LECCION 4 - EJERCICIO 5 (15.5) v.2005 - ELEMENTO TRIANGULAR LINEAL - TENSION PLANA - DEFINICION MATRIZ RIGIDEZ

INICIO

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```

1 - DEFINICION MATRIZ RIGIDEZ

```
StiffnessOf3NodePlaneStressTriangle[{{x1_, y1_}, {x2_, y2_}, {x3_, y3_}}, Emat_, {h_}] :=
  Module[{x21, x13, x32, y12, y31, y23, A, Be, Ke},
   A = Simplify[((x2 * y3 - x3 * y2) + (x3 * y1 - x1 * y3) + (x1 * y2 - x2 * y1)) / 2];
   {x21, x13, x32, y12, y31, y23} = {x2 - x1, x1 - x3, x3 - x2, y1 - y2, y3 - y1, y2 - y3};
   Be = {{y23, 0, y31, 0, y12, 0}, {0, x32, 0, x13, 0, x21}, {x32, y23, x13, y31, x21, y12}} / (2 * A);
   Ke = (A * h) * Transpose[Be].Emat.Be;
   Return[Ke];];
```

DEFINICION DEL AREA A - EC. (15.3)

```
DaT = 2A = det[1 1 1; x1 x2 x3; y1 y2 y3] = (x2 y3 - x3 y2) + (x3 y1 - x1 y3) + (x1 y2 - x2 y1).;
```

```
Show[DaT, ImageSize → 500]
```

$$2A = \det\begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} = (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3) + (x_1y_2 - x_2y_1).$$

```
A[x1_, y1_, x2_, y2_, x3_, y3_] = Simplify[((x2 * y3 - x3 * y2) + (x3 * y1 - x1 * y3) + (x1 * y2 - x2 * y1)) / 2]
```

$$\frac{1}{2}\,(x3\,(y1 - y2) + x1\,(y2 - y3) + x2\,(-y1 + y3))$$

DEFINICION DIFERENCIA COORDENADAS NODOS - EC. (15.11)

```
DdcN = [ζ1; ζ2; ζ3] = 1/(2A) [x2 y3 - x3 y2   y2 - y3   x3 - x2; x3 y1 - x1 y3   y3 - y1   x1 - x3; x1 y2 - x2 y1   y1 - y2   x2 - x1] [1; x; y]
  - 1/(2A) [2A23   y23   x32; 2A31   y31   x13; 2A12   y12   x21] [1; x; y]
  x32 = x3 - x2    y23 = y2 - y3;
```

```
Show[DdcN, ImageSize → 450]
```

$$\begin{bmatrix}\zeta_1\\ \zeta_2\\ \zeta_3\end{bmatrix} = \frac{1}{2A}\begin{bmatrix}x_2y_3 - x_3y_2 & y_2 - y_3 & x_3 - x_2\\ x_3y_1 - x_1y_3 & y_3 - y_1 & x_1 - x_3\\ x_1y_2 - x_2y_1 & y_1 - y_2 & x_2 - x_1\end{bmatrix}\begin{bmatrix}1\\ x\\ y\end{bmatrix}$$

$$= \frac{1}{2A}\begin{bmatrix}2A_{23} & y_{23} & x_{32}\\ 2A_{31} & y_{31} & x_{13}\\ 2A_{12} & y_{12} & x_{21}\end{bmatrix}\begin{bmatrix}1\\ x\\ y\end{bmatrix}$$

$$x_{jk} = x_j - x_k \qquad y_{jk} = y_j - y_k$$

```
{x21, x13, x32, y12, y31, y23} = {x2 - x1, x1 - x3, x3 - x2, y1 - y2, y3 - y1, y2 - y3}
```

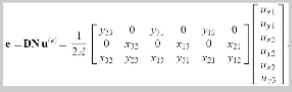
```
{-x1 + x2, x1 - x3, -x2 + x3, y1 - y2, -y1 + y3, y2 - y3}
```

```
x21
```

```
-x1 + x2
```

■ DEFINICION MATRIX Be - EC. (15.18)

```
DmbE = 
```



```
;
```

```
Show[DmbE, ImageSize → 450]
```

$$\mathbf{e} = \mathbf{D}\mathbf{N}\,\mathbf{u}^{(e)} = \frac{1}{2A}\begin{bmatrix}y_{23} & 0 & y_{31} & 0 & y_{12} & 0\\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21}\\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12}\end{bmatrix}\begin{bmatrix}u_{x1}\\ u_{y1}\\ u_{x2}\\ u_{y2}\\ u_{x3}\\ u_{y3}\end{bmatrix}$$

```
B[x1_, y1_, x2_, y2_, x3_, y3_] =
 {{y23, 0, y31, 0, y12, 0}, {0, x32, 0, x13, 0, x21}, {x32, y23, x13, y31, x21, y12}}
```

```
{{y2 - y3, 0, -y1 + y3, 0, y1 - y2, 0}, {0, -x2 + x3, 0, x1 - x3, 0, -x1 + x2},
 {-x2 + x3, y2 - y3, x1 - x3, -y1 + y3, -x1 + x2, y1 - y2}}
```

```
% // MatrixForm
```

$$\begin{pmatrix} y2 - y3 & 0 & -y1 + y3 & 0 & y1 - y2 & 0\\ 0 & -x2 + x3 & 0 & x1 - x3 & 0 & -x1 + x2\\ -x2 + x3 & y2 - y3 & x1 - x3 & -y1 + y3 & -x1 + x2 & y1 - y2\end{pmatrix}$$

```
Be[x1_, y1_, x2_, y2_, x3_, y3_] = B[x1, y1, x2, y2, x3, y3] / (2 * A[x1, y1, x2, y2, x3, y3])
```

$$\left\{\left\{\frac{y2-y3}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)},\ 0,\ \frac{-y1+y3}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)},\right.\right.$$
$$\left.0,\ \frac{y1-y2}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)},\ 0\right\},\ \left\{0,\ \frac{-x2+x3}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)},\right.$$
$$0,\ \frac{x1-x3}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)},\ 0,\ \left.\frac{-x1+x2}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)}\right\},$$
$$\left\{\frac{-x2+x3}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)},\ \frac{y2-y3}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)},\right.$$
$$\frac{x1-x3}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)},\ \frac{-y1+y3}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)},$$
$$\left.\left.\frac{-x1+x2}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)},\ \frac{y1-y2}{x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3)}\right\}\right\}$$

■ DEFINICION MATRIX E - EC. (15.19)

DmE = $\boldsymbol{\sigma} = \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix}E_{11} & E_{12} & E_{13}\\ E_{12} & E_{22} & E_{23}\\ E_{13} & E_{23} & E_{33}\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ 2e_{xy}\end{bmatrix} = \mathbf{Ee}$;

```
Show[DmE, ImageSize → 450]
```

$$\boldsymbol{\sigma} = \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix}E_{11} & E_{12} & E_{13}\\ E_{12} & E_{22} & E_{23}\\ E_{13} & E_{23} & E_{33}\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ 2e_{xy}\end{bmatrix} = \mathbf{Ee}$$

```
Emat = {{E11, E12, E13}, {E12, E22, E23}, {E13, E23, E33}};
```

■ DEFINICION MATRIX DE RIGIDEZ DEL ELEMENTO TRIANGULAR LINEAL

```
Ke[x1_, y1_, x2_, y2_, x3_, y3_, E11_, E12_, E13_, E22_, E23_, E33_, h_] =
  (A[x1, y1, x2, y2, x3, y3] * h) * Transpose[Be[x1, y1, x2, y2, x3, y3]].Emat.Be[x1, y1, x2, y2, x3, y3];
```

■ EJEMPLO UTILIZACION

```
Ke[x1, y1, x2, y2, x3, y3, E11, E12, E13, E22, E23, E33, h] /. {x1 → 0, y1 → 0, x2 → 3, y2 → 1,
   x3 → 2, y3 → 2, E11 → 100, E12 → 25, E13 → 0, E22 → 100, E23 → 0, E33 → 50, h → 1} // MatrixForm
```

$$\begin{pmatrix} \frac{75}{4} & \frac{75}{8} & -\frac{25}{2} & -\frac{25}{4} & -\frac{25}{4} & -\frac{25}{8} \\ \frac{75}{8} & \frac{75}{4} & \frac{25}{4} & \frac{25}{2} & -\frac{125}{8} & -\frac{125}{4} \\ -\frac{25}{2} & \frac{25}{4} & 75 & -\frac{75}{2} & -\frac{125}{2} & \frac{125}{4} \\ -\frac{25}{4} & \frac{25}{2} & -\frac{75}{2} & 75 & \frac{175}{4} & -\frac{175}{2} \\ -\frac{25}{4} & -\frac{125}{8} & -\frac{125}{2} & \frac{175}{4} & \frac{275}{4} & -\frac{225}{8} \\ -\frac{25}{8} & -\frac{125}{4} & \frac{125}{4} & -\frac{175}{2} & -\frac{225}{8} & \frac{475}{4} \end{pmatrix}$$

```
Ke[x1, y1, x2, y2, x3, y3, E11, E12, E13, E22, E23, E33, h] /. {x1 → 0, y1 → 0, x2 → 3, y2 → 1, x3 → 2, y3 → 2,
    E11 → 100, E12 → 25, E13 → 0, E21 → 25, E22 → 100, E23 → 0, E31 → 0, E32 → 0, E33 → 50, h → 1.0} // MatrixForm
```

$$\begin{pmatrix} 18.75 & 9.375 & -12.5 & -6.25 & -6.25 & -3.125 \\ 9.375 & 18.75 & 6.25 & 12.5 & -15.625 & -31.25 \\ -12.5 & 6.25 & 75. & -37.5 & -62.5 & 31.25 \\ -6.25 & 12.5 & -37.5 & 75. & 43.75 & -87.5 \\ -6.25 & -15.625 & -62.5 & 43.75 & 68.75 & -28.125 \\ -3.125 & -31.25 & 31.25 & -87.5 & -28.125 & 118.75 \end{pmatrix}$$

```
Show[Graphics[Line[{{0, 0}, {3, 1}, {2, 2}, {0, 0}}]], Axes -> True]
```