

LECCION 7 - EJERCICIO 6 (18.6) v.2005

■ INICIO


```
Off [General::"spell1"]  
Off [General::"spell"]
```

```
SetDirectory [NotebookDirectory []]
```

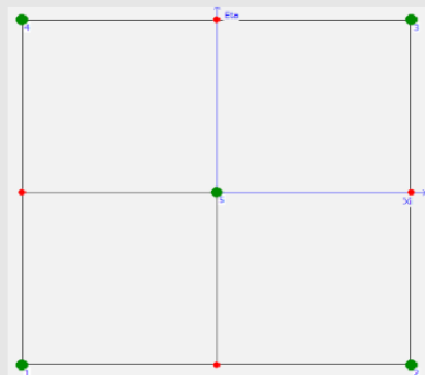
```
C:\#0-Modulos-M30x_MeF-10\#M306-m6-a3a-sws\08-Funciones-forma
```

■ DEFINICION ELEMENTO CUADRILATERO DE TRANSICION DE 5 NODOS

□ DEFINICION GRAFICA

```
CuaT5 =  ;
```

```
CuaT5r = Show [CuaT5, ImageSize -> 200]
```



□ COORDENADAS NATURALES NODOS

```
Cn = {{-1, -1}, {1, -1}, {1, 1}, {-1, 1}, {0, 0}};
```

```
NNodos = Dimensions [Cn] [[1]]
```

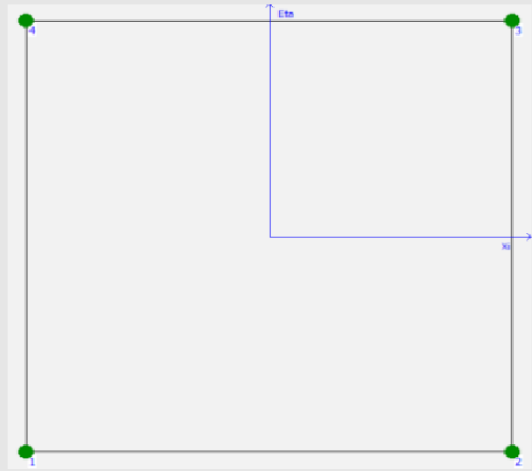
```
5
```

■ ELEMENTOS COMPLETOS NECESARIOS

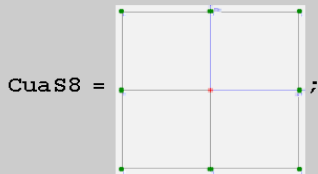
□ REGULAR DE 4 NODOS



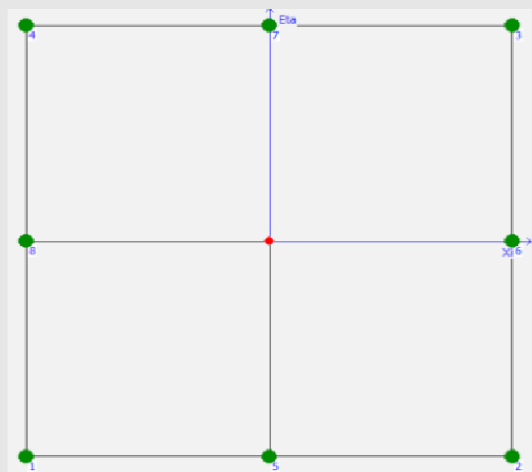
```
CuaR4r = Show[CuaR4, ImageSize -> 250]
```



□ SERENDIPITO DE 8 NODOS - 2 DIVISIONES POR LADO



```
CuaS8r = Show[CuaS8, ImageSize -> 250]
```



■ Curvas a Considerar

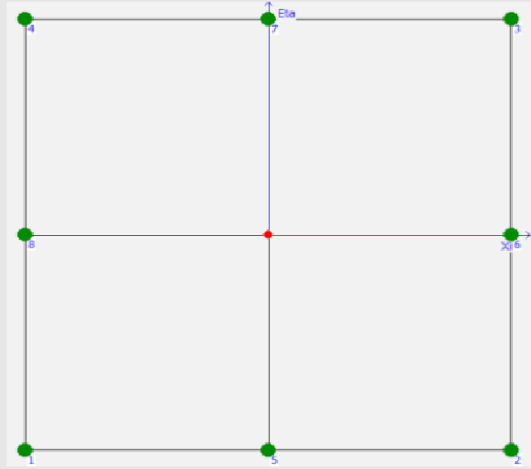
□ LADOS - CUADRILATERO REGULAR DE 4 NODOS

```
CuCR04N = Table[0, {i, 4}];
```

```
CuCR04N[[1]] = ( $\eta + 1$ ); CuCR04N[[2]] = ( $\xi - 1$ ); CuCR04N[[3]] = ( $\eta - 1$ ); CuCR04N[[4]] = ( $\xi + 1$ );
```

□ LADOS Y MEDIANAS - CUADRILATERO SERENDIPITO DE 8 NODOS

```
Show[CuaS8, ImageSize -> 250]
```



```
CuCS8N = Table[0, {i, 6}];
```

```
CuCS8N[[1]] = ( $\eta + 1$ );
```

```
CuCS8N[[2]] = ( $\xi - 1$ );
```

```
CuCS8N[[3]] = ( $\eta - 1$ );
```

```
CuCS8N[[4]] = ( $\xi + 1$ );
```

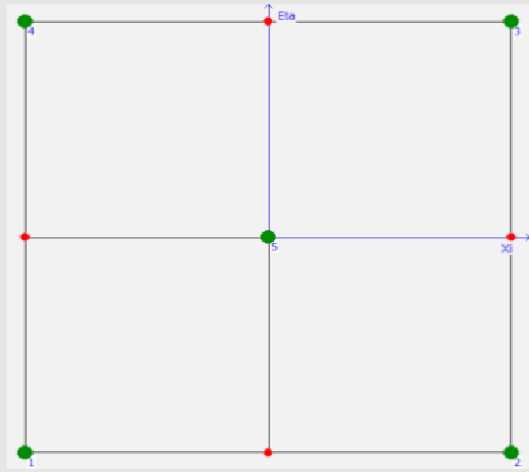
```
CuCS8N[[5]] = ( $\xi$ );
```

```
CuCS8N[[6]] = ( $\eta$ );
```

■ Definición Productos de Curvas en cada Nodo - # - NODOS NO ESQUINA

```
Nc = Table[{0, 0}, {i, NNodos}];
```

Show[CuaT5, ImageSize -> 250]



□ Tipo 1 - INTERIOR

```
Nc[[5]] = CuCS8N[[1]] * CuCS8N[[2]] * CuCS8N[[3]] * CuCS8N[[4]];
```

■ Obtención Funciones de Forma - NODOS NO ESQUINA

```
Clear[Nf]
```

```
Nfp = Table[0, {i, NNodos}];
```

```
Nf = Table[0, {i, NNodos}];
```

```
Do[
  Nfp[[i]] = a * Nc[[i]];
  eq = 1 == Nfp[[i]] /. {xi -> Cn[[i, 1]], eta -> Cn[[i, 2]]};
  as = a /. Solve[eq, a][[1]]; Print["Nodo ", i];
  Nf[[i]] = Simplify[Nfp[[i]] /. {a -> as}],
  {i, 5, NNodos}
];
```

Nodo 5

```
MatrixForm[Nf]
```

$$\begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \\ (-1 + \eta) & (1 + \eta) & (-1 + \xi) & (1 + \xi) \end{pmatrix}$$

■ Obtención Funciones de Forma - NODOS ESQUINA

Utilizamos las Funciones de Forma del Cuadrilatero de 4 Nodos.

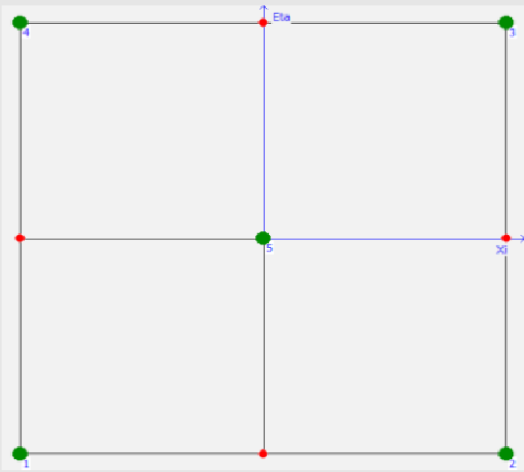
$$Nf_{FCR4} = \left\{ \frac{1}{4} (-1 + \eta) (-1 + \xi), -\frac{1}{4} (-1 + \eta) (1 + \xi), \frac{1}{4} (1 + \eta) (1 + \xi), -\frac{1}{4} (1 + \eta) (-1 + \xi) \right\};$$

□ **NODO 1 - Desarrollo - #**

```
Clear[a5];
```

```
Nf[[1]] = NfCR4[[1]] + a5 * Nf[[5]];
```

```
Show[CuaT5, ImageSize -> 250]
```



```
eq = 0 == Nf[[1]] /. {xi -> Cn[[5]][[1]], eta -> Cn[[5]][[2]]}
a5s = a5 /. Solve[eq, a5][[1]]
```

$$0 == \frac{1}{4} + a5$$

$$-\frac{1}{4}$$

```
Nf[[1]] = Simplify[Nf[[1]] /. {a5 -> a5s}];
```

```
Nf[[1]]
```

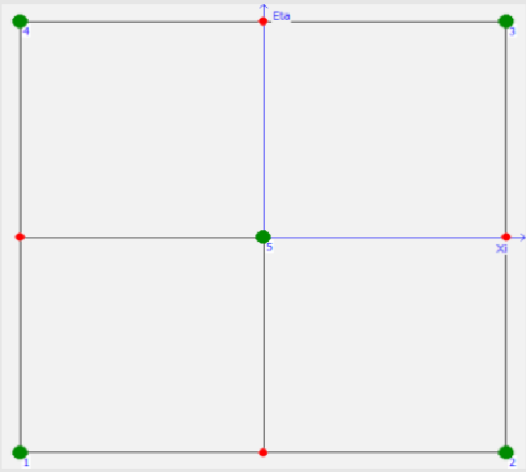
$$-\frac{1}{4} (-1 + \eta) (-1 + \xi) (\eta + \xi + \eta \xi)$$

□ **NODO 2 - Desarrollo - #**

```
Clear[a5];
```

```
Nf[[2]] = NfCR4[[2]] + a5 * Nf[[5]];
```

Show[CuaT5, ImageSize -> 250]



```
eq = 0 == Nf[[2]] /. {xi -> Cn[[5]][[1]], eta -> Cn[[5]][[2]]}
a5s = a5 /. Solve[eq, a5][[1]]
```

$$0 == \frac{1}{4} + a5$$

$$-\frac{1}{4}$$

```
Nf[[2]] = Simplify[Nf[[2]] /. {a5 -> a5s}]
```

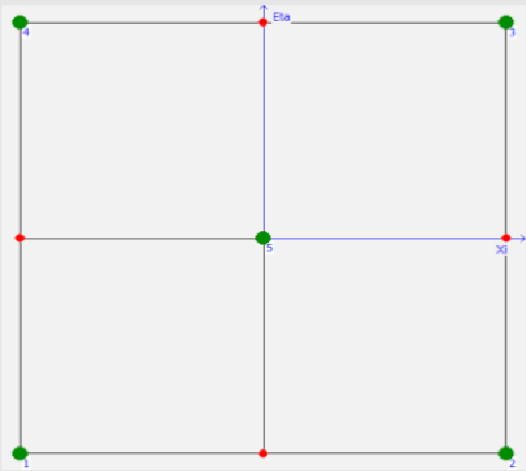
$$-\frac{1}{4} (-1 + \eta) (1 + \xi) (\eta (-1 + \xi) + \xi)$$

□ NODO 3 - Desarrollo - #

```
Clear[a5];
```

```
Nf[[3]] = NfCR4[[3]] + a5 * Nf[[5]];
```

Show[CuaT5, ImageSize -> 250]



```
eq = 0 == Nf[[3]] /. {ξ -> Cn[[5]][[1]], η -> Cn[[5]][[2]]}
a5s = a5 /. Solve[eq, a5][[1]]
```

$$0 == \frac{1}{4} + a5$$

$$-\frac{1}{4}$$

```
Nf[[3]] = Simplify[Nf[[3]] /. {a5 -> a5s}]
```

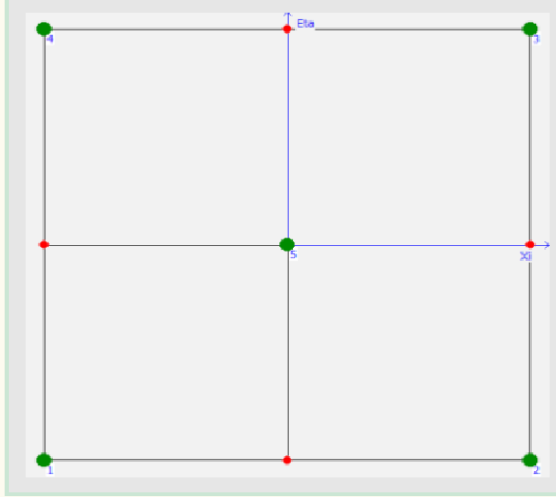
$$\frac{1}{4} (\eta + \eta^2 + \xi + \eta \xi + \xi^2 - \eta^2 \xi^2)$$

□ NODO 4 - Desarrollo - #

```
Clear[a5];
```

```
Nf[[4]] = NfCR4[[4]] + a5 * Nf[[5]];
```

```
Show[CuaT5, ImageSize -> 250]
```



```
eq = 0 == Nf[[4]] /. {ξ -> Cn[[5]][[1]], η -> Cn[[5]][[2]]}
a5s = a5 /. Solve[eq, a5][[1]]
```

$$0 == \frac{1}{4} + a5$$

$$-\frac{1}{4}$$

```
Nf[[4]] = Simplify[Nf[[4]] /. {a5 -> a5s}]
```

$$-\frac{1}{4} (1 + \eta) (-1 + \xi) (\eta - \xi + \eta \xi)$$

■ **Funciones de Forma de todos los Nodos.**

MatrixForm[Nf]

$$\begin{pmatrix} -\frac{1}{4} (-1+\eta) (-1+\xi) (\eta + \xi + \eta \xi) \\ -\frac{1}{4} (-1+\eta) (1+\xi) (\eta (-1+\xi) + \xi) \\ \frac{1}{4} (\eta + \eta^2 + \xi + \eta \xi + \xi^2 - \eta^2 \xi^2) \\ -\frac{1}{4} (1+\eta) (-1+\xi) (\eta - \xi + \eta \xi) \\ (-1+\eta) (1+\eta) (-1+\xi) (1+\xi) \end{pmatrix}$$

■ **Comprobación Suma Unidad - #**

$$\text{Suma} = \sum_{i=1}^{\text{NNodos}} \text{Nf}[[i]]$$

$$\begin{aligned} & (-1+\eta) (1+\eta) (-1+\xi) (1+\xi) - \frac{1}{4} (-1+\eta) (1+\xi) (\eta (-1+\xi) + \xi) - \\ & \frac{1}{4} (1+\eta) (-1+\xi) (\eta - \xi + \eta \xi) - \frac{1}{4} (-1+\eta) (-1+\xi) (\eta + \xi + \eta \xi) + \frac{1}{4} (\eta + \eta^2 + \xi + \eta \xi + \xi^2 - \eta^2 \xi^2) \end{aligned}$$

Simplify[%]

1

OK.

□ **Proceso para comprobar Valor Funciones de Forman en Nodos - en caso de Error**

```
Do[
  Print["NODO ", j];
  Do[
    Print[i, " ", Simplify[Nf[[j]] /. {ξ -> Cn[[i, 1]], η -> Cn[[i, 2]]}],
    {i, NNodos}
  ],
  {j, NNodos}
];
```

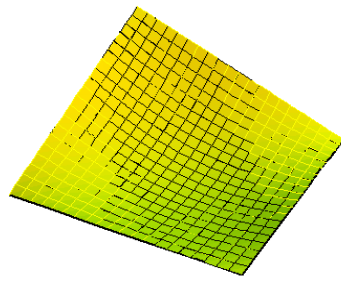
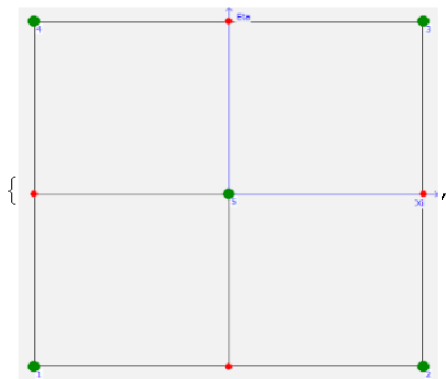
■ **Representación Gráfica.**

RESULTADOS INTERACTIVOS

```
Manipulate[{CuaT5r, Ng[[n]], Nf[[n]]}, {n, 1, Dimensions[Nf][[1]], 1}, {n, Range[Dimensions[Nf][[1]]]},  
FrameLabel -> {"FUNCION DE FORMA EN NODO n - CUADRILATERO TRANSICION 5 NODOS"},  
SaveDefinitions -> True]
```

n

n



$$\left\{ -\frac{1}{4} (1 + \eta) (-1 + \xi) (\eta - \xi + \eta \xi) \right\}$$

FUNCION DE FORMA EN NODO n - CUADRILATERO TRANSICION 5 NODOS