

LECCION 7 - EJERCICIO 4 (18.4) v.2005

■ INICIO

```
Off [General::"spell1"]  
Off [General::"spell"]
```

```
SetDirectory[NotebookDirectory[]]
```

```
C:\H0-Modulos-M30x_MeF-10\H306-m6-a3a-sws\08-Funciones-forma
```

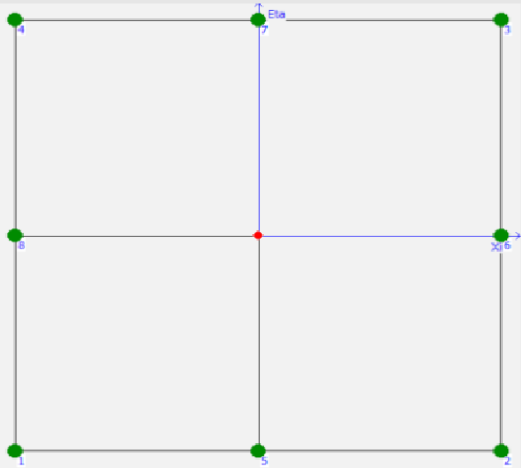
■ DEFINICION ELEMENTO CUADRILATERO SERENDIPITO DE 8 NODOS

□ DEFINICION GRAFICA

```
CuaS8 =
```



```
CuaS8r = Show[CuaS8, ImageSize -> 250]
```



□ COORDENADAS NATURALES NODOS

```
Cn = {{-1, -1}, {1, -1}, {1, 1}, {-1, 1}, {0, -1}, {1, 0}, {0, 1}, {-1, 0}};
```

```
NNodos = Dimensions[Cn][[1]]
```

```
8
```

FUNCIONES DE FORMA - METODO PRODUCTO DE CURVAS -

■ CURVAS A CONSIDERAR

```
Cu = Table[0, {i, 4}];
```

□ LADOS

```
Cu[[1]] = (η + 1); Cu[[2]] = (ξ - 1); Cu[[3]] = (η - 1); Cu[[4]] = (ξ + 1);
```

□ DIAGONALES

Línea que pasa por dos puntos y no es ni vertical ni horizontal:

$$Lc[\xi1_, \eta1_, \xi2_, \eta2_] = \eta - \eta1 - \left(\frac{\eta2 - \eta1}{\xi2 - \xi1} \right) * (\xi - \xi1);$$

```
Cdu = Table[0, {i, 4}];
```

```
Cdu[[1]] = Lc[Cn[[8, 1]], Cn[[8, 2]], Cn[[5, 1]], Cn[[5, 2]]]
```

```
1 + η + ξ
```

```
Cdu[[2]] = Lc[Cn[[5, 1]], Cn[[5, 2]], Cn[[6, 1]], Cn[[6, 2]]]
```

```
1 + η - ξ
```

```
Cdu[[3]] = Lc[Cn[[6, 1]], Cn[[6, 2]], Cn[[7, 1]], Cn[[7, 2]]]
```

```
-1 + η + ξ
```

```
Cdu[[4]] = Lc[Cn[[7, 1]], Cn[[7, 2]], Cn[[8, 1]], Cn[[8, 2]]]
```

```
-1 + η - ξ
```

■ DEFINICION PRODUCTOS DE CURVAS EN CADA NODO

```
Nc = Table[0, {i, NNodos}];
```

□ Tipo 1 - ESQUINA

```
Nc[[1]] = Cu[[2]] * Cu[[3]] * Cdu[[1]]
```

```
(-1 + η) (-1 + ξ) (1 + η + ξ)
```

```
Nc[[2]] = Cu[[3]] * Cu[[4]] * Cdu[[2]]
```

```
(-1 + η) (1 + η - ξ) (1 + ξ)
```

$$Nc[[3]] = Cu[[1]] * Cu[[4]] * Cdu[[3]]$$

$$(1 + \eta) (1 + \xi) (-1 + \eta + \xi)$$

$$Nc[[4]] = Cu[[1]] * Cu[[2]] * Cdu[[4]]$$

$$(1 + \eta) (-1 + \eta - \xi) (-1 + \xi)$$

□ **Tipo 2 - LADOS**

$$Nc[[5]] = Cu[[2]] * Cu[[3]] * Cu[[4]]$$

$$(-1 + \eta) (-1 + \xi) (1 + \xi)$$

$$Nc[[6]] = Cu[[1]] * Cu[[3]] * Cu[[4]]$$

$$(-1 + \eta) (1 + \eta) (1 + \xi)$$

$$Nc[[7]] = Cu[[1]] * Cu[[2]] * Cu[[4]]$$

$$(1 + \eta) (-1 + \xi) (1 + \xi)$$

$$Nc[[8]] = Cu[[1]] * Cu[[2]] * Cu[[3]]$$

$$(-1 + \eta) (1 + \eta) (-1 + \xi)$$

■ **OBTENCION FUNCIONES DE FORMA**

```
Clear[Nf]
```

```
Nfp = Table[0, {i, NNodos}];
```

```
Nf = Table[0, {i, NNodos}];
```

```
Do[
  Nfp[[i]] = a * Nc[[i]];
  eq = 1 == Nfp[[i]] /. {ξ -> Cn[[i, 1]], η -> Cn[[i, 2]]};
  as = a /. Solve[eq, a][[1]]; Print["Nodo ", i];
  Nf[[i]] = Simplify[Nfp[[i]] /. {a -> as}],
  {i, NNodos}
];
```

Nodo 1

Nodo 2

Nodo 3

Nodo 4

Nodo 5

Nodo 6

Nodo 7

Nodo 8

MatrixForm[Nf]

$$\begin{pmatrix} -\frac{1}{4}(-1+\eta)(-1+\xi)(1+\eta+\xi) \\ \frac{1}{4}(-1+\eta)(1+\eta-\xi)(1+\xi) \\ \frac{1}{4}(1+\eta)(1+\xi)(-1+\eta+\xi) \\ -\frac{1}{4}(1+\eta)(-1+\eta-\xi)(-1+\xi) \\ \frac{1}{2}(-1+\eta)(-1+\xi)(1+\xi) \\ -\frac{1}{2}(-1+\eta)(1+\eta)(1+\xi) \\ -\frac{1}{2}(1+\eta)(-1+\xi)(1+\xi) \\ \frac{1}{2}(-1+\eta)(1+\eta)(-1+\xi) \end{pmatrix}$$

■ COMPROBACION SUMA UNIDAD

$$\text{Suma} = \sum_{i=1}^{\text{NNodos}} \text{Nf}[[i]]$$

$$\begin{aligned} & \frac{1}{2}(-1+\eta)(1+\eta)(-1+\xi) - \frac{1}{4}(1+\eta)(-1+\eta-\xi)(-1+\xi) - \\ & \frac{1}{2}(-1+\eta)(1+\eta)(1+\xi) + \frac{1}{4}(-1+\eta)(1+\eta-\xi)(1+\xi) + \frac{1}{2}(-1+\eta)(-1+\xi)(1+\xi) - \\ & \frac{1}{2}(1+\eta)(-1+\xi)(1+\xi) + \frac{1}{4}(1+\eta)(1+\xi)(-1+\eta+\xi) - \frac{1}{4}(-1+\eta)(-1+\xi)(1+\eta+\xi) \end{aligned}$$

Simplify[%]

1

OK.

■ REPRESENTACION GRAFICA

- Función Representación Gráfica Funciones de Forma
- Representación Gráfica Funciones Forma Elemento.

```
Ng = Table[0, {i, NNodos}];
```

```
xy1 = {0, 0, 0}; xy2 = {3, 0, 0}; xy3 = {3, 3, 0};
xy4 = {0, 3, 0}; xyquad = N[{xy1, xy2, xy3, xy4, xy1}];
```

Control de Cuadrícula

```
Nsub = 15;
```

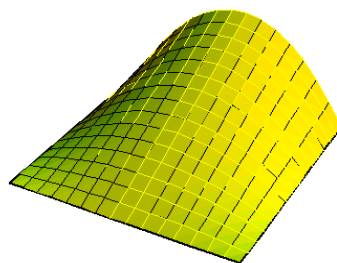
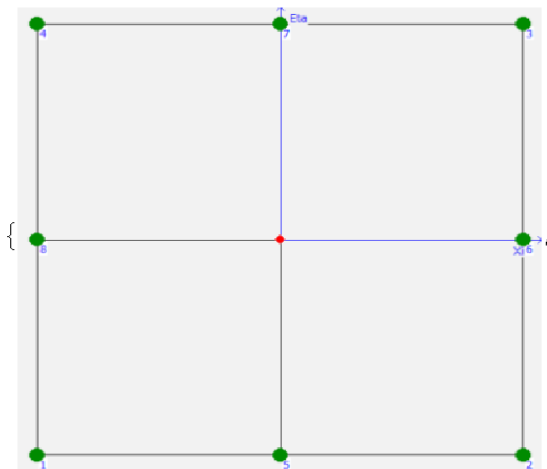
```
Do[
  fi[ξ_, η_] = Nf[[i]];
  Ng[[i]] = PlotQuadrilateralShapeFunction[xyquad, fi, Nsub, 1/2],
  {i, NNodos}
];
```

RESULTADOS INTERACTIVOS -

```
Manipulate[{CuaS8r, Ng[[n]], Nf[[n]]}, {n, 1, Dimensions[Nf][[1]], 1}, {n, Range[Dimensions[Nf][[1]]]},  
FrameLabel -> {"FUNCION DE FORMA EN NODO n - CUADRILATERO SERENDIPITO 8 NODOS"},  
SaveDefinitions -> True]
```

n

n



$$, -\frac{1}{2} (1 + \eta) (-1 + \xi) (1 + \xi) \}$$

FUNCION DE FORMA EN NODO n - CUADRILATERO SERENDIPITO 8 NODOS