LECCION 3 - EJERCICIO 2 (14.2) v.2005

DEFINICIONES

```
Off[General::"spell1"]
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```

DEFINICION CONSTANTES DE LAME - FUNCION M ELASTICIDAD Y C POISSON

$$\lambda 1 = \frac{\nu * En}{(1+\nu) * (1-2\nu)};$$

$$\mu 1 = \frac{En}{2 * (1+\nu)};$$

DEFINICION M ELASTICIDAD Y C POISSON - FUNCION CONSTANTES DE LAME

$$E1 = \frac{\mu * (3 * \lambda + 2 * \mu)}{\lambda + \mu};$$

$$\nu 1 = \frac{\lambda}{2 * (\lambda + \mu)};$$

MATRIZ DEFORMACIONES-TENSIONES - PROBLEMA DEFORMACION PLANA

$$EmDp = \frac{Em\ (1-\nu)}{(1+\nu) * (1-2\nu)} * \begin{pmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2\,(1-\nu)} \end{pmatrix};$$

MATRIZ DEFORMACIONES-TENSIONES - PROBLEMA TENSION PLANA

$$EmTp = \frac{Em}{(1-\nu^2)} * \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix};$$

DESARROLLO - MATRIZ E - DEFORMACION PLANA (DP) - SEPARACION

EXPRESION MATRIZ E - DP - FUNCON COEFICIENTE DE LAME

```
Emats = EmDp /. {Em → E1, ν → ν1};
```

```
Emats // MatrixForm
```

$$\begin{pmatrix} \frac{\mu\,(3\lambda+2\mu)\left(1-\frac{\lambda}{2(\lambda+\mu)}\right)}{(\lambda+\mu)\left(1-\frac{\lambda}{\lambda+\mu}\right)\left(1+\frac{\lambda}{2(\lambda+\mu)}\right)} & \frac{\lambda\,\mu\,(3\lambda+2\mu)}{2\,(\lambda+\mu)^2\left(1-\frac{\lambda}{\lambda+\mu}\right)\left(1+\frac{\lambda}{2(\lambda+\mu)}\right)} & 0 \\ \frac{\lambda\,\mu\,(3\lambda+2\mu)}{2\,(\lambda+\mu)^2\left(1-\frac{\lambda}{\lambda+\mu}\right)\left(1+\frac{\lambda}{2(\lambda+\mu)}\right)} & \frac{\mu\,(3\lambda+2\mu)\left(1-\frac{\lambda}{2(\lambda+\mu)}\right)}{(\lambda+\mu)\left(1-\frac{\lambda}{\lambda+\mu}\right)\left(1+\frac{\lambda}{2(\lambda+\mu)}\right)} & 0 \\ 0 & 0 & \frac{\mu\,(3\lambda+2\mu)}{2\,(\lambda+\mu)\left(1+\frac{\lambda}{2(\lambda+\mu)}\right)} \end{pmatrix}$$

```
Ematx = FullSimplify[Emats];
```

```
Ematx // MatrixForm
```

$$\begin{pmatrix} \lambda + 2\mu & \lambda & 0 \\ \lambda & \lambda + 2\mu & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

DOS MATRICES - Eλ y Eμ

```
Eλ = {{λ, λ, 0}, {λ, λ, 0}, {0, 0, 0}};
```

```
Eλm = Eλ / λ;
```

```
Eλm // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
Simplify[Ematx - Eλ] // MatrixForm
```

$$\begin{pmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

```
Eμ = Ematx - Eλ;
```

```
Eμ // MatrixForm
```

$$\begin{pmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

```
Eμm = Eμ / μ;
```

```
Eμm // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

COMPROBACION

```
Ematx == μ * Eμm + λ * Eλm
```

```
True
```

DESARROLLO - MATRIZ E - TENSION PLANA (DP) - SEPARACION

EXPRESION MATRIZ E - DP - FUNCON COEFICIENTE DE LAME

```
Clear[Eμm, Eλm];
```

```
Emats = EmTp /. {Em → E1, ν → ν1};
```

```
Emats // MatrixForm
```

$$\begin{pmatrix} \frac{\mu\,(3\lambda+2\mu)}{(\lambda+\mu)\left(1-\frac{\lambda^2}{4(\lambda+\mu)^2}\right)} & \frac{\lambda\,\mu\,(3\lambda+2\mu)}{2(\lambda+\mu)^2\left(1-\frac{\lambda^2}{4(\lambda+\mu)^2}\right)} & 0 \\ \frac{\lambda\,\mu\,(3\lambda+2\mu)}{2(\lambda+\mu)^2\left(1-\frac{\lambda^2}{4(\lambda+\mu)^2}\right)} & \frac{\mu\,(3\lambda+2\mu)}{(\lambda+\mu)\left(1-\frac{\lambda^2}{4(\lambda+\mu)^2}\right)} & 0 \\ 0 & 0 & \frac{\mu\,(3\lambda+2\mu)\left(1-\frac{\lambda}{2(\lambda+\mu)}\right)}{2(\lambda+\mu)\left(1-\frac{\lambda^2}{4(\lambda+\mu)^2}\right)} \end{pmatrix}$$

```
Ematx = FullSimplify[Emats];
```

```
Ematx // MatrixForm
```

$$\begin{pmatrix} \frac{4\mu(\lambda+\mu)}{\lambda+2\mu} & \frac{2\lambda\mu}{\lambda+2\mu} & 0 \\ \frac{2\lambda\mu}{\lambda+2\mu} & \frac{4\mu(\lambda+\mu)}{\lambda+2\mu} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

```
Expand[Ematx] // MatrixForm
```

$$\begin{pmatrix} \frac{4\lambda\mu}{\lambda+2\mu}+\frac{4\mu^2}{\lambda+2\mu} & \frac{2\lambda\mu}{\lambda+2\mu} & 0 \\ \frac{2\lambda\mu}{\lambda+2\mu} & \frac{4\lambda\mu}{\lambda+2\mu}+\frac{4\mu^2}{\lambda+2\mu} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

DOS MATRICES - A y B

$$A = \begin{pmatrix} \frac{4\mu^2}{\lambda+2\mu} & 0 & 0 \\ 0 & \frac{4\mu^2}{\lambda+2\mu} & 0 \\ 0 & 0 & \mu \end{pmatrix};$$

```
Simplify[Ematx - A] // MatrixForm
```

$$\begin{pmatrix} \frac{4\lambda\mu}{\lambda+2\mu} & \frac{2\lambda\mu}{\lambda+2\mu} & 0 \\ \frac{2\lambda\mu}{\lambda+2\mu} & \frac{4\lambda\mu}{\lambda+2\mu} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
B = Simplify[Ematx - A];
```

```
B // MatrixForm
```

$$\begin{pmatrix} \frac{4\lambda\mu}{\lambda+2\mu} & \frac{2\lambda\mu}{\lambda+2\mu} & 0 \\ \frac{2\lambda\mu}{\lambda+2\mu} & \frac{4\lambda\mu}{\lambda+2\mu} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

DEFINICION λm y de la matriz Eλm

$$\lambda m = \frac{2\lambda\mu}{\lambda+2\mu};$$

```
Eλm = B / λm;
```

```
Eλm // MatrixForm
```

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

DETALLE

$$\text{Simplify}\left[\frac{2\,\lambda\,\mu}{\lambda+2\,\mu}+\frac{4\,\mu^2}{\lambda+2\,\mu}\right]$$

$2\,\mu$

```
Clear[λm]
```

$$\lambda m+\frac{4\,\mu^2}{\lambda+2\,\mu} == \text{Simplify}\left[\frac{2\,\lambda\,\mu}{\lambda+2\,\mu}+\frac{4\,\mu^2}{\lambda+2\,\mu}\right]$$

$$\lambda m+\frac{4\,\mu^2}{\lambda+2\,\mu} == 2\,\mu$$

DEFINICION de la matriz Eμm

```
A // MatrixForm
```

$$\begin{pmatrix} \frac{4\mu^2}{\lambda+2\mu} & 0 & 0 \\ 0 & \frac{4\mu^2}{\lambda+2\mu} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

$$\text{E}\mu\text{m} = \text{A} \;/.\; \left\{\frac{4\,\mu^2}{\lambda+2\,\mu} \text{->} 2\,\mu-\lambda m\right\};$$

```
Eμm // MatrixForm
```

$$\begin{pmatrix} -\lambda m+2\,\mu & 0 & 0 \\ 0 & -\lambda m+2\,\mu & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

COMPROBACION

$$\text{Ematx} == \text{E}\mu\text{m}+\lambda m * \text{E}\lambda\text{m} \;/.\; \left\{\lambda m \text{->} \frac{2\,\lambda\,\mu}{\lambda+2\,\mu}\right\}$$

$$\left\{\left\{\frac{4\,\mu\,(\lambda+\mu)}{\lambda+2\,\mu}, \frac{2\,\lambda\,\mu}{\lambda+2\,\mu}, 0\right\}, \left\{\frac{2\,\lambda\,\mu}{\lambda+2\,\mu}, \frac{4\,\mu\,(\lambda+\mu)}{\lambda+2\,\mu}, 0\right\}, \{0, 0, \mu\}\right\} ==$$
$$\left\{\left\{2\,\mu+\frac{2\,\lambda\,\mu}{\lambda+2\,\mu}, \frac{2\,\lambda\,\mu}{\lambda+2\,\mu}, 0\right\}, \left\{\frac{2\,\lambda\,\mu}{\lambda+2\,\mu}, 2\,\mu+\frac{2\,\lambda\,\mu}{\lambda+2\,\mu}, 0\right\}, \{0, 0, \mu\}\right\}$$

```
Simplify[%]
```

True