

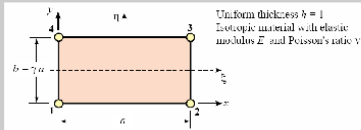
LECCION 6 - EJERCICIO 1 (17.1) v.2005

■ INICIO

```
Off [General::"spell1"]
```

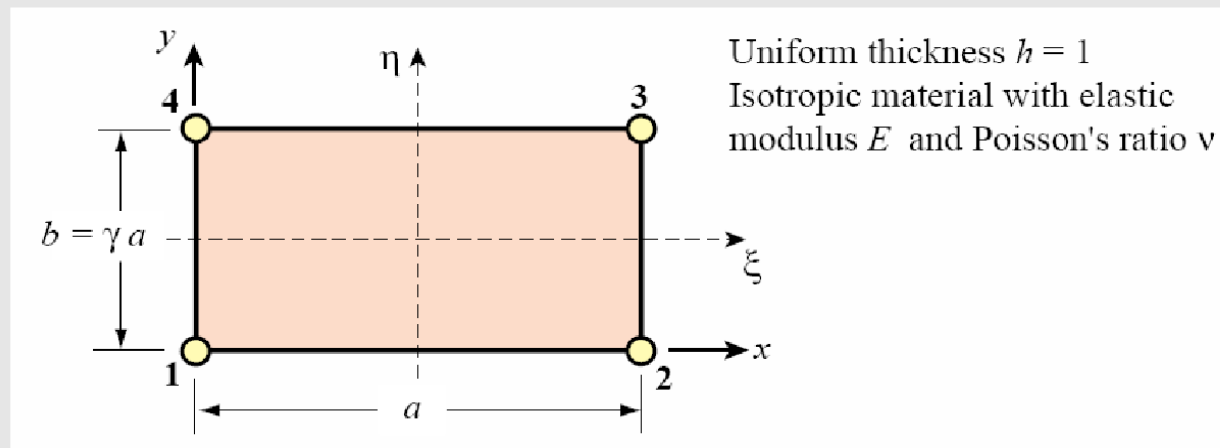
```
Off [General::"spell"]
```

■ DEFINICION ELEMENTO CUADRILATERO



```
Cua4 = ;
```

```
Show[Cua4, ImageSize -> 600]
```



□ PROPIEDADES DEL MATERIAL

```
Em = 48;
```

```
nu = 0;
```

□ DIMENSIONES GEOMETRICAS

```
a = 4;
```

```
b = 2;
```

□ DATOS FABRICACION - ESPESOR CONSTANTE

```
h = 1;
```

□ COORDENADAS NODOS

```
ncoor = {{0, 0}, {a, 0}, {a, b}, {0, b}};
```

```
ncoor // MatrixForm
```

```

$$\begin{pmatrix} 0 & 0 \\ 4 & 0 \\ 4 & 2 \\ 0 & 2 \end{pmatrix}$$

```

▣ MATRIZ DE PROPIEDADES DEL MATERIAL

```
Emat = Em / (1 - nu^2) * {{1, nu, 0}, {nu, 1, 0}, {0, 0, (1 - nu) / 2}};
```

```
Emat // MatrixForm
```

$$\begin{pmatrix} 48 & 0 & 0 \\ 0 & 48 & 0 \\ 0 & 0 & 24 \end{pmatrix}$$

■ FORMULACION ELEMENTO CUADRILATERO 4 NODOS - PROBLEMA TENSION PLANA

```
Quad4IsoPMembraneStiffness[ncoor_, mprop_, fprop_, options_] := Module[{i, k, p = 2, numer = False, Emat,
  th = 1, h, qcoor, c, w, Nf, dNx, dNy, Jdet, B, Ke = Table[0, {8}, {8}]}, Emat = mprop[[1]];
  If[Length[options] == 2, {numer, p} = options, {numer} = options];
  If[Length[fprop] > 0, th = fprop[[1]]];
  If[p < 1 || p > 4, Print["p out of range"]; Return[Null]];
  For[k = 1, k ≤ p * p, k++, {qcoor, w} = QuadGaussRuleInfo[{p, numer}, k];
    {Nf, dNx, dNy, Jdet} = Quad4IsoPShapeFunDer[ncoor, qcoor];
    If[Length[th] == 0, h = th, h = th.Nf]; c = w * Jdet * h;
    B = {Flatten[Table[{dNx[[i]], 0}, {i, 4}]],
      Flatten[Table[{0, dNy[[i]]}, {i, 4}]], Flatten[Table[{dNy[[i]], dNx[[i]]}, {i, 4}]]};
    Ke += Simplify[c * Transpose[B].(Emat.B)]; Return[Simplify[Ke]]];
```

```
Quad4IsoPShapeFunDer[ncoor_, qcoor_] :=
  Module[{Nf, dNx, dNy, dNξ, dNη, i, J11, J12, J21, J22, Jdet, ξ, η, x, y}, {ξ, η} = qcoor;
  Nf = {(1 - ξ) * (1 - η), (1 + ξ) * (1 - η), (1 + ξ) * (1 + η), (1 - ξ) * (1 + η)} / 4;
  dNξ = {-(1 - η), (1 - η), (1 + η), -(1 + η)} / 4;
  dNη = {-(1 - ξ), -(1 + ξ), (1 + ξ), (1 - ξ)} / 4;
  x = Table[ncoor[[i, 1]], {i, 4}]; y = Table[ncoor[[i, 2]], {i, 4}];
  J11 = dNξ.x; J21 = dNξ.y; J12 = dNη.x; J22 = dNη.y;
  Jdet = Simplify[J11 * J22 - J12 * J21];
  dNx = (J22 * dNξ - J21 * dNη) / Jdet; dNx = Simplify[dNx];
  dNy = (-J12 * dNξ + J11 * dNη) / Jdet; dNy = Simplify[dNy];
  Return[{Nf, dNx, dNy, Jdet}];
```

```
QuadGaussRuleInfo[{rule_, numer_}, point_] := Module[
  {xi, eta, p1, p2, i1, i2, w1, w2, k, info = Null}, If[Length[rule] == 2, {p1, p2} = rule, p1 = p2 = rule];
  If[Length[point] == 2, {i1, i2} = point, k = point; i2 = Floor[(k - 1) / p1] + 1; i1 = k - p1 * (i2 - 1)];
  {xi, w1} = LineGaussRuleInfo[{p1, numer}, i1];
  {eta, w2} = LineGaussRuleInfo[{p2, numer}, i2];
  info = {{xi, eta}, w1 * w2};
  If[numer, Return[N[info]], Return[Simplify[info]]];
```

```
LineGaussRuleInfo[{rule_, numer_}, point_] :=
  Module[{g2 = {-1, 1} / Sqrt[3], w3 = {5 / 9, 8 / 9, 5 / 9}, g3 = {-Sqrt[3 / 5], 0, Sqrt[3 / 5]},
  w4 = {(1 / 2) - Sqrt[5 / 6] / 6, (1 / 2) + Sqrt[5 / 6] / 6, (1 / 2) + Sqrt[5 / 6] / 6, (1 / 2) - Sqrt[5 / 6] / 6},
  g4 = {-Sqrt[(3 + 2 * Sqrt[6 / 5]) / 7], -Sqrt[(3 - 2 * Sqrt[6 / 5]) / 7],
  Sqrt[(3 - 2 * Sqrt[6 / 5]) / 7], Sqrt[(3 + 2 * Sqrt[6 / 5]) / 7]}, i, info = Null}, i = point;
  If[rule == 1, info = {0, 2}];
  If[rule == 2, info = {g2[[i]], 1}];
  If[rule == 3, info = {g3[[i]], w3[[i]]}];
  If[rule == 4, info = {g4[[i]], w4[[i]]}];
  If[numer, Return[N[info]], Return[Simplify[info]]];
```

■ REGLA DE GAUSS p = 1 (1 x 1)

```
p = 1;
```

```
Ke = Quad4IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
```

```
Chop[Ke] // MatrixForm
```

$$\begin{pmatrix} 18. & 6. & 6. & -6. & -18. & -6. & -6. & 6. \\ 6. & 27. & 6. & 21. & -6. & -27. & -6. & -21. \\ 6. & 6. & 18. & -6. & -6. & -6. & -18. & 6. \\ -6. & 21. & -6. & 27. & 6. & -21. & 6. & -27. \\ -18. & -6. & -6. & 6. & 18. & 6. & 6. & -6. \\ -6. & -27. & -6. & -21. & 6. & 27. & 6. & 21. \\ -6. & -6. & -18. & 6. & 6. & 6. & 18. & -6. \\ 6. & -21. & 6. & -27. & -6. & 21. & -6. & 27. \end{pmatrix}$$

□ COMPROBACION SUFICIENCIA DE RANGO

```
Chop[Eigenvalues[N[Ke ]]]
```

```
{96., 60., 24., 0, 0, 0, 0, 0}
```

NO TIENE SUFICIENCIA DE RANGO

■ REGLA DE GAUSS $p = 2$ (2 x 2)

```
p = 2;
```

```
Ke = Quad4IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
```

```
Chop[Ke] // MatrixForm
```

$$\begin{pmatrix} 24. & 6. & 0 & -6. & -12. & -6. & -12. & 6. \\ 6. & 36. & 6. & 12. & -6. & -18. & -6. & -30. \\ 0 & 6. & 24. & -6. & -12. & -6. & -12. & 6. \\ -6. & 12. & -6. & 36. & 6. & -30. & 6. & -18. \\ -12. & -6. & -12. & 6. & 24. & 6. & 0 & -6. \\ -6. & -18. & -6. & -30. & 6. & 36. & 6. & 12. \\ -12. & -6. & -12. & 6. & 0 & 6. & 24. & -6. \\ 6. & -30. & 6. & -18. & -6. & 12. & -6. & 36. \end{pmatrix}$$

□ COMPROBACION SUFICIENCIA DE RANGO

```
Chop[Eigenvalues[N[Ke ]]]
```

```
{96., 60., 36., 24., 24., 0, 0, 0}
```

TIENE SUFICIENCIA DE RANGO

■ REGLA DE GAUSS $p = 3$ (3 x 3)

```
p = 3;
```

```
Ke = Quad4IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
```

```
Chop[Ke] // MatrixForm
```

$$\begin{pmatrix} 24. & 6. & 0 & -6. & -12. & -6. & -12. & 6. \\ 6. & 36. & 6. & 12. & -6. & -18. & -6. & -30. \\ 0 & 6. & 24. & -6. & -12. & -6. & -12. & 6. \\ -6. & 12. & -6. & 36. & 6. & -30. & 6. & -18. \\ -12. & -6. & -12. & 6. & 24. & 6. & 0 & -6. \\ -6. & -18. & -6. & -30. & 6. & 36. & 6. & 12. \\ -12. & -6. & -12. & 6. & 0 & 6. & 24. & -6. \\ 6. & -30. & 6. & -18. & -6. & 12. & -6. & 36. \end{pmatrix}$$

□ COMPROBACION SUFICIENCIA DE RANGO

```
Chop[Eigenvalues[N[Ke]]]
```

```
{96., 60., 36., 24., 24., 0, 0, 0}
```

TIENE SUFICIENCIA DE RANGO

■ REGLA DE GAUSS $p = 4$ (4 x 4)

```
p = 4;
```

```
Ke = Quad4IsoPMembraneStiffness[ncoor, {Emat, 0, 0}, {h}, {True, p}];
```

```
Chop[Ke] // MatrixForm
```

$$\begin{pmatrix} 24. & 6. & 0 & -6. & -12. & -6. & -12. & 6. \\ 6. & 36. & 6. & 12. & -6. & -18. & -6. & -30. \\ 0 & 6. & 24. & -6. & -12. & -6. & -12. & 6. \\ -6. & 12. & -6. & 36. & 6. & -30. & 6. & -18. \\ -12. & -6. & -12. & 6. & 24. & 6. & 0 & -6. \\ -6. & -18. & -6. & -30. & 6. & 36. & 6. & 12. \\ -12. & -6. & -12. & 6. & 0 & 6. & 24. & -6. \\ 6. & -30. & 6. & -18. & -6. & 12. & -6. & 36. \end{pmatrix}$$

□ COMPROBACION SUFICIENCIA DE RANGO

```
Chop[Eigenvalues[N[Ke]]]
```

```
{96., 60., 36., 24., 24., 0, 0, 0}
```

TIENE SUFICIENCIA DE RANGO

■ CONCLUSION

A PARTIR DE $p=2$ SE ALCANZA LA SUFICIENCIA DE RANGO Y LOS ELEMENTOS DE LA MATRIZ NO VARIAN