UNIVERSIDAD POLITECNICA DE VALENCIA

DEPARTAMENTO DE INGENIERIA MECANICA Y DE MATERIALES

ELEMENTOS FINITOS

(E.T.S.I.I.V)

MODELADO POR ELEMENTOS FINITOS

LECCION 2.- MALLADO, CONDICIONES DE CONTORNO Y CARGAS

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Topics in Chapter 8

General Modeling Rules

Finite Element Mesh Layouts

Distributed Loads

Displacement BCs

suppressing rigid body motions
taking advantage of symmetry and antisymmetry

General FEM Modeling Rules

- **Use the simplest elements that will do the job**
- ***Never, never, never* use complicated or special elements unless you are absolutely sure of what you are doing**
- **Use the coarsest mesh that will capture the dominant behavior of the physical model, particularly in *design* situations**

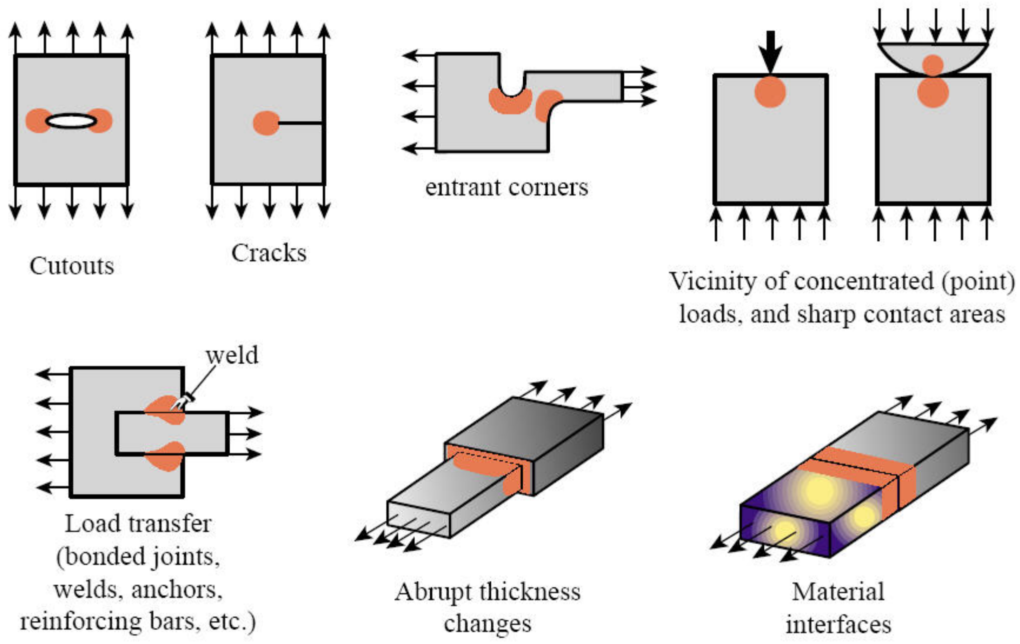
3 word summary: *Keep It Simple*

Another Justification for Simplicity

In product design situations several FEM models of increasing refinement will be set up as design evolves

Ergo, do not overkill at the beginning

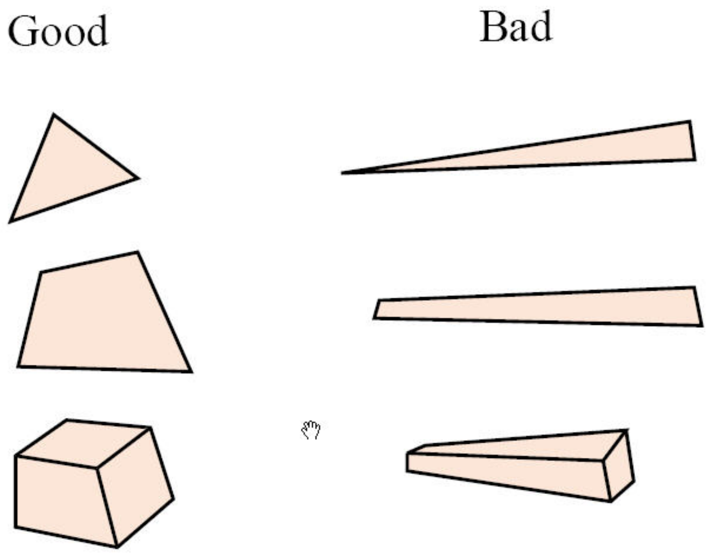
Where Finer Meshes Should be Used



001 SOBRE LA FORMA DE LOS ELEMENTOS CURSO 2004-5

Introduction to FEM

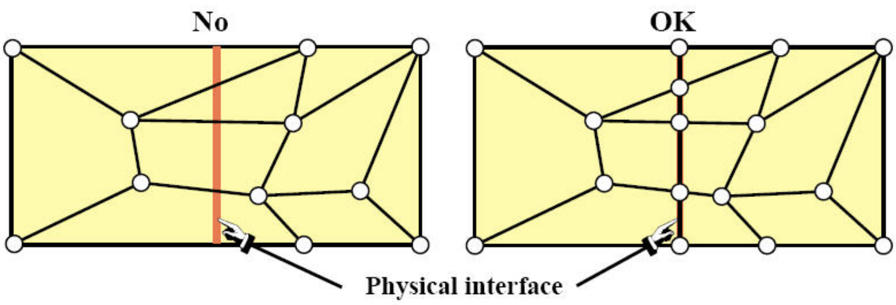
Avoid 2D/3D Elements of Bad Aspect Ratio



001 MALLADO EN INTERFASES DE MATERIALES CURSO 2004-5

Introduction to FEM

Elements Must Not Cross Interfaces



001 PREFERENCIAS SOBRE FORMAS ELEMENTOS CURSO 2004-5

Introduction to FEM

Element Geometry Preferences

Other things being equal, prefer

in 2D: Quadrilaterals over Triangles

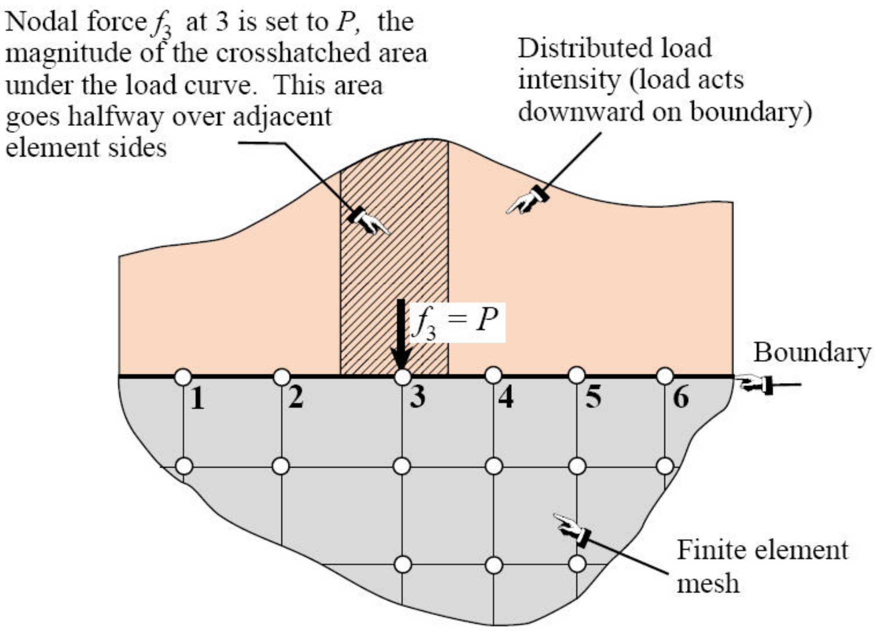
in 3D: Bricks over Wedges
Wedges over Tetrahedra

(Elements do not file discrimination suits)

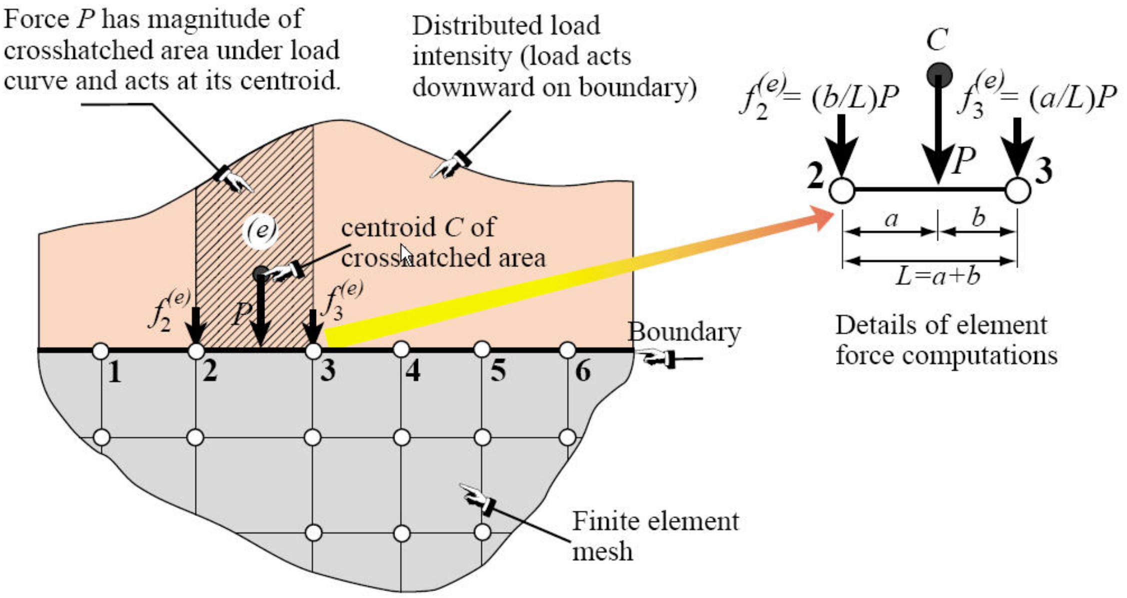
001 CONVERSION CARGAS DISTRIBUIDAS A CARGAS NODALES EQUIVALENTES CURSO 2004-5

Introduction to FEM

Node by Node (NbN) Distributed Load Lumping



Element by Element (EbE) Distributed Load Lumping



Boundary Conditions (BCs)

- **The most difficult topic for FEM program users**

Two types { **Essential** / **Natural** }

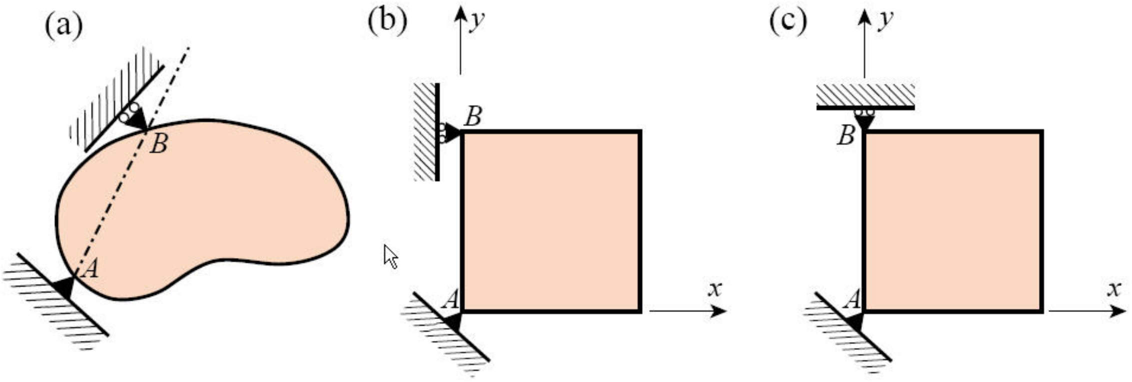
Boundary Conditions
Essential vs. Natural

Recipe:

1. **If a BC involves one or more DOF in a** *direct* **way, it is** *essential* **and goes to the Left Hand Side (LHS) of Ku = f**

2. **Otherwise it is** *natural* **and goes to the Right Hand Side (RHS) of Ku = f**

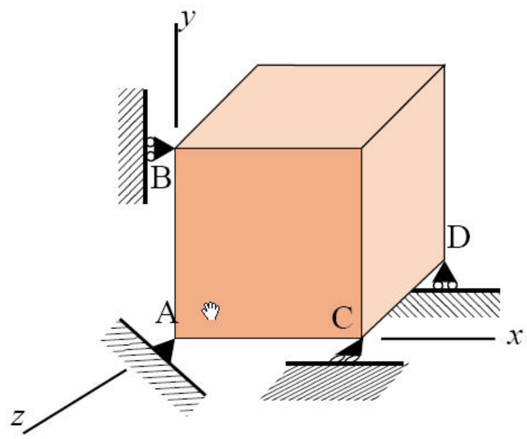
Minimum Support Conditions to Suppress Rigid Body Motions in 2D



001 SUPRESION MODOS CUERPO RIGIDO EN 3D CURSO 2004-5

Introduction to FEM

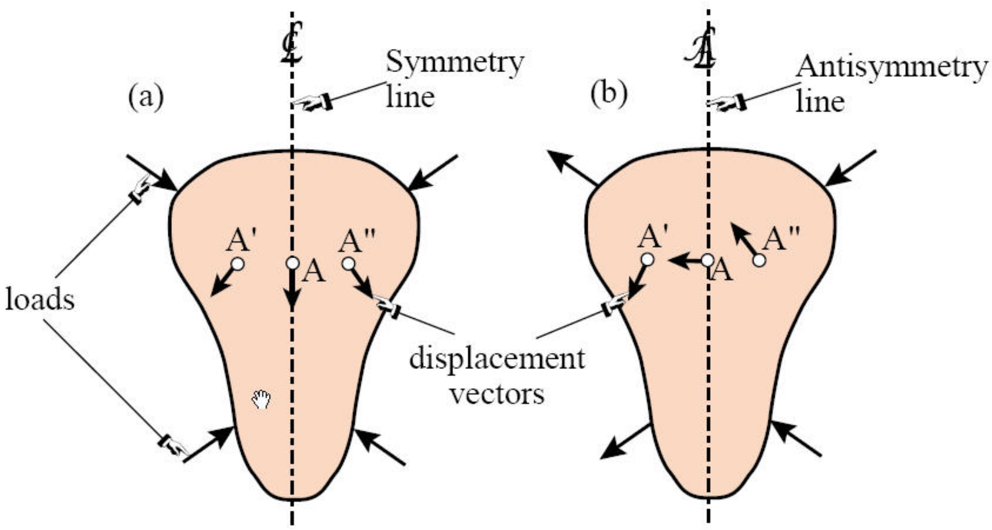
Minimum Support Conditions to Supress Rigid Body Motions in 3D



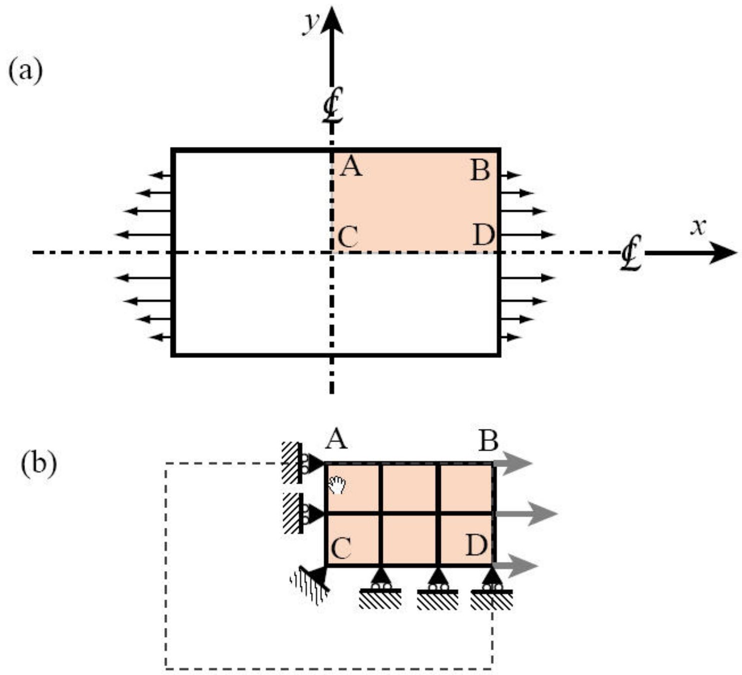
001 CONDICIONES DE SIMETRIA Y ANTISIMETRIA EN 2D CURSO 2004-5

Introduction to FEM

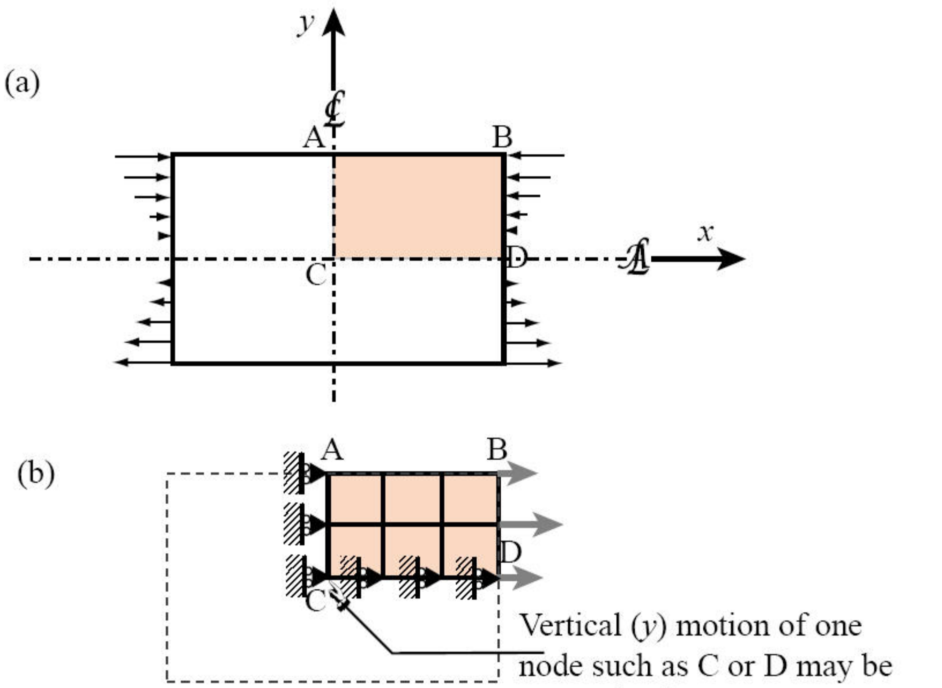
Visualizing Symmetry and Antisymmetry Conditions in 2D



Example of Application of Symmetry BCs



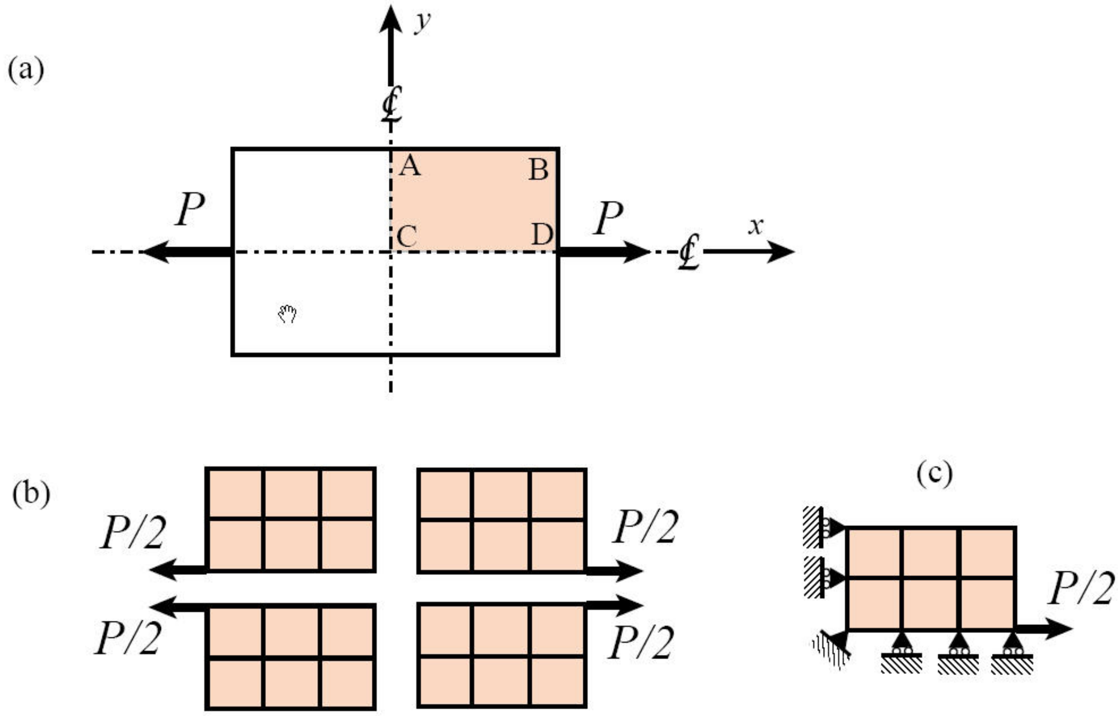
Example of Application of Antisymmetry BCs



001 DISTRIBUCION CARGAS NODALES EN CONDICIONES CONTORNO DE SIMETRIA — CURSO 2004-5

Introduction to FEM

"Breaking Up" Point Loads at Symmetry BCs

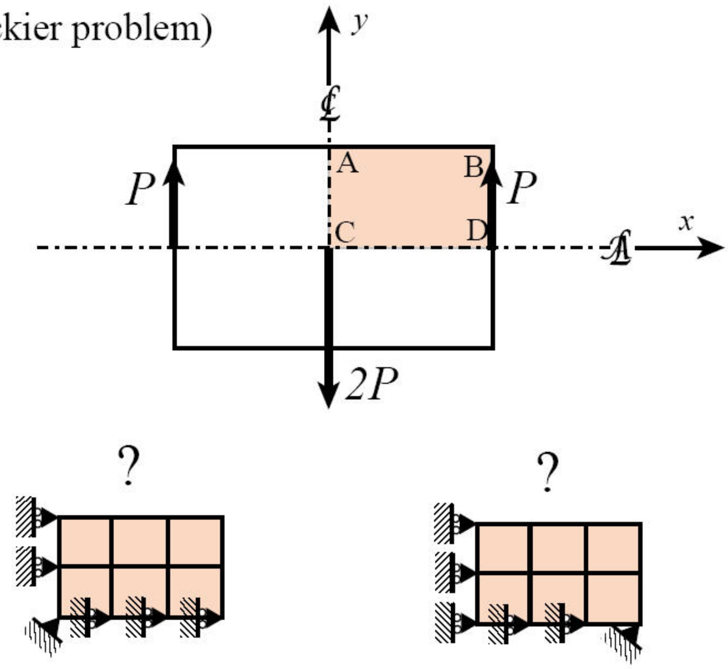


001 DISTRIBUCION CARGAS NODALES EN CONDICIONES CONTORNO DE ANTISIMETRIA — CURSO 2004-5

Introduction to FEM

"Breaking Up" Point Loads at Antisymmetry BCs

(A trickier problem)



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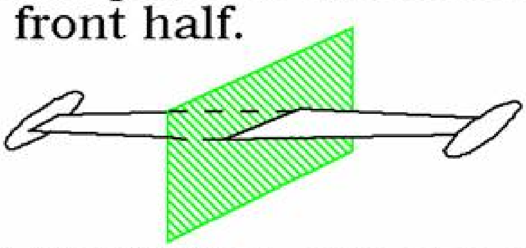
SYMMETRIES

A. REFLECTIVE SYMMETRY

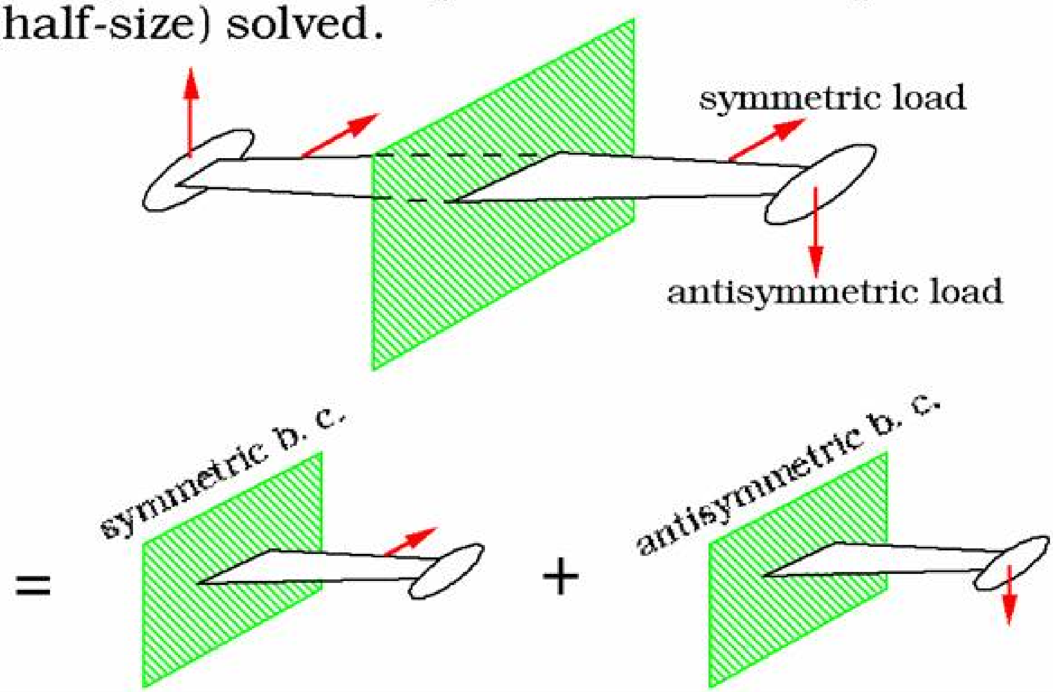
A body has a reflective plane if the half of the body behind the plane is identical to a mirror image of the front half.



In structural mechanics, one can exploit the symmetry if one has symmetry of:

- geometry
- material
- displacement boundary conditions

It is not necessary for the loads to be symmetric. They can be decomposed into symmetric and antisymmetric components, and two problems (half-size) solved.



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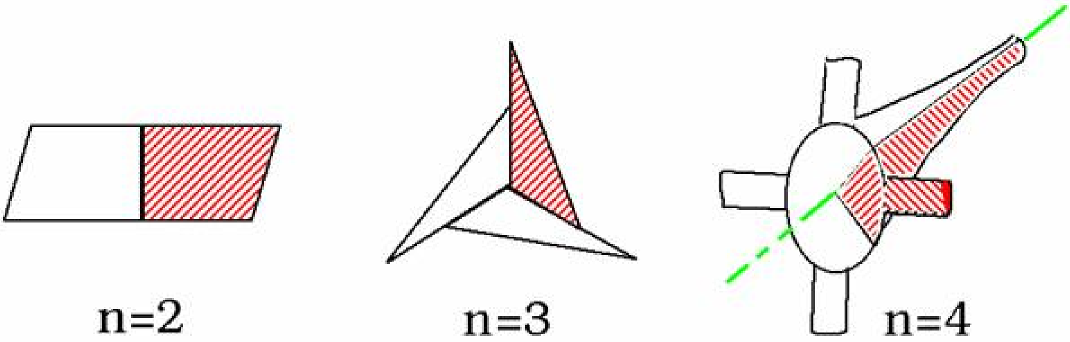
B. ROTATIONAL SYMMETRY

The body has a straight axis of symmetry. A fundamental region can generate the entire body by n replications, equally spaced about the axis:



The NASTRAN® codes have a strong rotational (cyclic) symmetry capability.

(10)

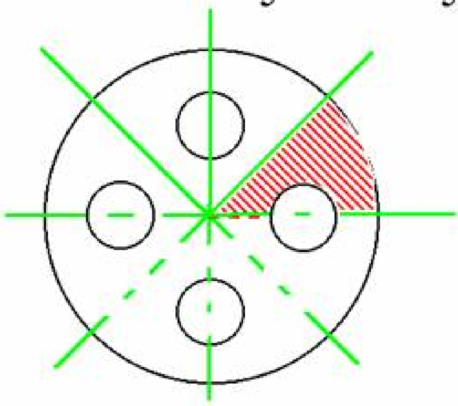
C. DIHEDRAL SYMMETRY

(11)

Combine rotational and reflective symmetry.

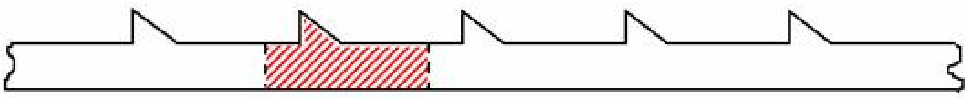
One only needs to model half of one cyclic region.

There are multiple reflective planes.



D. TRANSLATIONAL SYMMETRY

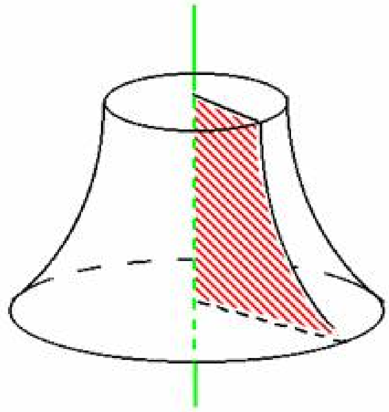
The fundamental pattern is repeated on a straight line:



E. AXISYMMETRY

A subcase of rotational (cyclic) symmetry:

- straight axis of rotation.
- body is swept out by rotation of one segment.
- lathe pieces are examples.
- can have skew due to winding process for shells, and material anisotropy.



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BOUNDARY CONDITIONS FOR REFLECTIVE SYMMETRY

A. "ELASTICITY" ELEMENTS

Consider elements that have only translational degrees of freedom (elasticity elements).

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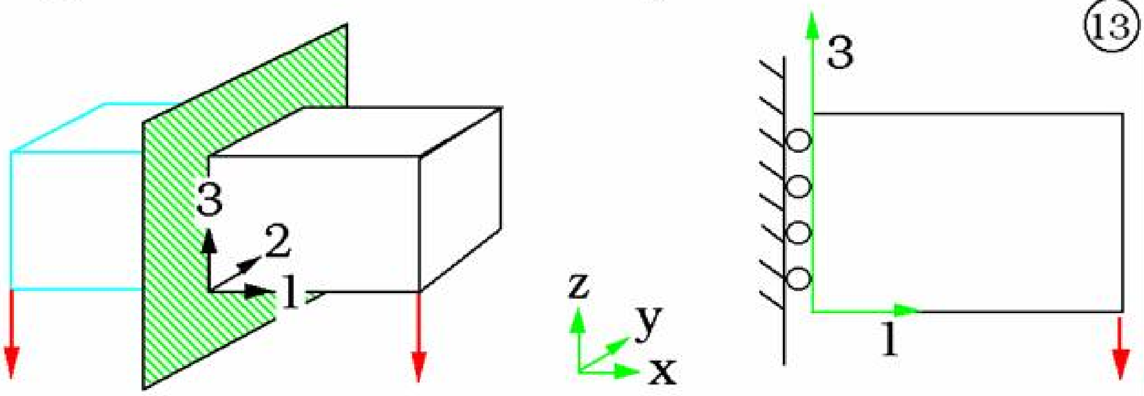
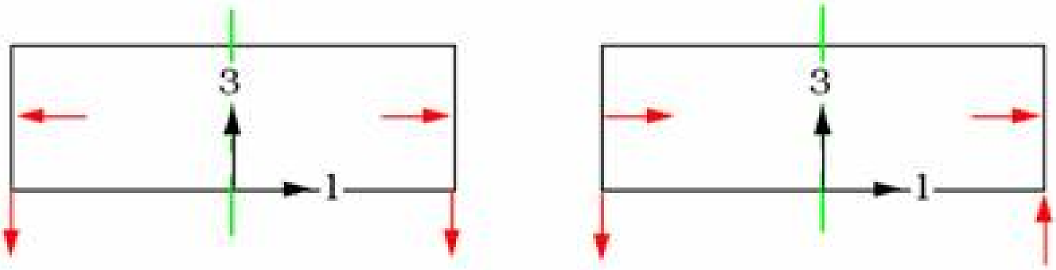


001 **CONDICIONES DE CONTORNO SIMETRIA REFLEXIVA EN ELEMENTOS ELASTICOS** CURSO 2004-5



Symmetric B. C.:

$T_1 = 0$
$F_2 = 0$
$F_3 = 0$

Antisymmetric B. C.:

$F_1 = 0$
$T_2 = 0$
$T_3 = 0$

It is easiest to find the antisymmetric conditions as the complement of the symmetric case.

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001 **MODOS DE CUERPO RIGIDO – DEFINICION** CURSO 2004-5

RIGID BODY MODES

A. DEFINITION

(16)

A rigid body mode is a displacement field which causes no strain energy. An elastic body undergoes neither direct nor shear strain.

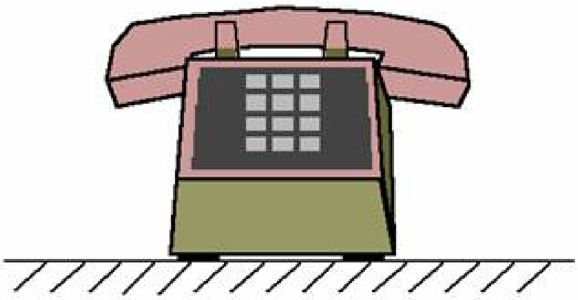
In contrast, straining modes cause strain energy to be developed. Straining modes can, however, include some rigid body motion.

Elastic bodies in three dimensions have 6 rigid body modes, 3 translations and 3 rotations:

surge (skid)	roll
sideslip	pitch
plunge	yaw

001 MODOS DE CUERPO RIGIDO – DEFINICION CURSO 2004-5

Bodies constrained to move in 2-D, such as a desk telephone, have only 3 rigid body modes.



There are two translations and one rotation.

If the receiver falls off, one has a "mechanism."

A broken spring on a dial phone, would also cause a mechanism

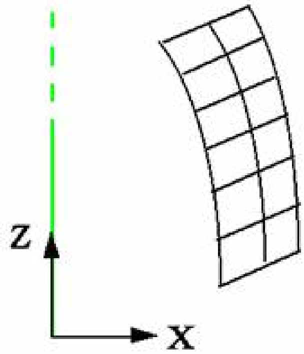
(17)

001 MODOS DE CUERPO RIGIDO - AXISIMETRIA CURSO 2004-5

The axisymmetric case is special and must be handled carefully. The NASTRAN® codes embed axisymmetry into a 3-D space. Other codes restrict the space, typically to the xz plane.

If the element is a special axisymmetric element restricted to the xz plane, there is only one rigid body mode---translation in the z direction.

Hoop stresses prevent both x translation and rotation in the xz plane as rigid body modes.



(18)

B. RULES OF THUMB

(19)

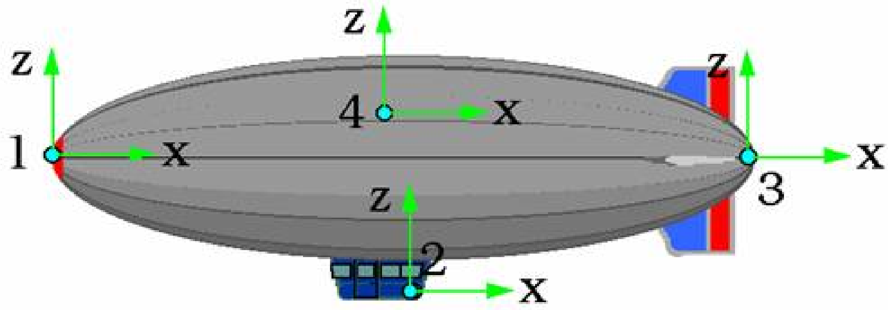
BODY CONSTRAINED TO LIE IN:	SUBCASE	# OF RIGID BODY MODES
3-D		6
2-D	plane stress plane strain	3
	axisymmetry	1
1-D		1

Codes vary widely in how two-dimensional cases are handled. Some codes (SAP) treat 2-D with special elements. Other codes (NASTRAN®) embed plane stress and plane strain into 3-D.

C. SUPPRESSION OF RIGID BODY MODES

(20)

Consider a vehicle in flight which is to be stress analyzed. The analyst must constrain 6 degrees of freedom to remove 6 rigid body modes. If this is not done, the static analysis will fail!



Node 2 (beam/plate elements): (easy)
$T_1, T_2, T_3, R_1, R_2, R_3 = 0$

Node 1 (solid): $T_1, T_2, T_3 = 0$
Node 3 (solid): $T_2, T_3 = 0$
Node 4 (membrane): $T_2 = 0$
} (good)

Node 1 (solid): $T_1, T_2, T_3 = 0$
Node 3 (solid): $T_1, T_2, T_3 = 0$
} (fails)

This last attempt fails because it does not constrain the rolling rigid body mode and it prevents elastic strain from nose to tail! This also "grounds" live loads in the longitudinal direction:

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001 **EJERCICIO 1** CURSO 2004-5

EXERCISE 8.1

[D:10] The plate structure shown in Figure E8.1 is loaded and deforms in the plane of the figure. The applied load at D and the supports at I and N extend over a fairly narrow area. Give a list of what you think are the likely "trouble spots" that would require a locally finer finite element mesh to capture high stress gradients.

Identify those spots by its letter and a reason. For example, D: vicinity of point load.

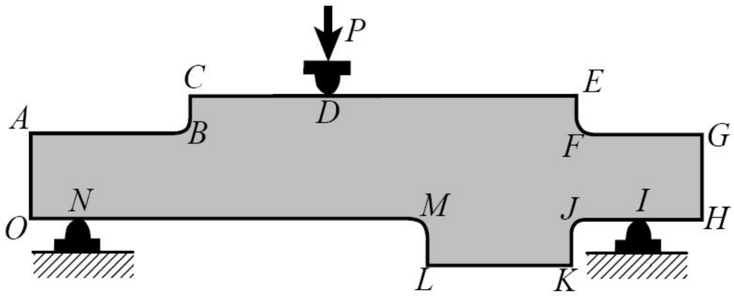

Figure E8.1. Plate structure for Exercise 8.1.

001 **EJERCICIO 2** CURSO 2004-5

EXERCISE 8.2

[D:15] Part of a two-dimensional FE mesh has been set up as indicated in Figure E8.2. Region $ABCD$ is still unmeshed. Draw a *transition mesh* within that region that correctly merges with the regular grids shown, uses 4-node quadrilateral elements (quadrilaterals with corner nodes only), and *avoids triangles*. *Note*: There are several (equally acceptable) solutions.

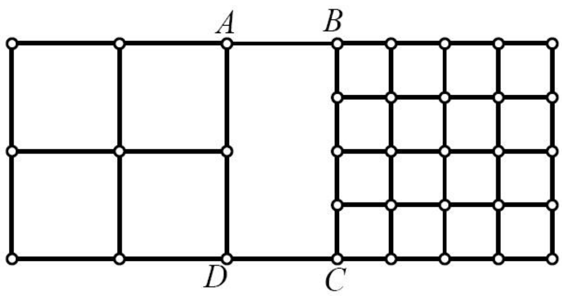

Figure E8.2. Plate structure for Exercise 8.2.

EXERCISE 8.3

[A:15] Compute the "lumped" nodal forces f_1, f_2, f_3 and f_4 equivalent to the linearly-varying distributed surface load q for the finite element layout defined in Figure E8.3. Use both NbN and EbE lumping. For example, $f_1 = 3q/8$ for NbN. Check that $f_1 + f_2 + f_3 + f_4 = 6q$ for both schemes (why?). Note that q is given as a force per unit of vertical length.

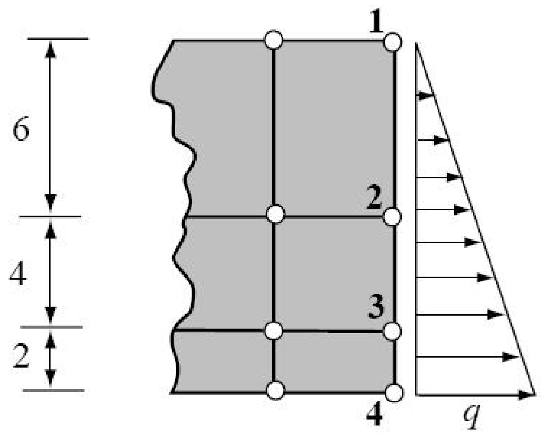

Figure E8.3. Mesh layout for Exercise 8.3.

EXERCISE 8.4

[D:15] Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in Figure E8.4. They are: (a) a circular disk under two diametrically opposite point forces (the famous “Brazilian test” for concrete); (b) the same disk under two diametrically opposite force pairs; (c) a clamped semiannulus under a force pair oriented as shown; (d) a stretched rectangular plate with a central circular hole. Finally (e) and (f) are half-planes under concentrated loads.[6]

Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines. *Note: Do all sketches on your paper, not on the printed figures.*

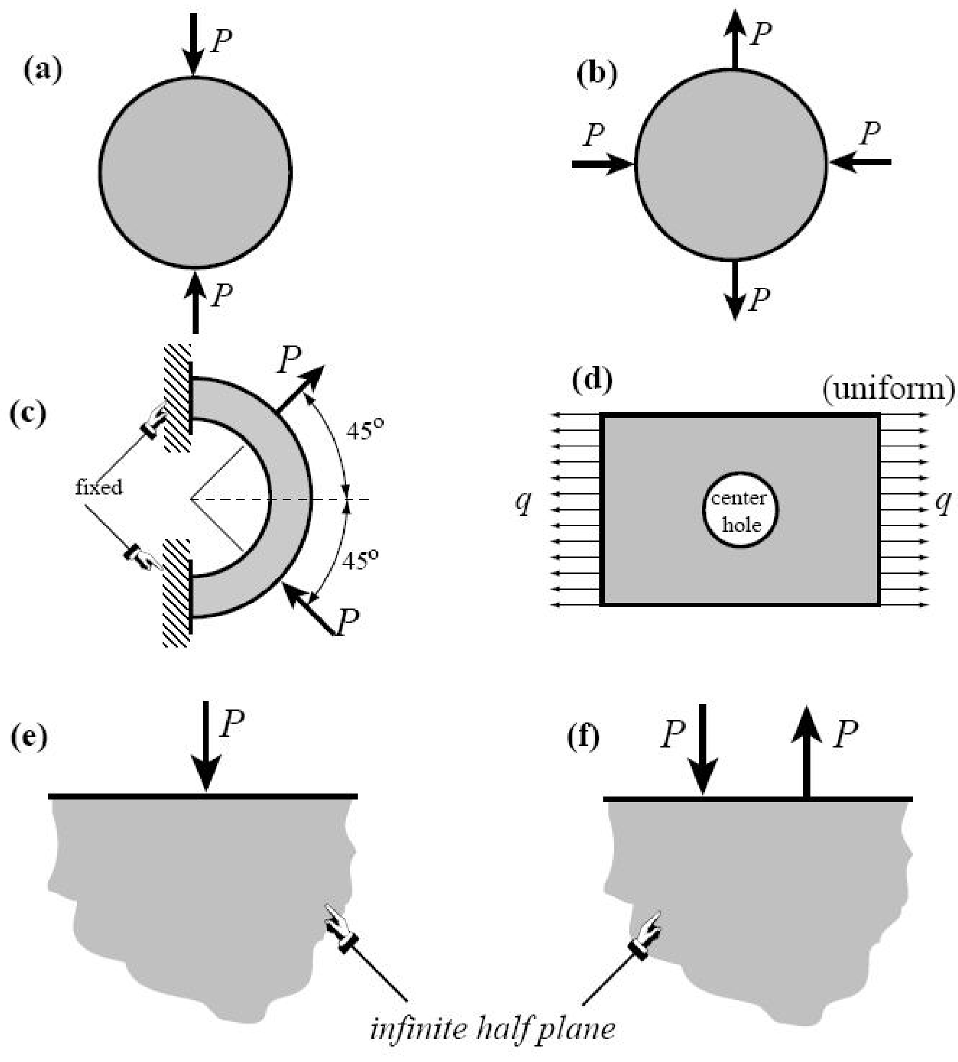

Figure E8.4. Problems for Exercise 8.4.