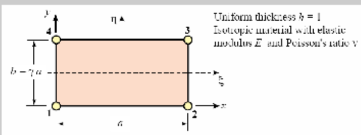
LECCION 6 - EJERCICIO 2 (17.2) v.2005

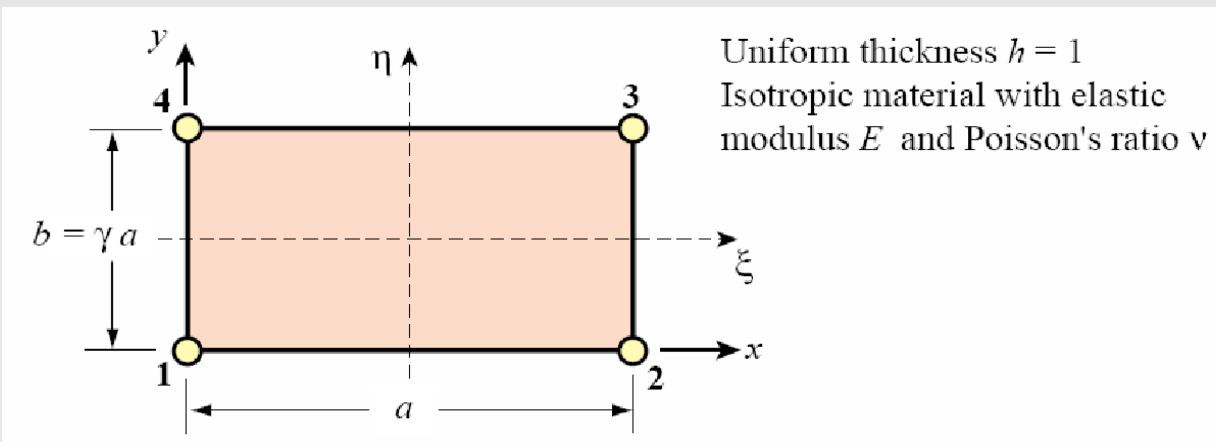
INICIO

```
Off[General::"spell1"]
Off[General::"spell"]
```

DEFINICION ELEMENTO CUADRILATERO

Cua4 =  ;

```
Show[Cua4, ImageSize → 600]
```



NO SE DEFINEN DATOS NUMERICOS - SE ELIMINAN LOS PREVIAMENTE DEFINIDOS

```
ClearAll[Em, ν, a, b, h];
```

RELACION ENTRE LONGITUD Y ALTURA DEL ELEMENTO

```
b = γ * a;
```

COORDENADAS NODOS

```
ncoor = {{0, 0}, {a, 0}, {a, b}, {0, b}};
```

MATRIZ DE PROPIEDADES DEL MATERIAL

```
Emat = Em / (1 - ν^2) * {{1, ν, 0}, {ν, 1, 0}, {0, 0, (1 - ν) / 2}};
```

FORMULACION ELEMENTO CUADRILATERO 4 NODOS - PROBLEMA TENSION PLANA

```
Quad4IsoPMembraneStiffness[ncoor_, mprop_, fprop_, options_] := Module[{i, k, p = 2, numer = False, Emat,
    th = 1, h, qcoor, c, w, Nf, dNx, dNy, Jdet, B, Ke = Table[0, {8}, {8}]}, Emat = mprop[[1]];
   If[Length[options] == 2, {numer, p} = options, {numer} = options];
   If[Length[fprop] > 0, th = fprop[[1]]];
   If[p < 1 || p > 4, Print["p out of range"]; Return[Null]];
   For[k = 1, k ≤ p * p, k++, {qcoor, w} = QuadGaussRuleInfo[{p, numer}, k];
    {Nf, dNx, dNy, Jdet} = Quad4IsoPShapeFunDer[ncoor, qcoor];
    If[Length[th] == 0, h = th, h = th.Nf]; c = w * Jdet * h;
    B = {Flatten[Table[{dNx[[i]], 0}, {i, 4}]],
      Flatten[Table[{0, dNy[[i]]}, {i, 4}]], Flatten[Table[{dNy[[i]], dNx[[i]]}, {i, 4}]]};
    Ke += Simplify[c * Transpose[B].(Emat.B)];]; Return[Simplify[Ke]]];
```

```
Quad4IsoPShapeFunDer[ncoor_, qcoor_] :=
  Module[{Nf, dNx, dNy, dNξ, dNη, i, J11, J12, J21, J22, Jdet, ξ, η, x, y}, {ξ, η} = qcoor;
   Nf = {(1 - ξ) * (1 - η), (1 + ξ) * (1 - η), (1 + ξ) * (1 + η), (1 - ξ) * (1 + η)} / 4;
   dNξ = {- (1 - η), (1 - η), (1 + η), - (1 + η)} / 4;
   dNη = {- (1 - ξ), - (1 + ξ), (1 + ξ), (1 - ξ)} / 4;
   x = Table[ncoor[[i, 1]], {i, 4}]; y = Table[ncoor[[i, 2]], {i, 4}];
   J11 = dNξ.x; J21 = dNξ.y; J12 = dNη.x; J22 = dNη.y;
   Jdet = Simplify[J11 * J22 - J12 * J21];
   dNx = (J22 * dNξ - J21 * dNη) / Jdet; dNx = Simplify[dNx];
   dNy = (-J12 * dNξ + J11 * dNη) / Jdet; dNy = Simplify[dNy];
   Return[{Nf, dNx, dNy, Jdet}]];
```

```
QuadGaussRuleInfo[{rule_, numer_}, point_] := Module[
   {xi, eta, p1, p2, i1, i2, w1, w2, k, info = Null}, If[Length[rule] == 2, {p1, p2} = rule, p1 = p2 = rule];
   If[Length[point] == 2, {i1, i2} = point, k = point; i2 = Floor[(k - 1) / p1] + 1; i1 = k - p1 * (i2 - 1)];
   {xi, w1} = LineGaussRuleInfo[{p1, numer}, i1];
   {eta, w2} = LineGaussRuleInfo[{p2, numer}, i2];
   info = {{xi, eta}, w1 * w2};
   If[numer, Return[N[info]], Return[Simplify[info]]];];
```

```
LineGaussRuleInfo[{rule_, numer_}, point_] :=
  Module[{g2 = {-1, 1} / Sqrt[3], w3 = {5 / 9, 8 / 9, 5 / 9}, g3 = {-Sqrt[3 / 5], 0, Sqrt[3 / 5]},
    w4 = {(1 / 2) - Sqrt[5 / 6] / 6, (1 / 2) + Sqrt[5 / 6] / 6, (1 / 2) + Sqrt[5 / 6] / 6, (1 / 2) - Sqrt[5 / 6] / 6},
    g4 = {-Sqrt[(3 + 2 * Sqrt[6 / 5]) / 7], -Sqrt[(3 - 2 * Sqrt[6 / 5]) / 7],
      Sqrt[(3 - 2 * Sqrt[6 / 5]) / 7], Sqrt[(3 + 2 * Sqrt[6 / 5]) / 7]}, i, info = Null}, i = point;
   If[rule == 1, info = {0, 2}];
   If[rule == 2, info = {g2[[i]], 1}];
   If[rule == 3, info = {g3[[i]], w3[[i]]}];
   If[rule == 4, info = {g4[[i]], w4[[i]]}];
   If[numer, Return[N[info]], Return[Simplify[info]]];];
```

REGLA DE GAUSS p = 2 (2 x 2) - PROPORCIONA SUFICIENCIA DE RANGO

```
p = 2;
```

```
Ke = Quad4IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {False, p}];
```

```
Ke // MatrixForm
```

$$\begin{pmatrix}
\frac{\text{Em h}\,(-1-2\gamma^2+\nu)}{6\gamma\,(-1+\nu^2)} & \frac{\text{Em h}}{8-8\nu} & \frac{\text{Em h}\,(-1+4\gamma^2+\nu)}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(1-3\nu)}{8\,(-1+\nu^2)} & \frac{\text{Em h}\,(1+2\gamma^2-\nu)}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}}{8\,(-1+\nu)} & -\frac{\text{Em h}\,(-1+\gamma^2+\nu)}{6\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(-1}{8\,(-1-} \\
\frac{\text{Em h}}{8-8\nu} & \frac{\text{Em h}\,(-2+\gamma^2\,(-1+\nu))}{6\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(-1+3\nu)}{8\,(-1+\nu^2)} & -\frac{\text{Em h}\,(1+\gamma^2\,(-1+\nu))}{6\gamma\,(-1+\nu^2)} & \frac{\text{Em h}}{8\,(-1+\nu)} & \frac{\text{Em h}\,(2-\gamma^2\,(-1+\nu))}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(1-3\nu)}{8\,(-1+\nu^2)} & \frac{\text{Em h}\,(4+\gamma^2}{12\gamma\,(-1+\nu} \\
\frac{\text{Em h}\,(-1+4\gamma^2+\nu)}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(-1+3\nu)}{8\,(-1+\nu^2)} & \frac{\text{Em h}\,(-1-2\gamma^2+\nu)}{6\gamma\,(-1+\nu^2)} & \frac{\text{Em h}}{8\,(-1+\nu)} & -\frac{\text{Em h}\,(-1+\gamma^2+\nu)}{6\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(1-3\nu)}{8\,(-1+\nu^2)} & \frac{\text{Em h}\,(1+2\gamma^2-\nu)}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em}}{8-8} \\
\frac{\text{Em h}\,(1-3\nu)}{8\,(-1+\nu^2)} & -\frac{\text{Em h}\,(1+\gamma^2\,(-1+\nu))}{6\gamma\,(-1+\nu^2)} & \frac{\text{Em h}}{8\,(-1+\nu)} & \frac{\text{Em h}\,(-2+\gamma^2\,(-1+\nu))}{6\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(-1+3\nu)}{8\,(-1+\nu^2)} & \frac{\text{Em h}\,(4+\gamma^2\,(-1+\nu))}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}}{8-8\nu} & \frac{\text{Em h}\,(2-\gamma^2}{12\gamma\,(-1+\nu} \\
\frac{\text{Em h}\,(1+2\gamma^2-\nu)}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}}{8\,(-1+\nu)} & -\frac{\text{Em h}\,(-1+\gamma^2+\nu)}{6\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(-1+3\nu)}{8\,(-1+\nu^2)} & \frac{\text{Em h}\,(-1-2\gamma^2+\nu)}{6\gamma\,(-1+\nu^2)} & \frac{\text{Em h}}{8-8\nu} & \frac{\text{Em h}\,(-1+4\gamma^2+\nu)}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(1}{8\,(-1-} \\
\frac{\text{Em h}}{8\,(-1+\nu)} & \frac{\text{Em h}\,(2-\gamma^2\,(-1+\nu))}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(1-3\nu)}{8\,(-1+\nu^2)} & \frac{\text{Em h}\,(4+\gamma^2\,(-1+\nu))}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}}{8-8\nu} & \frac{\text{Em h}\,(-2+\gamma^2\,(-1+\nu))}{6\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(-1+3\nu)}{8\,(-1+\nu^2)} & -\frac{\text{Em h}\,(1+\gamma}{6\gamma\,(-} \\
-\frac{\text{Em h}\,(-1+\gamma^2+\nu)}{6\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(1-3\nu)}{8\,(-1+\nu^2)} & \frac{\text{Em h}\,(1+2\gamma^2-\nu)}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}}{8-8\nu} & \frac{\text{Em h}\,(-1+4\gamma^2+\nu)}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(-1+3\nu)}{8\,(-1+\nu^2)} & \frac{\text{Em h}\,(-1-2\gamma^2+\nu)}{6\gamma\,(-1+\nu^2)} & \frac{\text{Em}}{8\,(-1} \\
\frac{\text{Em h}\,(-1+3\nu)}{8\,(-1+\nu^2)} & \frac{\text{Em h}\,(4+\gamma^2\,(-1+\nu))}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}}{8-8\nu} & \frac{\text{Em h}\,(2-\gamma^2\,(-1+\nu))}{12\gamma\,(-1+\nu)\,(1+\nu)} & \frac{\text{Em h}\,(1-3\nu)}{8\,(-1+\nu^2)} & -\frac{\text{Em h}\,(1+\gamma^2\,(-1+\nu))}{6\gamma\,(-1+\nu^2)} & \frac{\text{Em h}}{8\,(-1+\nu)} & \frac{\text{Em h}\,(-2+\gamma^2}{6\gamma\,(-1+\nu}
\end{pmatrix}$$

ESCALAMOS LA MATRIZ DIVIENDO POR EL FACTOR DE ESCALA

```
scaledKe = Simplify[Ke * (24 * γ * (1 - ν^2) / (Em * h))];
```

RESULTADO A COMPARAR - (E17.4)

```
Print["Ke=", Em * h / (24 * γ * (1 - ν^2)), "*\n", scaledKe // MatrixForm];
```

$$\text{Ke}=\frac{\text{Em h}}{24\,\gamma\,(1-\nu^2)}*$$

$$\begin{pmatrix}
4+8\gamma^2-4\nu & 3\gamma\,(1+\nu) & -2\,(-1+4\gamma^2+\nu) & 3\gamma\,(-1+3\nu) & -2-4\gamma^2+2\nu & -3\gamma\,(1+\nu) & 4\,(-1+\gamma^2 \\
3\gamma\,(1+\nu) & 8-4\gamma^2\,(-1+\nu) & \gamma\,(3-9\nu) & 4+4\gamma^2\,(-1+\nu) & -3\gamma\,(1+\nu) & -4+2\gamma^2\,(-1+\nu) & 3\gamma\,(-1+ \\
-2\,(-1+4\gamma^2+\nu) & \gamma\,(3-9\nu) & 4+8\gamma^2-4\nu & -3\gamma\,(1+\nu) & 4\,(-1+\gamma^2+\nu) & 3\gamma\,(-1+3\nu) & -2-4\gamma^2 \\
3\gamma\,(-1+3\nu) & 4+4\gamma^2\,(-1+\nu) & -3\gamma\,(1+\nu) & 8-4\gamma^2\,(-1+\nu) & \gamma\,(3-9\nu) & -8-2\gamma^2\,(-1+\nu) & 3\gamma\,(1+ \\
-2-4\gamma^2+2\nu & -3\gamma\,(1+\nu) & 4\,(-1+\gamma^2+\nu) & \gamma\,(3-9\nu) & 4+8\gamma^2-4\nu & 3\gamma\,(1+\nu) & -2\,(-1+4 \\
-3\gamma\,(1+\nu) & -4+2\gamma^2\,(-1+\nu) & 3\gamma\,(-1+3\nu) & -8-2\gamma^2\,(-1+\nu) & 3\gamma\,(1+\nu) & 8-4\gamma^2\,(-1+\nu) & \gamma\,(3-9 \\
4\,(-1+\gamma^2+\nu) & 3\gamma\,(-1+3\nu) & -2-4\gamma^2+2\nu & 3\gamma\,(1+\nu) & -2\,(-1+4\gamma^2+\nu) & \gamma\,(3-9\nu) & 4+8\gamma^2- \\
\gamma\,(3-9\nu) & -8-2\gamma^2\,(-1+\nu) & 3\gamma\,(1+\nu) & -4+2\gamma^2\,(-1+\nu) & 3\gamma\,(-1+3\nu) & 4+4\gamma^2\,(-1+\nu) & -3\gamma\,(1
\end{pmatrix}$$