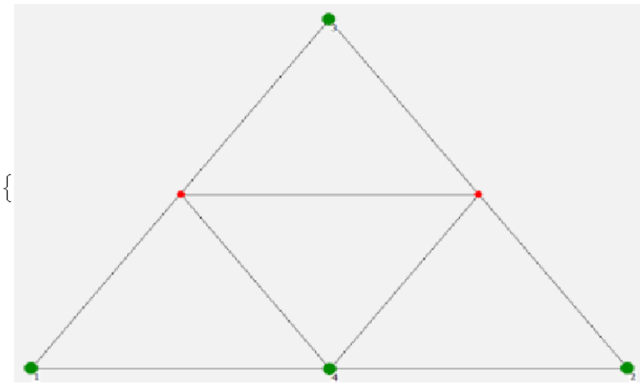
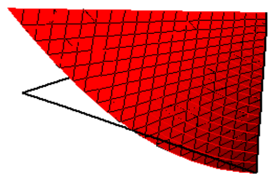
```
Do[
  fi[ξ1_, ξ2_, ξ3_] = Nf[[i]];
  Ng[[i]] = PlotTriangleShapeFunction[xytrig, fi, Nsub, 1 / 2],
  {i, NNodos}
 ];
```

RESULTADOS INTERACTIVOS

```
Manipulate[{TriT4r, Ng[[n]], Nf[[n]]}, {n, 1, Dimensions[Nf][[1]], 1}, {n, Range[Dimensions[Nf][[1]]]},
 FrameLabel → {"FUNCION DE FORMA EN NODO n - TRIANGULO TRANSICION 4 NODOS"}, SaveDefinitions → True]
```

n

n 1 2 3 4

{  ,  , ξ1 − 2 ξ1 ξ2}

FUNCION DE FORMA EN NODO n – TRIANGULO TRANSICION 4 NODOS

COMPROBACION COMPATIBILIDAD - N1 - LADO 1-2

CONDICION EN LADO

```
{ξ3 → 0};
```

GRADO FUNCION DE FORMA N1 EN LADO 1-2 -> CUADRATICA

```
Nf[[1]]
```

ξ1 − 2 ξ1 ξ2

```
Expand[Nf[[1]] /. {ξ3 → 0}]
```

ξ1 − 2 ξ1 ξ2

UNA FUNCION CUADRATICA SE DEFINE POR 3 NODOS - EN AL LADO 1-2 HAY 3 ---> SE CUMPLE LA CONDICION DE COMPATIBILIDAD

COMPROBACION COMPATIBILIDAD - N1 - LADO 1-3

CONDICION EN LADO

```
{ξ2 → 0};
```

GRADO FUNCION DE FORMA N1 EN LADO 1-2 -> LINEAR

```
Nf[[1]]
```

$\xi1 - 2\,\xi1\,\xi2$

```
Expand[Nf[[1]] /. {ξ2 → 0}]
```

$\xi1$

UNA FUNCION LINEAR SE DEFINE POR 2 NODOS - EN AL LADO 1-3 HAY 2 ---> SE CUMPLE LA CONDICION DE COMPATIBILIDAD