LECCION 3 - EJERCICIO 2 (14.2) v.2006

Mechanics ▸ Elasticity ▸ Elastic Properties ▾

Lamé Constants

The constants λ and μ arising in strain-stress relationships. They are given in terms of other solid properties as

$$\begin{aligned}
\lambda &\equiv \frac{\nu E}{(1+\nu)(1-2\nu)} \\
&= K - \tfrac{2}{3}G \\
&= \frac{2\nu G}{1-2\nu} \\
&= 3K\frac{\nu}{1+\nu} \\
&= \rho(v_p^2 - 2v_s^2) \\
\mu &\equiv \frac{E}{2(1+\nu)} \\
&= \tfrac{3}{2}(K-\lambda) \\
&= \lambda\frac{1-2\nu}{2\nu} \\
&= 3K\frac{1-2\nu}{2+2\nu} \\
&= \rho v_s^2,
\end{aligned}$$

where E is Young's modulus, ν is the Poisson ratio, G is the shear modulus, K is the bulk modulus, ρ is the density, v_p is P-wave speed, and v_s

SEE ALSO: Bulk Modulus, Poisson Ratio, Shear Modulus, Strain, Stress, Young's Modulus

- Inicio

```
Off[General::"spell1"]
Off[General::"spell"]
```

```
<< StructuralMechanics`
```

```
opts = {
    YoungsModulus → Em,
 LameCoefficients -> {λ, μ},
    BulkModulus -> k,
 PoissonsRatio -> ν,
    ShearModulus -> G};
```

1

```
ConvertCoefficients[{Em, ν}, {λ, μ}, ##] & @@ opts
```

$$\left\{\text{Em} \to \frac{\mu\,(3\,\lambda + 2\,\mu)}{\lambda + \mu},\ \nu \to \frac{\lambda}{2\,(\lambda + \mu)}\right\}$$

2

```
ConvertCoefficients[{Em, ν}, {λ, k}, ##] & @@ opts
```

$$\left\{\text{Em} \to \frac{9\,k\,(k - \lambda)}{3\,k - \lambda},\ \nu \to \frac{\lambda}{3\,k - \lambda}\right\}$$

3

```
ConvertCoefficients[{Em, ν}, {k, μ}, ##] & @@ opts
```

$$\left\{\text{Em} \to \frac{9\,k\,\mu}{3\,k + \mu},\ \nu \to \frac{3\,k - 2\,\mu}{2\,(3\,k + \mu)}\right\}$$

4

```
ConvertCoefficients[{Em, ν}, {k, G}, ##] & @@ opts
```

$$\left\{\text{Em} \to \frac{9\,G\,k}{G + 3\,k},\ \nu \to -\frac{2\,G - 3\,k}{2\,(G + 3\,k)}\right\}$$

5

```
ConvertCoefficients[{Em, ν}, {λ, G}, ##] & @@ opts
```

$$\left\{\text{Em} \to \frac{G\,(2\,G + 3\,\lambda)}{G + \lambda},\ \nu \to \frac{\lambda}{2\,(G + \lambda)}\right\}$$

6

```
ConvertCoefficients[{Em, ν}, {λ, ν}, ##] & @@ opts
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. More…
```

$$\left\{\text{Em} \to -\frac{\lambda\,(1 + \nu)\,(-1 + 2\,\nu)}{\nu}\right\}$$

7

```
ConvertCoefficients[{Em, ν}, {μ, ν}, ##] & @@ opts
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. More…
```

$$\{\text{Em} \to 2\,\mu\,(1 + \nu)\}$$

8

```
ConvertCoefficients[{Em, ν}, {k, ν}, ##] & @@ opts
```

Solve::svars : Equations may not give solutions for all "solve" variables. More…

```
{Em → -3 k (-1 + 2 ν)}
```

9

```
ConvertCoefficients[{Em, ν}, {G, ν}, ##] & @@ opts
```

Solve::svars : Equations may not give solutions for all "solve" variables. More…

```
{Em → 2 G (1 + ν)}
```

```
opts = {
    YoungsModulus → Em,
 LameCoefficients -> {λ, μ},
    BulkModulus -> k,
 PoissonsRatio -> ν,
    ShearModulus -> G};
```

PLANE STRESS - 1

$$\text{Emat} = \frac{\text{Em}}{\left(1-\nu^2\right)} * \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix};$$

PLANE STRESS - 1

```
Emat1 = Emat /. ConvertCoefficients[{Em, ν}, {λ, μ}, ##] & @@ opts;
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{\mu\,(3\lambda+2\mu)}{(\lambda+\mu)\left(1-\frac{\lambda^2}{4(\lambda+\mu)^2}\right)} & \frac{\lambda\mu\,(3\lambda+2\mu)}{2(\lambda+\mu)^2\left(1-\frac{\lambda^2}{4(\lambda+\mu)^2}\right)} & 0 \\ \frac{\lambda\mu\,(3\lambda+2\mu)}{2(\lambda+\mu)^2\left(1-\frac{\lambda^2}{4(\lambda+\mu)^2}\right)} & \frac{\mu\,(3\lambda+2\mu)}{(\lambda+\mu)\left(1-\frac{\lambda^2}{4(\lambda+\mu)^2}\right)} & 0 \\ 0 & 0 & \frac{\mu\,(3\lambda+2\mu)\left(1-\frac{\lambda}{2(\lambda+\mu)}\right)}{2(\lambda+\mu)\left(1-\frac{\lambda^2}{4(\lambda+\mu)^2}\right)} \end{pmatrix}$$

```
Emat1 = FullSimplify[Emat1];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{4\mu\,(\lambda+\mu)}{\lambda+2\mu} & \frac{2\lambda\mu}{\lambda+2\mu} & 0 \\ \frac{2\lambda\mu}{\lambda+2\mu} & \frac{4\mu\,(\lambda+\mu)}{\lambda+2\mu} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

```
Expand[Emat1];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{4\lambda\mu}{\lambda+2\mu}+\frac{4\mu^2}{\lambda+2\mu} & \frac{2\lambda\mu}{\lambda+2\mu} & 0 \\ \frac{2\lambda\mu}{\lambda+2\mu} & \frac{4\lambda\mu}{\lambda+2\mu}+\frac{4\mu^2}{\lambda+2\mu} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

PLANE STRESS - 2

```
Emat2 = Emat /. ConvertCoefficients[{Em, ν}, {λ, k}, ##] & @@ opts;
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{9k(k-\lambda)}{(3k-\lambda)\left(1-\frac{\lambda^2}{(3k-\lambda)^2}\right)} & \frac{9k(k-\lambda)\lambda}{(3k-\lambda)^2\left(1-\frac{\lambda^2}{(3k-\lambda)^2}\right)} & 0 \\ \frac{9k(k-\lambda)\lambda}{(3k-\lambda)^2\left(1-\frac{\lambda^2}{(3k-\lambda)^2}\right)} & \frac{9k(k-\lambda)}{(3k-\lambda)\left(1-\frac{\lambda^2}{(3k-\lambda)^2}\right)} & 0 \\ 0 & 0 & \frac{9k(k-\lambda)\left(1-\frac{\lambda}{3k-\lambda}\right)}{2(3k-\lambda)\left(1-\frac{\lambda^2}{(3k-\lambda)^2}\right)} \end{pmatrix}$$

```
Emat2 = FullSimplify[Emat2];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{3(k-\lambda)(3k-\lambda)}{3k-2\lambda} & \frac{3(k-\lambda)\lambda}{3k-2\lambda} & 0 \\ \frac{3(k-\lambda)\lambda}{3k-2\lambda} & \frac{3(k-\lambda)(3k-\lambda)}{3k-2\lambda} & 0 \\ 0 & 0 & \frac{3(k-\lambda)}{2} \end{pmatrix}$$

```
Expand[Emat2];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{9k^2}{3k-2\lambda}-\frac{12k\lambda}{3k-2\lambda}+\frac{3\lambda^2}{3k-2\lambda} & \frac{3k\lambda}{3k-2\lambda}-\frac{3\lambda^2}{3k-2\lambda} & 0 \\ \frac{3k\lambda}{3k-2\lambda}-\frac{3\lambda^2}{3k-2\lambda} & \frac{9k^2}{3k-2\lambda}-\frac{12k\lambda}{3k-2\lambda}+\frac{3\lambda^2}{3k-2\lambda} & 0 \\ 0 & 0 & \frac{3k}{2}-\frac{3\lambda}{2} \end{pmatrix}$$

PLANE STRESS - 3

```
Emat3 = Emat /. ConvertCoefficients[{Em, ν}, {k, μ}, ##] & @@ opts;
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{9k\mu}{(3k+\mu)\left(1-\frac{(3k-2\mu)^2}{4(3k+\mu)^2}\right)} & \frac{9k(3k-2\mu)\mu}{2(3k+\mu)^2\left(1-\frac{(3k-2\mu)^2}{4(3k+\mu)^2}\right)} & 0 \\ \frac{9k(3k-2\mu)\mu}{2(3k+\mu)^2\left(1-\frac{(3k-2\mu)^2}{4(3k+\mu)^2}\right)} & \frac{9k\mu}{(3k+\mu)\left(1-\frac{(3k-2\mu)^2}{4(3k+\mu)^2}\right)} & 0 \\ 0 & 0 & \frac{9k\mu\left(1-\frac{3k-2\mu}{2(3k+\mu)}\right)}{2(3k+\mu)\left(1-\frac{(3k-2\mu)^2}{4(3k+\mu)^2}\right)} \end{pmatrix}$$

```
Emat3 = FullSimplify[Emat3];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{4\mu\,(3k+\mu)}{3k+4\mu} & \frac{2\,(3k-2\mu)\,\mu}{3k+4\mu} & 0 \\ \frac{2\,(3k-2\mu)\,\mu}{3k+4\mu} & \frac{4\mu\,(3k+\mu)}{3k+4\mu} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

```
Expand[Emat3];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{12k\mu}{3k+4\mu}+\frac{4\mu^2}{3k+4\mu} & \frac{6k\mu}{3k+4\mu}-\frac{4\mu^2}{3k+4\mu} & 0 \\ \frac{6k\mu}{3k+4\mu}-\frac{4\mu^2}{3k+4\mu} & \frac{12k\mu}{3k+4\mu}+\frac{4\mu^2}{3k+4\mu} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

■ PLANE STRESS - 4

```
Emat4 = Emat /. ConvertCoefficients[{Em, ν}, {k, G}, ##] &@@ opts;
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{9Gk}{(G+3k)\left(1-\frac{(2G-3k)^2}{4(G+3k)^2}\right)} & -\frac{9G(2G-3k)k}{2(G+3k)^2\left(1-\frac{(2G-3k)^2}{4(G+3k)^2}\right)} & 0 \\ -\frac{9G(2G-3k)k}{2(G+3k)^2\left(1-\frac{(2G-3k)^2}{4(G+3k)^2}\right)} & \frac{9Gk}{(G+3k)\left(1-\frac{(2G-3k)^2}{4(G+3k)^2}\right)} & 0 \\ 0 & 0 & \frac{9Gk\left(1+\frac{2G-3k}{2(G+3k)}\right)}{2(G+3k)\left(1-\frac{(2G-3k)^2}{4(G+3k)^2}\right)} \end{pmatrix}$$

```
Emat4 = FullSimplify[Emat4];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{4G(G+3k)}{4G+3k} & 2G\left(1-\frac{6G}{4G+3k}\right) & 0 \\ 2G\left(1-\frac{6G}{4G+3k}\right) & \frac{4G(G+3k)}{4G+3k} & 0 \\ 0 & 0 & G \end{pmatrix}$$

```
Expand[Emat4];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{4G^2}{4G+3k}+\frac{12Gk}{4G+3k} & 2G-\frac{12G^2}{4G+3k} & 0 \\ 2G-\frac{12G^2}{4G+3k} & \frac{4G^2}{4G+3k}+\frac{12Gk}{4G+3k} & 0 \\ 0 & 0 & G \end{pmatrix}$$

■ PLANE STRESS - 5

```
Emat5 = Emat /. ConvertCoefficients[{Em, ν}, {λ, G}, ##] &@@ opts;
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{G\,(2\,G+3\,\lambda)}{(G+\lambda)\,\left(1-\frac{\lambda^2}{4\,(G+\lambda)^2}\right)} & \frac{G\,\lambda\,(2\,G+3\,\lambda)}{2\,(G+\lambda)^2\,\left(1-\frac{\lambda^2}{4\,(G+\lambda)^2}\right)} & 0 \\ \frac{G\,\lambda\,(2\,G+3\,\lambda)}{2\,(G+\lambda)^2\,\left(1-\frac{\lambda^2}{4\,(G+\lambda)^2}\right)} & \frac{G\,(2\,G+3\,\lambda)}{(G+\lambda)\,\left(1-\frac{\lambda^2}{4\,(G+\lambda)^2}\right)} & 0 \\ 0 & 0 & \frac{G\,(2\,G+3\,\lambda)\,\left(1-\frac{\lambda}{2\,(G+\lambda)}\right)}{2\,(G+\lambda)\,\left(1-\frac{\lambda^2}{4\,(G+\lambda)^2}\right)} \end{pmatrix}$$

```
Emat5 = FullSimplify[Emat4];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{4\,G\,(G+3\,k)}{4\,G+3\,k} & 2\,G\left(1-\frac{6\,G}{4\,G+3\,k}\right) & 0 \\ 2\,G\left(1-\frac{6\,G}{4\,G+3\,k}\right) & \frac{4\,G\,(G+3\,k)}{4\,G+3\,k} & 0 \\ 0 & 0 & G \end{pmatrix}$$

```
Expand[Emat5];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{4\,G^2}{4\,G+3\,k}+\frac{12\,G\,k}{4\,G+3\,k} & 2\,G-\frac{12\,G^2}{4\,G+3\,k} & 0 \\ 2\,G-\frac{12\,G^2}{4\,G+3\,k} & \frac{4\,G^2}{4\,G+3\,k}+\frac{12\,G\,k}{4\,G+3\,k} & 0 \\ 0 & 0 & G \end{pmatrix}$$

■ PLANE STRESS - 6

```
Emat6 = Emat /. ConvertCoefficients[{Em, ν}, {λ, ν}, ##] &@@ opts;
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. More…
```

```
% // MatrixForm
```

$$\begin{pmatrix} -\frac{\lambda\,(1+\nu)\,(-1+2\,\nu)}{\nu\,\left(1-\nu^2\right)} & -\frac{\lambda\,(1+\nu)\,(-1+2\,\nu)}{1-\nu^2} & 0 \\ -\frac{\lambda\,(1+\nu)\,(-1+2\,\nu)}{1-\nu^2} & -\frac{\lambda\,(1+\nu)\,(-1+2\,\nu)}{\nu\,\left(1-\nu^2\right)} & 0 \\ 0 & 0 & -\frac{\lambda\,(1-\nu)\,(1+\nu)\,(-1+2\,\nu)}{2\,\nu\,\left(1-\nu^2\right)} \end{pmatrix}$$

```
Emat6 = FullSimplify[Emat6];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \lambda\left(\frac{1}{-1+\nu}+\frac{1}{\nu}\right) & \lambda\left(2+\frac{1}{-1+\nu}\right) & 0 \\ \lambda\left(2+\frac{1}{-1+\nu}\right) & \lambda\left(\frac{1}{-1+\nu}+\frac{1}{\nu}\right) & 0 \\ 0 & 0 & \frac{1}{2}\,\lambda\left(-2+\frac{1}{\nu}\right) \end{pmatrix}$$

```
Expand[Emat6];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{\lambda}{-1+\nu}+\frac{\lambda}{\nu} & 2\lambda+\frac{\lambda}{-1+\nu} & 0 \\ 2\lambda+\frac{\lambda}{-1+\nu} & \frac{\lambda}{-1+\nu}+\frac{\lambda}{\nu} & 0 \\ 0 & 0 & -\lambda+\frac{\lambda}{2\nu} \end{pmatrix}$$

■ PLANE STRESS - 7

```
Emat7 = Emat /. ConvertCoefficients[{Em, ν}, {μ, ν}, ##] &@@ opts;
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. More…
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{2\mu(1+\nu)}{1-\nu^2} & \frac{2\mu\nu(1+\nu)}{1-\nu^2} & 0 \\ \frac{2\mu\nu(1+\nu)}{1-\nu^2} & \frac{2\mu(1+\nu)}{1-\nu^2} & 0 \\ 0 & 0 & \frac{\mu(1-\nu)(1+\nu)}{1-\nu^2} \end{pmatrix}$$

```
Emat7 = FullSimplify[Emat7];
```

```
% // MatrixForm
```

$$\begin{pmatrix} -\frac{2\mu}{-1+\nu} & -\frac{2\mu\nu}{-1+\nu} & 0 \\ -\frac{2\mu\nu}{-1+\nu} & -\frac{2\mu}{-1+\nu} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

```
Expand[Emat7];
```

```
% // MatrixForm
```

$$\begin{pmatrix} -\frac{2\mu}{-1+\nu} & -\frac{2\mu\nu}{-1+\nu} & 0 \\ -\frac{2\mu\nu}{-1+\nu} & -\frac{2\mu}{-1+\nu} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

■ PLANE STRESS - 8

```
Emat8 = Emat /. ConvertCoefficients[{Em, ν}, {k, ν}, ##] &@@ opts;
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. More…
```

```
% // MatrixForm
```

$$\begin{pmatrix} -\frac{3k(-1+2\nu)}{1-\nu^2} & -\frac{3k\nu(-1+2\nu)}{1-\nu^2} & 0 \\ -\frac{3k\nu(-1+2\nu)}{1-\nu^2} & -\frac{3k(-1+2\nu)}{1-\nu^2} & 0 \\ 0 & 0 & -\frac{3k(1-\nu)(-1+2\nu)}{2(1-\nu^2)} \end{pmatrix}$$

```
Emat8 = FullSimplify[Emat8];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{3k(-1+2\nu)}{-1+\nu^2} & \frac{3k\nu(-1+2\nu)}{-1+\nu^2} & 0 \\ \frac{3k\nu(-1+2\nu)}{-1+\nu^2} & \frac{3k(-1+2\nu)}{-1+\nu^2} & 0 \\ 0 & 0 & k\left(-3+\frac{9}{2(1+\nu)}\right) \end{pmatrix}$$

```
Expand[Emat8];
```

```
% // MatrixForm
```

$$\begin{pmatrix} -\frac{3k}{-1+\nu^2}+\frac{6k\nu}{-1+\nu^2} & -\frac{3k\nu}{-1+\nu^2}+\frac{6k\nu^2}{-1+\nu^2} & 0 \\ -\frac{3k\nu}{-1+\nu^2}+\frac{6k\nu^2}{-1+\nu^2} & -\frac{3k}{-1+\nu^2}+\frac{6k\nu}{-1+\nu^2} & 0 \\ 0 & 0 & -3k+\frac{9k}{2(1+\nu)} \end{pmatrix}$$

■ PLANE STRESS - 9

```
Emat9 = Emat /. ConvertCoefficients[{Em, ν}, {G, ν}, ##] &@@ opts;
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. More…
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{2G(1+\nu)}{1-\nu^2} & \frac{2G\nu(1+\nu)}{1-\nu^2} & 0 \\ \frac{2G\nu(1+\nu)}{1-\nu^2} & \frac{2G(1+\nu)}{1-\nu^2} & 0 \\ 0 & 0 & \frac{G(1-\nu)(1+\nu)}{1-\nu^2} \end{pmatrix}$$

```
Emat9 = FullSimplify[Emat9];
```

```
% // MatrixForm
```

$$\begin{pmatrix} -\frac{2G}{-1+\nu} & -\frac{2G\nu}{-1+\nu} & 0 \\ -\frac{2G\nu}{-1+\nu} & -\frac{2G}{-1+\nu} & 0 \\ 0 & 0 & G \end{pmatrix}$$

```
Expand[Emat9];
```

```
% // MatrixForm
```

$$\begin{pmatrix} -\frac{2G}{-1+\nu} & -\frac{2G\nu}{-1+\nu} & 0 \\ -\frac{2G\nu}{-1+\nu} & -\frac{2G}{-1+\nu} & 0 \\ 0 & 0 & G \end{pmatrix}$$

PLANE STRAIN

$$\text{Emat} = \frac{\text{Em} * (1 - \nu)}{(1 + \nu) * (1 - 2 * \nu)} * \begin{pmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2*\nu}{2*(1-\nu)} \end{pmatrix};$$

PLANE STRAIN - 10

```
Emat10 = Emat /. ConvertCoefficients[{Em, ν}, {λ, μ}, ##] &@@ opts;
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{\mu\,(3\,\lambda+2\,\mu)\,\left(1-\frac{\lambda}{2\,(\lambda+\mu)}\right)}{(\lambda+\mu)\,\left(1-\frac{\lambda}{\lambda+\mu}\right)\,\left(1+\frac{\lambda}{2\,(\lambda+\mu)}\right)} & \frac{\lambda\,\mu\,(3\,\lambda+2\,\mu)}{2\,(\lambda+\mu)^2\,\left(1-\frac{\lambda}{\lambda+\mu}\right)\,\left(1+\frac{\lambda}{2\,(\lambda+\mu)}\right)} & 0 \\ \frac{\lambda\,\mu\,(3\,\lambda+2\,\mu)}{2\,(\lambda+\mu)^2\,\left(1-\frac{\lambda}{\lambda+\mu}\right)\,\left(1+\frac{\lambda}{2\,(\lambda+\mu)}\right)} & \frac{\mu\,(3\,\lambda+2\,\mu)\,\left(1-\frac{\lambda}{2\,(\lambda+\mu)}\right)}{(\lambda+\mu)\,\left(1-\frac{\lambda}{\lambda+\mu}\right)\,\left(1+\frac{\lambda}{2\,(\lambda+\mu)}\right)} & 0 \\ 0 & 0 & \frac{\mu\,(3\,\lambda+2\,\mu)}{2\,(\lambda+\mu)\,\left(1+\frac{\lambda}{2\,(\lambda+\mu)}\right)} \end{pmatrix}$$

```
Emat10 = FullSimplify[Emat10];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \lambda + 2\,\mu & \lambda & 0 \\ \lambda & \lambda + 2\,\mu & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

```
Expand[Emat10];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \lambda + 2\,\mu & \lambda & 0 \\ \lambda & \lambda + 2\,\mu & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

PLANE STRAIN - 11

```
Emat11 = Emat /. ConvertCoefficients[{Em, ν}, {λ, k}, ##] &@@ opts;
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{9\,k\,(k-\lambda)\,\left(1-\frac{\lambda}{3\,k-\lambda}\right)}{(3\,k-\lambda)\,\left(1-\frac{2\,\lambda}{3\,k-\lambda}\right)\,\left(1+\frac{\lambda}{3\,k-\lambda}\right)} & \frac{9\,k\,(k-\lambda)\,\lambda}{(3\,k-\lambda)^2\,\left(1-\frac{2\,\lambda}{3\,k-\lambda}\right)\,\left(1+\frac{\lambda}{3\,k-\lambda}\right)} & 0 \\ \frac{9\,k\,(k-\lambda)\,\lambda}{(3\,k-\lambda)^2\,\left(1-\frac{2\,\lambda}{3\,k-\lambda}\right)\,\left(1+\frac{\lambda}{3\,k-\lambda}\right)} & \frac{9\,k\,(k-\lambda)\,\left(1-\frac{\lambda}{3\,k-\lambda}\right)}{(3\,k-\lambda)\,\left(1-\frac{2\,\lambda}{3\,k-\lambda}\right)\,\left(1+\frac{\lambda}{3\,k-\lambda}\right)} & 0 \\ 0 & 0 & \frac{9\,k\,(k-\lambda)}{2\,(3\,k-\lambda)\,\left(1+\frac{\lambda}{3\,k-\lambda}\right)} \end{pmatrix}$$

```
Emat11 = FullSimplify[Emat11];
```

```
% // MatrixForm
```

$$\begin{pmatrix} 3\,k-2\,\lambda & \lambda & 0 \\ \lambda & 3\,k-2\,\lambda & 0 \\ 0 & 0 & \frac{3\,(k-\lambda)}{2} \end{pmatrix}$$

```
Expand[Emat11];
```

```
% // MatrixForm
```

$$\begin{pmatrix} 3\,k-2\,\lambda & \lambda & 0 \\ \lambda & 3\,k-2\,\lambda & 0 \\ 0 & 0 & \frac{3\,k}{2}-\frac{3\,\lambda}{2} \end{pmatrix}$$

■ PLANE STRAIN - 12

```
Emat12 = Emat /. ConvertCoefficients[{Em, v}, {k, μ}, ##] & @@ opts;
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{9\,k\,\mu\left(1-\frac{3\,k-2\,\mu}{2\,(3\,k+\mu)}\right)}{(3\,k+\mu)\left(1-\frac{3\,k-2\,\mu}{3\,k+\mu}\right)\left(1+\frac{3\,k-2\,\mu}{2\,(3\,k+\mu)}\right)} & \frac{9\,k\,(3\,k-2\,\mu)\,\mu}{2\,(3\,k+\mu)^2\left(1-\frac{3\,k-2\,\mu}{3\,k+\mu}\right)\left(1+\frac{3\,k-2\,\mu}{2\,(3\,k+\mu)}\right)} & 0 \\ \frac{9\,k\,(3\,k-2\,\mu)\,\mu}{2\,(3\,k+\mu)^2\left(1-\frac{3\,k-2\,\mu}{3\,k+\mu}\right)\left(1+\frac{3\,k-2\,\mu}{2\,(3\,k+\mu)}\right)} & \frac{9\,k\,\mu\left(1-\frac{3\,k-2\,\mu}{2\,(3\,k+\mu)}\right)}{(3\,k+\mu)\left(1-\frac{3\,k-2\,\mu}{3\,k+\mu}\right)\left(1+\frac{3\,k-2\,\mu}{2\,(3\,k+\mu)}\right)} & 0 \\ 0 & 0 & \frac{9\,k\,\mu}{2\,(3\,k+\mu)\left(1+\frac{3\,k-2\,\mu}{2\,(3\,k+\mu)}\right)} \end{pmatrix}$$

```
Emat12 = FullSimplify[Emat12];
```

```
% // MatrixForm
```

$$\begin{pmatrix} k+\frac{4\,\mu}{3} & k-\frac{2\,\mu}{3} & 0 \\ k-\frac{2\,\mu}{3} & k+\frac{4\,\mu}{3} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

```
Expand[Emat12];
```

```
% // MatrixForm
```

$$\begin{pmatrix} k+\frac{4\,\mu}{3} & k-\frac{2\,\mu}{3} & 0 \\ k-\frac{2\,\mu}{3} & k+\frac{4\,\mu}{3} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

■ PLANE STRAIN - 13

```
Emat13 = Emat /. ConvertCoefficients[{Em, v}, {k, G}, ##] & @@ opts;
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{9\,G\,k\left(1+\frac{2\,G-3\,k}{2\,(G+3\,k)}\right)}{(G+3\,k)\left(1-\frac{2\,G-3\,k}{2\,(G+3\,k)}\right)\left(1+\frac{2\,G-3\,k}{G+3\,k}\right)} & -\frac{9\,G\,(2\,G-3\,k)\,k}{2\,(G+3\,k)^2\left(1-\frac{2\,G-3\,k}{2\,(G+3\,k)}\right)\left(1+\frac{2\,G-3\,k}{G+3\,k}\right)} & 0 \\ -\frac{9\,G\,(2\,G-3\,k)\,k}{2\,(G+3\,k)^2\left(1-\frac{2\,G-3\,k}{2\,(G+3\,k)}\right)\left(1+\frac{2\,G-3\,k}{G+3\,k}\right)} & \frac{9\,G\,k\left(1+\frac{2\,G-3\,k}{2\,(G+3\,k)}\right)}{(G+3\,k)\left(1-\frac{2\,G-3\,k}{2\,(G+3\,k)}\right)\left(1+\frac{2\,G-3\,k}{G+3\,k}\right)} & 0 \\ 0 & 0 & \frac{9\,G\,k}{2\,(G+3\,k)\left(1-\frac{2\,G-3\,k}{2\,(G+3\,k)}\right)} \end{pmatrix}$$

```
Emat13 = FullSimplify[Emat13];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{4\,G}{3}+k & -\frac{2\,G}{3}+k & 0 \\ -\frac{2\,G}{3}+k & \frac{4\,G}{3}+k & 0 \\ 0 & 0 & G \end{pmatrix}$$

```
Expand[Emat13];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{4\,G}{3}+k & -\frac{2\,G}{3}+k & 0 \\ -\frac{2\,G}{3}+k & \frac{4\,G}{3}+k & 0 \\ 0 & 0 & G \end{pmatrix}$$

■ PLANE STRAIN - 14

```
Emat14 = Emat /. ConvertCoefficients[{Em, ν}, {λ, G}, ##] & @@ opts;
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{G\,(2\,G+3\,\lambda)\left(1-\frac{\lambda}{2\,(G+\lambda)}\right)}{(G+\lambda)\left(1-\frac{\lambda}{G+\lambda}\right)\left(1+\frac{\lambda}{2\,(G+\lambda)}\right)} & \frac{G\,\lambda\,(2\,G+3\,\lambda)}{2\,(G+\lambda)^2\left(1-\frac{\lambda}{G+\lambda}\right)\left(1+\frac{\lambda}{2\,(G+\lambda)}\right)} & 0 \\ \frac{G\,\lambda\,(2\,G+3\,\lambda)}{2\,(G+\lambda)^2\left(1-\frac{\lambda}{G+\lambda}\right)\left(1+\frac{\lambda}{2\,(G+\lambda)}\right)} & \frac{G\,(2\,G+3\,\lambda)\left(1-\frac{\lambda}{2\,(G+\lambda)}\right)}{(G+\lambda)\left(1-\frac{\lambda}{G+\lambda}\right)\left(1+\frac{\lambda}{2\,(G+\lambda)}\right)} & 0 \\ 0 & 0 & \frac{G\,(2\,G+3\,\lambda)}{2\,(G+\lambda)\left(1+\frac{\lambda}{2\,(G+\lambda)}\right)} \end{pmatrix}$$

```
Emat14 = FullSimplify[Emat14];
```

```
% // MatrixForm
```

$$\begin{pmatrix} 2\,G+\lambda & \lambda & 0 \\ \lambda & 2\,G+\lambda & 0 \\ 0 & 0 & G \end{pmatrix}$$

```
Expand[Emat14];
```

```
% // MatrixForm
```

$$\begin{pmatrix} 2\,G+\lambda & \lambda & 0 \\ \lambda & 2\,G+\lambda & 0 \\ 0 & 0 & G \end{pmatrix}$$

PLANE STRAIN - 15

```
Emat15 = Emat /. ConvertCoefficients[{Em, ν}, {λ, ν}, ##] & @@ opts;
```

Solve::svars : Equations may not give solutions for all "solve" variables. More…

```
% // MatrixForm
```

$$\begin{pmatrix} -\frac{\lambda\,(1-\nu)\,(-1+2\,\nu)}{(1-2\,\nu)\,\nu} & -\frac{\lambda\,(-1+2\,\nu)}{1-2\,\nu} & 0 \\ -\frac{\lambda\,(-1+2\,\nu)}{1-2\,\nu} & -\frac{\lambda\,(1-\nu)\,(-1+2\,\nu)}{(1-2\,\nu)\,\nu} & 0 \\ 0 & 0 & -\frac{\lambda\,(-1+2\,\nu)}{2\,\nu} \end{pmatrix}$$

```
Emat15 = FullSimplify[Emat15];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \lambda\left(-1+\frac{1}{\nu}\right) & \lambda & 0 \\ \lambda & \lambda\left(-1+\frac{1}{\nu}\right) & 0 \\ 0 & 0 & \frac{1}{2}\lambda\left(-2+\frac{1}{\nu}\right) \end{pmatrix}$$

```
Expand[Emat15];
```

```
% // MatrixForm
```

$$\begin{pmatrix} -\lambda+\frac{\lambda}{\nu} & \lambda & 0 \\ \lambda & -\lambda+\frac{\lambda}{\nu} & 0 \\ 0 & 0 & -\lambda+\frac{\lambda}{2\,\nu} \end{pmatrix}$$

PLANE STRAIN - 16

```
Emat16 = Emat /. ConvertCoefficients[{Em, ν}, {μ, ν}, ##] & @@ opts;
```

Solve::svars : Equations may not give solutions for all "solve" variables. More…

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{2\,\mu\,(1-\nu)}{1-2\,\nu} & \frac{2\,\mu\,\nu}{1-2\,\nu} & 0 \\ \frac{2\,\mu\,\nu}{1-2\,\nu} & \frac{2\,\mu\,(1-\nu)}{1-2\,\nu} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

```
Emat16 = FullSimplify[Emat16];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \mu\left(1+\frac{1}{1-2\,\nu}\right) & \frac{2\,\mu\,\nu}{1-2\,\nu} & 0 \\ \frac{2\,\mu\,\nu}{1-2\,\nu} & \mu\left(1+\frac{1}{1-2\,\nu}\right) & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

```
Expand[Emat16];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \mu+\frac{\mu}{1-2\,\nu} & \frac{2\,\mu\,\nu}{1-2\,\nu} & 0 \\ \frac{2\,\mu\,\nu}{1-2\,\nu} & \mu+\frac{\mu}{1-2\,\nu} & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

■ PLANE STRAIN - 17

```
Emat17 = Emat /. ConvertCoefficients[{Em, ν}, {k, ν}, ##] & @@ opts;
```

Solve::svars : Equations may not give solutions for all "solve" variables. More…

```
% // MatrixForm
```

$$\begin{pmatrix} -\frac{3\,k\,(1-\nu)\,(-1+2\,\nu)}{(1-2\,\nu)\,(1+\nu)} & -\frac{3\,k\,\nu\,(-1+2\,\nu)}{(1-2\,\nu)\,(1+\nu)} & 0 \\ -\frac{3\,k\,\nu\,(-1+2\,\nu)}{(1-2\,\nu)\,(1+\nu)} & -\frac{3\,k\,(1-\nu)\,(-1+2\,\nu)}{(1-2\,\nu)\,(1+\nu)} & 0 \\ 0 & 0 & -\frac{3\,k\,(-1+2\,\nu)}{2\,(1+\nu)} \end{pmatrix}$$

```
Emat17 = FullSimplify[Emat17];
```

```
% // MatrixForm
```

$$\begin{pmatrix} -\frac{3\,k\,(-1+\nu)}{1+\nu} & \frac{3\,k\,\nu}{1+\nu} & 0 \\ \frac{3\,k\,\nu}{1+\nu} & -\frac{3\,k\,(-1+\nu)}{1+\nu} & 0 \\ 0 & 0 & k\left(-3+\frac{9}{2\,(1+\nu)}\right) \end{pmatrix}$$

```
Expand[Emat17];
```

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{3\,k}{1+\nu}-\frac{3\,k\,\nu}{1+\nu} & \frac{3\,k\,\nu}{1+\nu} & 0 \\ \frac{3\,k\,\nu}{1+\nu} & \frac{3\,k}{1+\nu}-\frac{3\,k\,\nu}{1+\nu} & 0 \\ 0 & 0 & -3\,k+\frac{9\,k}{2\,(1+\nu)} \end{pmatrix}$$

■ PLANE STRAIN - 18

```
Emat18 = Emat /. ConvertCoefficients[{Em, ν}, {G, ν}, ##] & @@ opts;
```

Solve::svars : Equations may not give solutions for all "solve" variables. More…

```
% // MatrixForm
```

$$\begin{pmatrix} \frac{2\,G\,(1-\nu)}{1-2\,\nu} & \frac{2\,G\,\nu}{1-2\,\nu} & 0 \\ \frac{2\,G\,\nu}{1-2\,\nu} & \frac{2\,G\,(1-\nu)}{1-2\,\nu} & 0 \\ 0 & 0 & G \end{pmatrix}$$

```
Emat18 = FullSimplify[Emat18];
```

```
% // MatrixForm
```

$$\begin{pmatrix} G\left(1+\frac{1}{1-2\,\nu}\right) & \frac{2\,G\,\nu}{1-2\,\nu} & 0 \\ \frac{2\,G\,\nu}{1-2\,\nu} & G\left(1+\frac{1}{1-2\,\nu}\right) & 0 \\ 0 & 0 & G \end{pmatrix}$$

```
Expand[Emat18];
```

```
% // MatrixForm
```

$$\begin{pmatrix} G+\frac{G}{1-2\,\nu} & \frac{2\,G\,\nu}{1-2\,\nu} & 0 \\ \frac{2\,G\,\nu}{1-2\,\nu} & G+\frac{G}{1-2\,\nu} & 0 \\ 0 & 0 & G \end{pmatrix}$$

More ...

```
SetDirectory[NotebookDirectory[]]
```

```
C:\#0-Modulos-M30x_MeF-10\#M302-m6-a2a-sw7\04-Tension-plana\T-Elasticidad-v2006
```

```
Export["L02TB2V6.dat", Emat18]
```

```
L02TB2V6.dat
```

```
<< "L02TB2V6.dat"
```

```
Emat18
```