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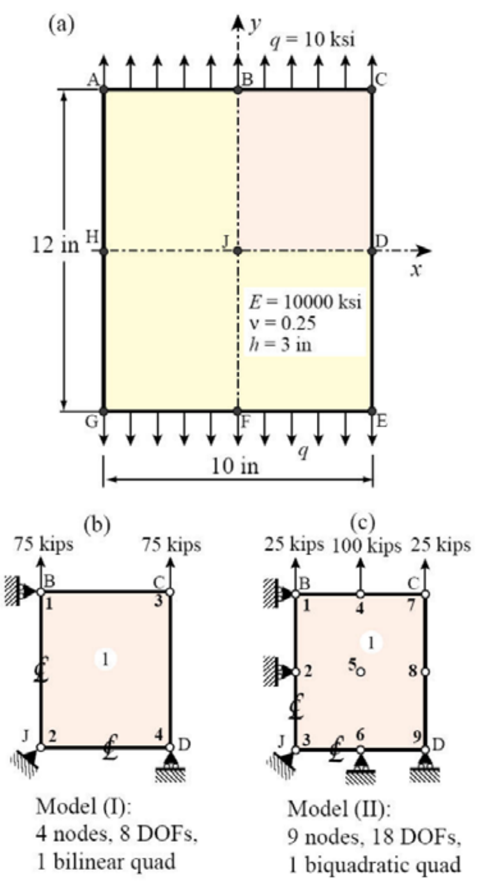
Estas ACTIVIDADES DE CLASE deberá realizarse descargando los documentos *NB* disponibles en las páginas web, completandolos adecuadamente, denominandolos de la forma especificada y subiendolos a tu cuenta de entrega personal. En este documento *PDF* habrá que contestar a las *PREGUNTAS* que planteo a lo largo de la grabación en video correspondiente a la clase.

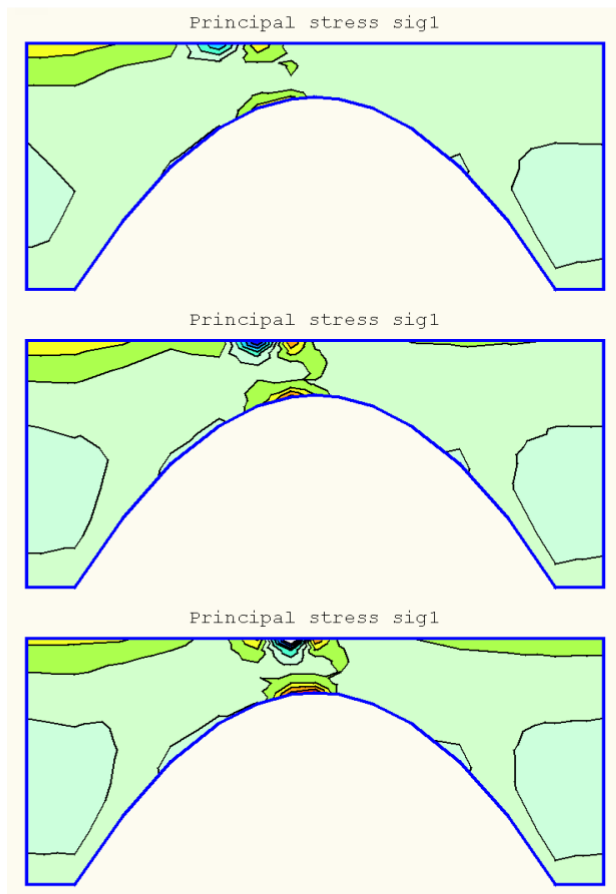
Para familiarizarnos con la ***Realización de un Análisis por Elementos Finitos bajo condiciones de Tensión Plana y con el Calculo de las Tensiones***, su definición, su terminología y su planteamiento; durante las explicaciones en clase habrá que completar este documento PDF.

Estas son imágenes de algunos de los ejercicios considerados en las ACTIVIDADES de esta CLASE:

10-C10-Mathematica-C1



08-C8-Mathematica-C2



PREGUNTAS Y TUS CONTESTACIONES:

DOCUMENTO PDF A COMPLETAR:

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PROGRAMA DE TENSION PLANA EN MATHEMATICA

001	PASOS A SEGUIR PARA REALIZAR UN ANALISIS POR EF	CURSO 2004-5

Preprocessing:

Processing:

Postprocessing:

001	ESTRUCTURA DE DATOS PARA LA DEFINICION DEL PROBLEMA	CURSO 2004-5

Geometry Data Set:

Element Data Set:

Degree of Freedom Data Set:

Miscellaneous Processing Data Set:

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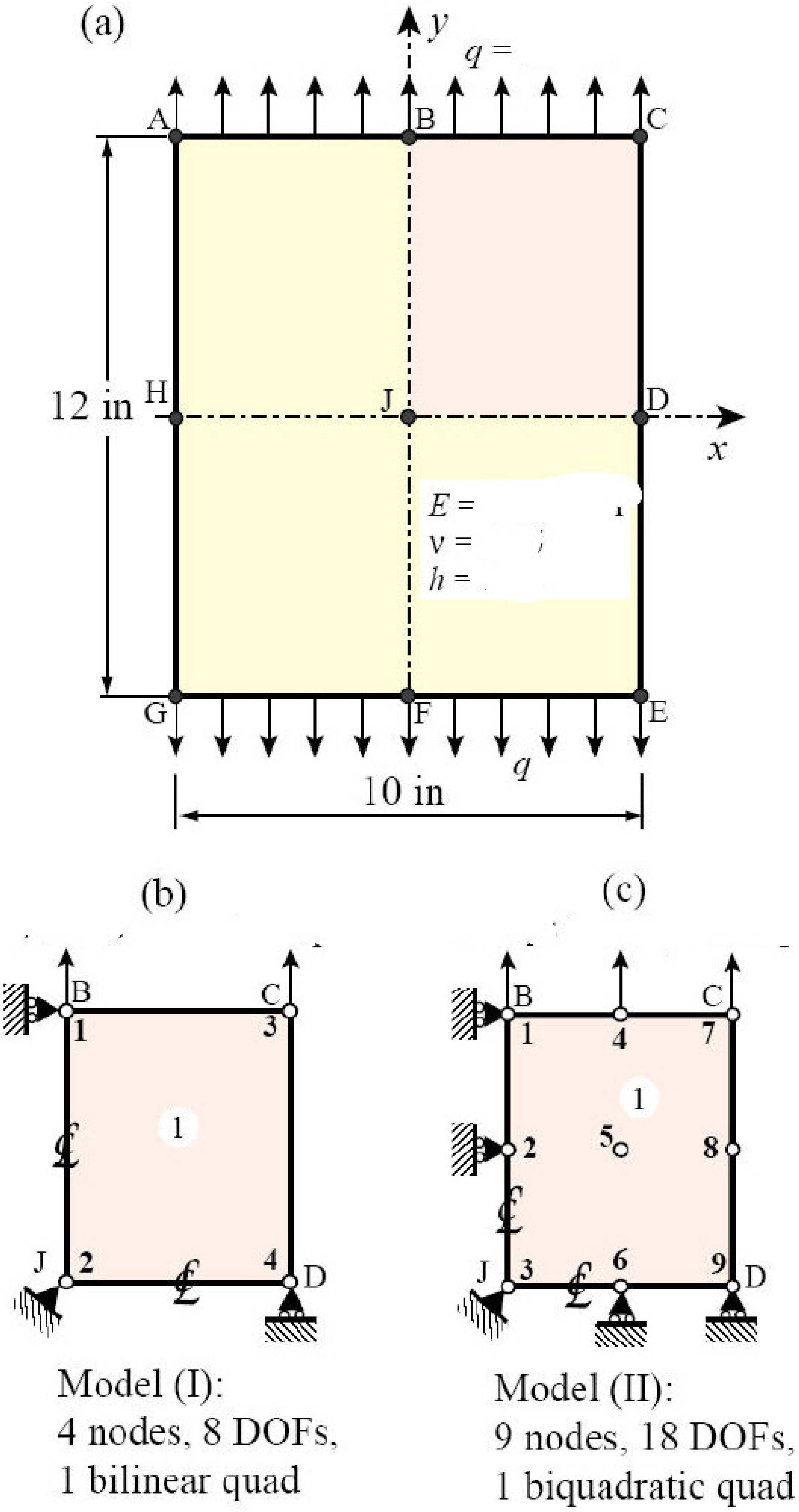
001 EJEMPLO PREEPARACION DATOS MODELO CON UN SOLO ELEMENTO CURSO 2004-5

Figure 27.1 is a rectangular plate in plane stress under uniform uniaxial loading. Its analytical solution is $\sigma_{yy} = q$, $\sigma_{xx} = \sigma_{xy} = 0$, $u_y = qy/E$, $u_x = -\nu qx/E$. This problem should be solved exactly by *any* finite element mesh. In particular, the two one-element models shown on the right of that figure?



Global node numbers shown

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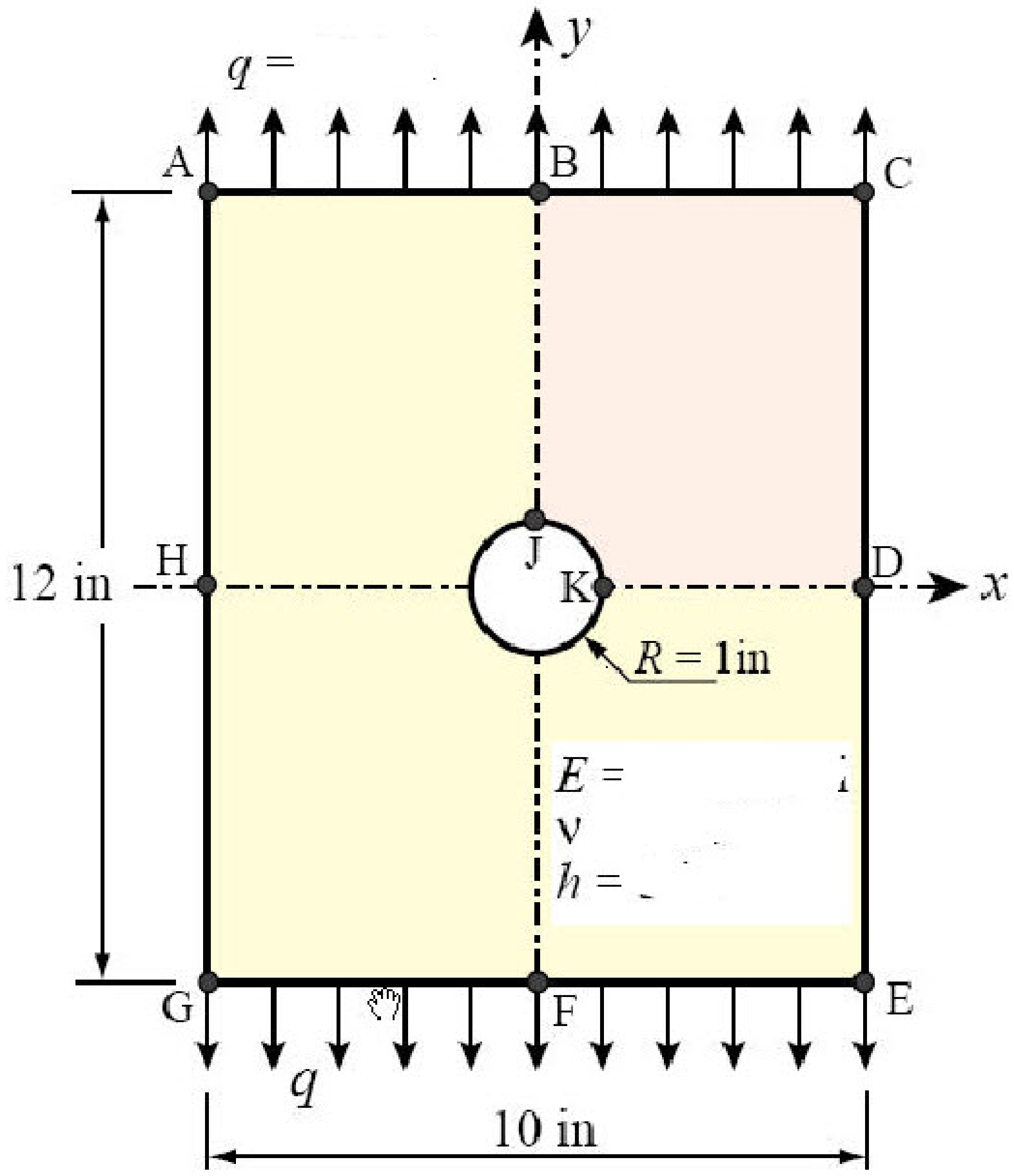
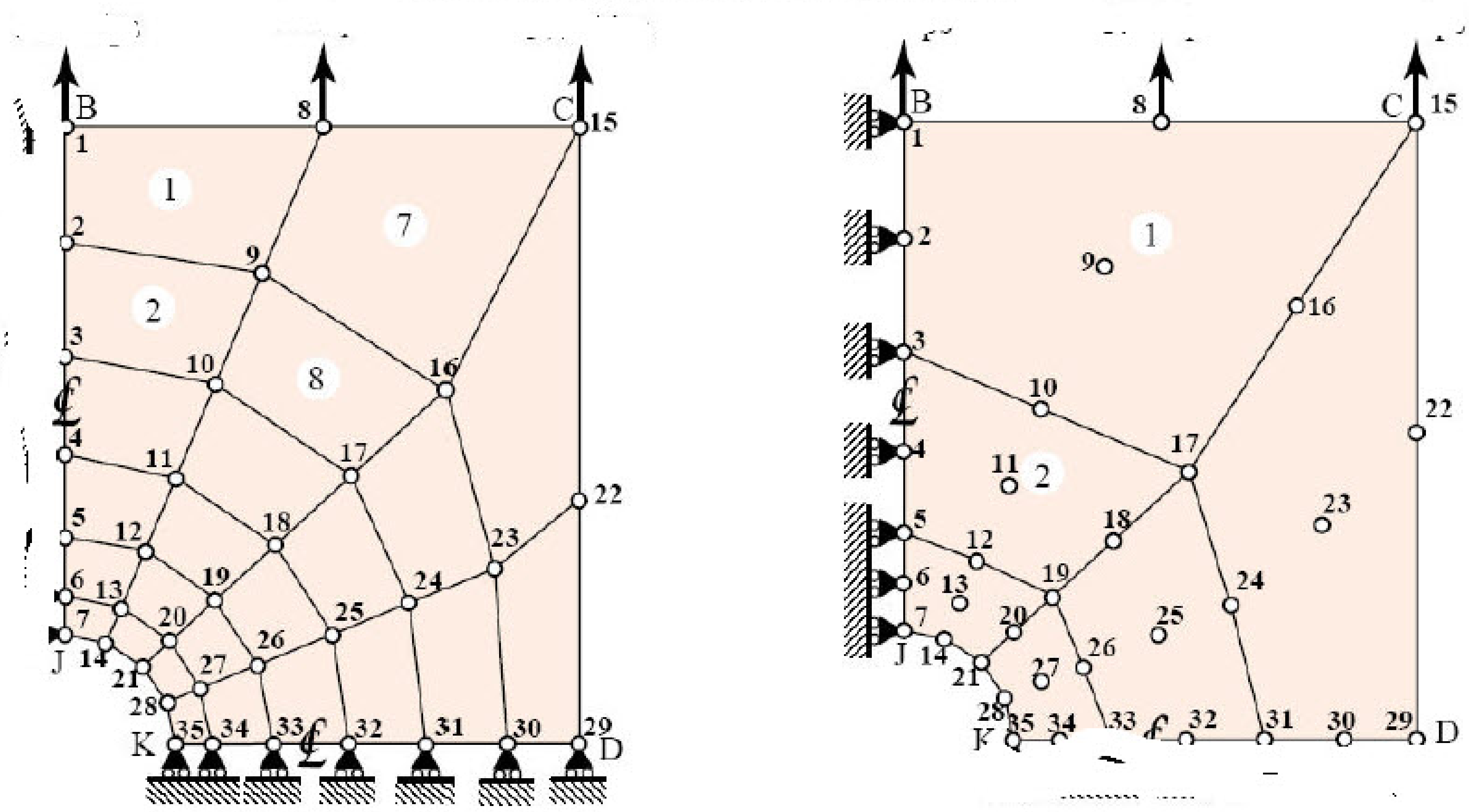
001 EJEMPLO PREPARACION DATOS CURSO 2004-5

MODELO CON MALLA GENERADA



Node 8 is exactly midway between 1 and 15



Model (I): 35 nodes, 70 DOFs, 24 bilinear quads

Model (II): 35 nodes, 70 DOFs, 6 biquadratic quads

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001 **DEFINICION DEL MODELO** **CURSO 2004-5**

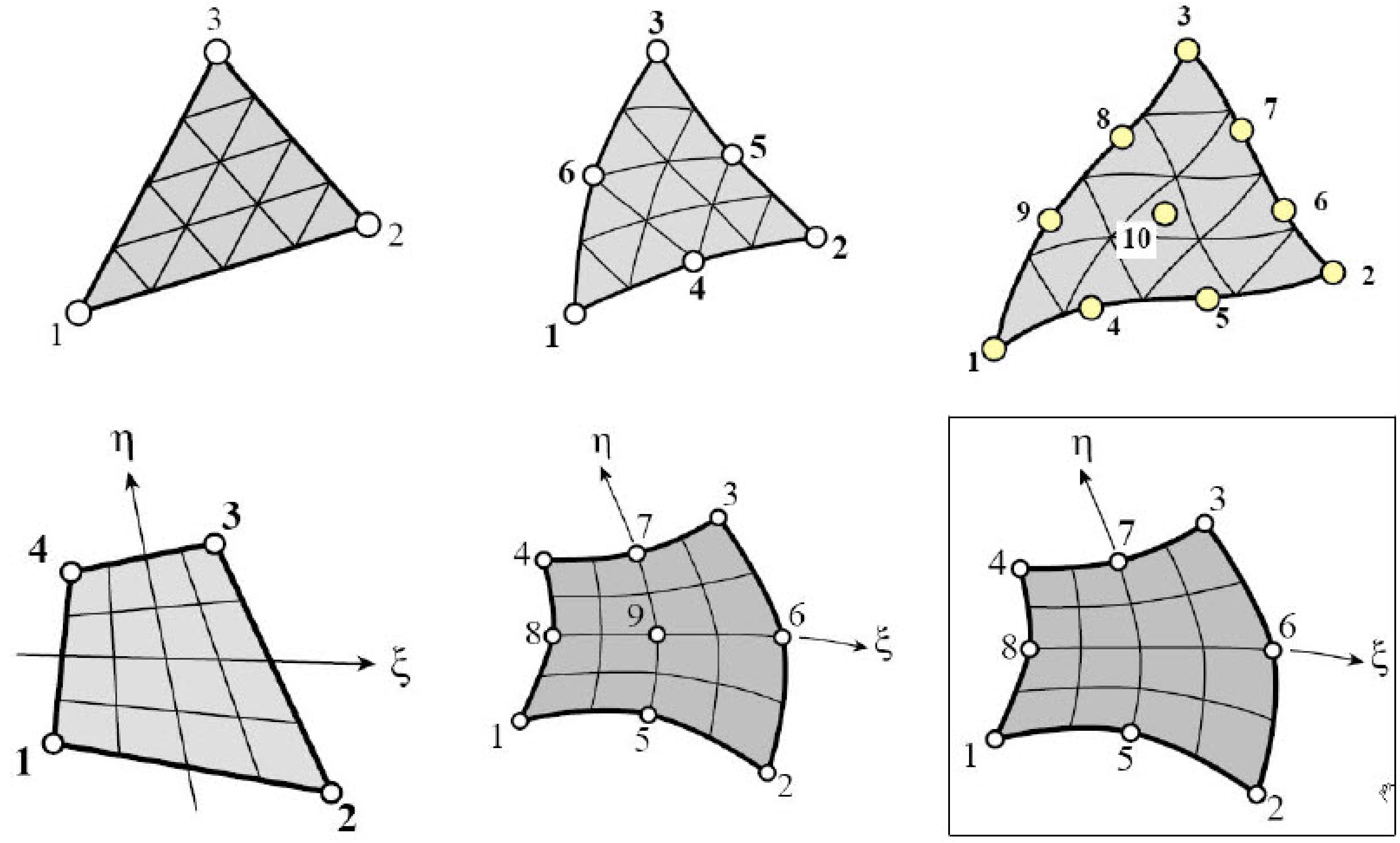
COORDENADAS NODALES

$$= \{\{x_1, y_1\}, \{x_2, y_2\}, \ \ldots \ \{x_N, y_N\}\}$$

TIPO DE ELEMENTO

$$= \{\{\mathtt{etyp}^{(1)}\}, \ \{\mathtt{etyp}^{(2)}\}, \ \ldots \ \{\mathtt{etyp}^{(N_e)}\}\}$$

Identifier	Plane Stress Model
"Trig3"	3-node linear triangle
"Trig6"	6-node quadratic triangle
"Trig10"	10-node cubic triangle
"Quad4"	4-node blinear quad
"Quad9"	9-node biquadratic quad



CONECTIVIDAD ELEMENTOS

$$= \{\{\mathtt{eNL}^{(1)}\}, \ \{\mathtt{eNL}^{(2)}\}, \ \ldots \ \{\mathtt{enL}^{(N_e)}\}\}$$

Element boundaries must be traversed counterclockwise but you can start at any corner. Numbering elements with midnodes requires more care: first list corners counterclockwise, followed by midpoints (first midpoint is the one that follows first corner when going counterclockwise). When elements have an interior node, as in the 9-node biquadratic quadrilateral, that node goes last.

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001 **DEFINICION DEL MODELO (CONT.)** CURSO 2004-5

PROPIEDADES DEL MATERIAL

$$= \{\{E^{(1)}, \nu^{(1)}\}, \{E^{(2)}, \nu^{(2)}\}, \ldots \{E^{(N_e)}, \nu^{(N_e)}\}\}$$

In the common case in which all elements have the same E and ν, this list can be easily generated by a `Table` instruction.

```
ElemMaterial=Table[{Em,nu},{numele}]
             =Length[EleemeNodeLists]
```

PROPIEDADES GEOMETRICAS

$$= \{\{h(1)\}, \{h^{(2)}\}, \ldots \{h^{(N_e)}\}\}$$

INDICADORES DE LIBERTAD – DISPLAZAMIENTO/FUERZA

`FreedomTags` labels each node degree of freedom as to whether the load or the displacement is specified. The configuration of this list is similar to that of `NodeCoordinates`:

$$=\{\{tag_{x1}, tag_{y1}\},\{tag_{x2}, tag_{y2}\}, \ldots \{tag_{xN}, tag_{yN}\}\}$$

The tag value is 0 if the force is specified and 1 if the displacement is specified. When there are a lot of nodes, the quickest way to specify this list is to start from all zeros, and then insert the boundary conditions appropriately.

VALORES DE LIBERTAD – DISPLAZAMIENTO/FUERZA

`FreedomValues` has the same node by node configuration as `FreedomTags`. It lists the specified values of the applied node force component if the corresponding tag is zero, and of the prescribed displacement component if the tag is one. Typically most of the list entries are zero.

OPCIONES DE PROCESADO

```
={True};
```

This specifies floating point numerical computations.

VISUALIZACION DE LA DISCRETIZACION (MALLA)

```
=6/5;
Plot2DElementsAndNodes[NodeCoordinates,ElemNodeLists,aspect,
  "Plate with circular hole - 4-node quad model",True,True];
```

Here aspect is the plot frame aspect ratio (y dimension over x dimension), and the last two `True` argument values specify that node labels and element labels, respectively, be shown.

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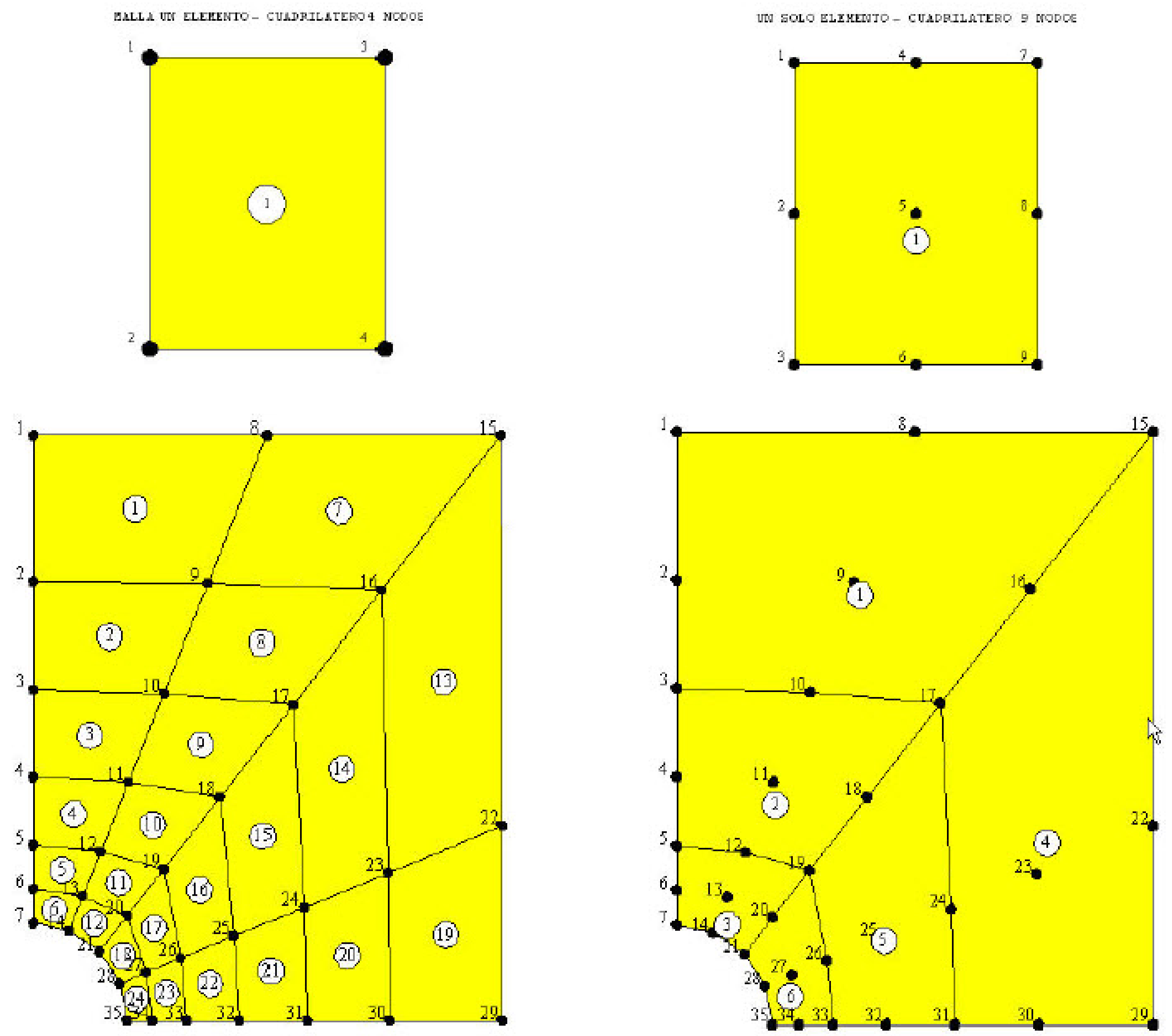
001 VISUALIZACION MALLAS EJEMPLOS CURSO 2004-5



001 PROCESADO DEL MODELO CURSO 2004-5

{ ., ., ;}= ...n[NodeCoordinates,ElemTypes,
ElemNodeLists,ElemMaterial,ElemFabrication,
ProcessingOptions,FreedomTags,FreedomValues];

```
MembraneSolution[nodcoor_,eletyp_,elenod_,elemat_,
  elefab_,eleopt_,doftag_,dofval_]:= Module[{K,Kmod,u,f,sig,j,n,ns,
  supdof,supval,numnod=Length[nodcoor],numele=Length[elenod]},
  u=f=sig={};
  K=MembraneMasterStiffness[nodcoor,
          eletyp,elenod,elemat,elefab,eleopt];  K=N[K];
  ns=0; Do [Do [If[doftag[[n,j]]>0,ns++],{j,1,2}],{n,1,numnod}];
  supdof=supval=Table[0,{ns}];
  k=0; Do [Do [If[doftag[[n,j]]>0,k++;supdof[[k]]=2*(n-1)+j;
         supval[[k]]=dofval[[n,j]]],{j,1,2}],{n,1,numnod}];
  f=ModifiedNodeForces[supdof,supval,K,Flatten[dofval]];
  Kmod=ModifiedMasterStiffness[supdof,K];
  u=Simplify[Inverse[Kmod].f]; u=Chop[u];
  f=Simplify[K.u]; f=Chop[f];
  sig=MembraneNodalStresses[nodcoor,
        eletyp,elenod,elemat,elefab,eleopt,u];
  sig=Chop[sig];
  Return[{u,f,sig}];
];
```

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001 PROCESADO DEL MODELO (CONT.) CURSO 2004-5

1 - ENSAMBLADO DE LA MATRIZ RIGIDEZ GLOBAL

The function begins by assembling the free-free master stiffness matrix K by calling the module `MembraneMasterStiffness`. this module is listed in Figure 27.5. As a study of its code reveals, it can handle the five element types described in the previous section. The modules that compute the element stiffness matrices have been studied in previous Chapters and need not be listed here.

```
                              [nodcoor_,eletyp_,elenod_,
  elemat_,elefab_,eleopt_]:=
  Module[{numele=Length[elenod],numnod=Length[nodcoor],numer,
   ne,eNL,eftab,neldof,i,n,Em,v,Emat,th,ncoor,Ke,K},
  K=Table[0,{2*numnod},{2*numnod}];  numer=eleopt[[1]];
  For [ne=1,ne<=numele,ne++,
       {type}=eletyp[[ne]];
       If [type!="Trig3"&&type!="Trig6"&&type!="Trig10"&&
           type!="Quad4"&&type!="Quad9",
           Print["Illegal element type,ne=",ne]; Return[Null]];
       eNL=elenod[[ne]]; n=Length[eNL];
       eftab=Flatten[Table[{2*eNL[[i]]-1,2*eNL[[i]]},{i,n}]];
       ncoor=Table[nodcoor[[eNL[[i]]]],{i,n}];
       {Em,v}=elemat[[ne]];
       Emat=Em/(1-v^2)*{{1,v,0},{v,1,0},{0,0,(1-v)/2}};
       {th}=elefab[[ne]];
       If [type=="Trig3", Ke=Trig3IsoPMembraneStiffness[ncoor,
          {Emat,0,0},{th},{numer}] ];
       If [type=="Quad4", Ke=Quad4IsoPMembraneStiffness[ncoor,
          {Emat,0,0},{th},{numer,2}] ];
       If [type=="Trig6", Ke=Trig6IsoPMembraneStiffness[ncoor,
          {Emat,0,0},{th},{numer,3}] ];
       If [type=="Quad9", Ke=Quad9IsoPMembraneStiffness[ncoor,
          {Emat,0,0},{th},{numer,3}] ];
       If [type=="Trig10",Ke=Trig10IsoPMembraneStiffness[ncoor,
          {Emat,0,0},{th},{numer,3}] ];
       neldof=Length[Ke];
       For [i=1,i<=neldof,i++,ii=eftab[[i]];
         For [j=i,j<=neldof,j++,jj=eftab[[j]];
             K[[jj,ii]]=K[[ii,jj]]+=Ke[[i,j]]
         ];
       ];
     ];
  Return[K];
];
```

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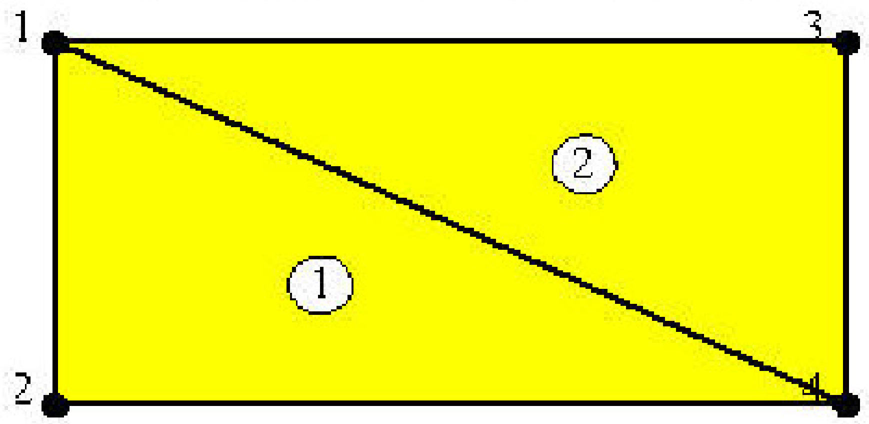
001 **PROCESADO DEL MODELO (CONT.)** CURSO 2004-5

EJEMPLO ENSAMBLADO DE LAS MATRICES DE DOS TRAINGULOS

MALLA DOS ELEMENTOS - TRAINGULOS 3 NODOS



Out[37]//MatrixForm=

	1	2	3	4	7	8
1	$\frac{24}{5}$	0	$-\frac{24}{5}$	$-\frac{12}{5}$	0	$\frac{12}{5}$
2	0	$\frac{64}{5}$	$-\frac{8}{5}$	$-\frac{64}{5}$	$\frac{8}{5}$	0
3	$-\frac{24}{5}$	$-\frac{8}{5}$	8	4	$-\frac{16}{5}$	$-\frac{12}{5}$
4	$-\frac{12}{5}$	$-\frac{64}{5}$	4	14	$-\frac{8}{5}$	$-\frac{6}{5}$
7	0	$\frac{8}{5}$	$-\frac{16}{5}$	$-\frac{8}{5}$	$\frac{16}{5}$	0
8	$\frac{12}{5}$	0	$-\frac{12}{5}$	$-\frac{6}{5}$	0	$\frac{6}{5}$

Out[36]//MatrixForm=

	1	2	7	8	5	6
1	2	0	0	2	-2	-2
2	0	$\frac{1}{2}$	1	0	-1	$-\frac{1}{2}$
7	0	1	2	0	-2	-1
8	2	0	0	8	-2	-8
5	-2	-1	-2	-2	4	3
6	-2	$-\frac{1}{2}$	-1	-8	3	$\frac{17}{2}$

	1	2	3	4	5	6	7	8
1	$\frac{34}{5}$	0	$-\frac{24}{5}$	$-\frac{12}{5}$	-2	-2	0	$\frac{22}{5}$
2	0	$\frac{133}{10}$	$-\frac{8}{5}$	$-\frac{64}{5}$	-1	$-\frac{1}{2}$		0
3	$-\frac{24}{5}$	$-\frac{8}{5}$	8	4	0	0	$-\frac{16}{5}$	$-\frac{12}{5}$
4	$-\frac{12}{5}$	$-\frac{64}{5}$	4	14	0	0	$-\frac{8}{5}$	$-\frac{6}{5}$
5	-2	-1	0	0	4	3	-2	-2
6	-2	$-\frac{1}{2}$	0	0	3	$\frac{17}{2}$	-1	-8
7	0	$\frac{13}{5}$	$-\frac{16}{5}$	$-\frac{8}{5}$	-2	-1		0
8	$\frac{22}{5}$	0	$-\frac{12}{5}$	$-\frac{6}{5}$	-2	-8	0	$\frac{46}{5}$

MATRIZ RIGIDEZ GLOBAL

APELLIDOS, NOMBRE: ____E-MAIL(UPV):___________________

001 **PROCESADO DEL MODELO (CONT.)** **CURSO 2004-5**

2 - APLICACIÓN CONDICIONES DE CONTORNO Y CARGAS

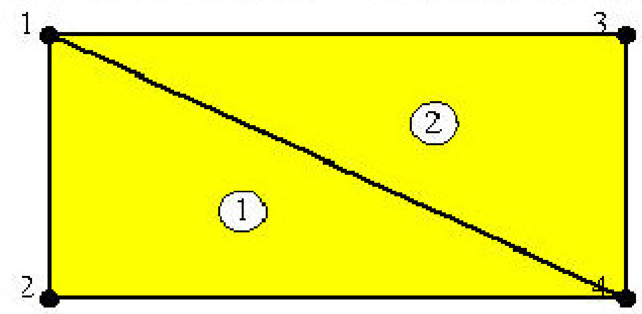
The displacement boundary conditions are applied by modules `ModifiedmasterStiffness` and `ModifiedNodeForces`, which return the modified stiffness matrix $\hat{\mathbf{K}}$ and the modified force vector $\hat{\mathbf{f}}$ in `Khat` and `fnat`, respectively.

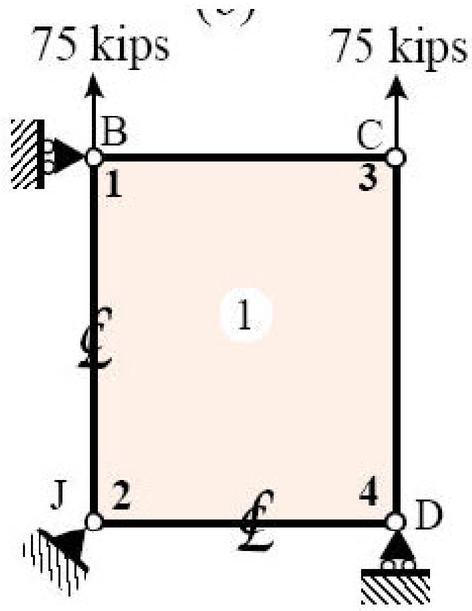
```
ModifiedMasterStiffness[pdof_, Km_] := Module[{i, j, k, K}, K = Km;
  For[k = 1, k ≤ Length[pdof], k++, i = pdof[[k]];
   For[j = 1, j ≤ Length[K], j++, K[[i, j]] = K[[j, i]] = 0]; K[[i, i]] = 1;]; Return[K];];
```

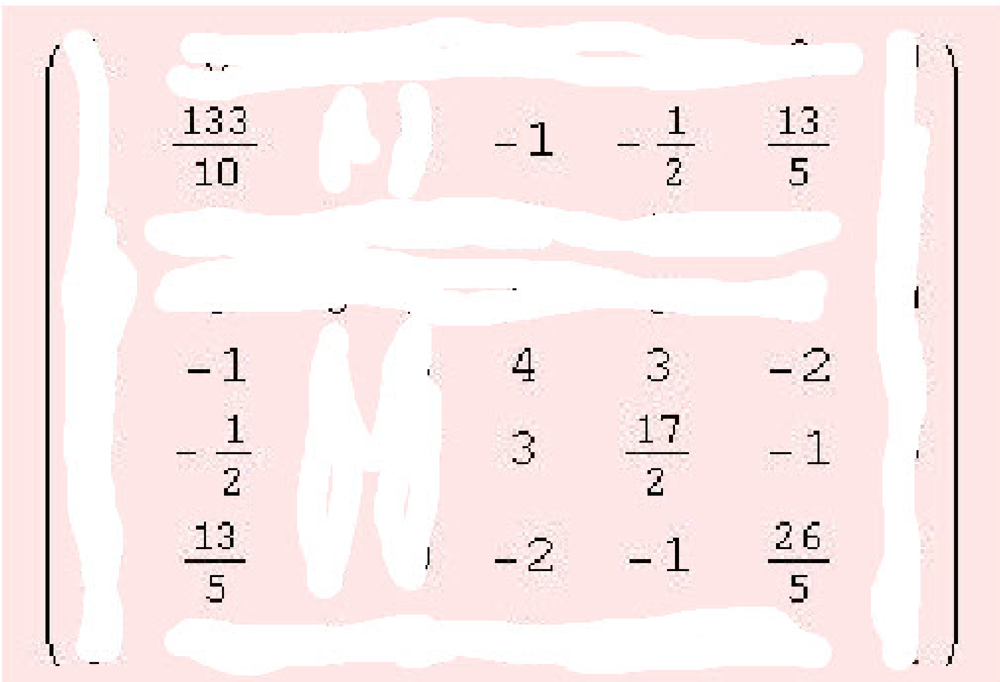
```
ModifiedNodeForces[pdof_, pdofv_, Km_, nfv_] :=
  Module[{i, j, k, l, d, kk = Length[pdof], n = Length[Km], fixed, rhs}, rhs = nfv;
   d = pdofv; fixed = Table[False, {n}];
   Do[i = pdof[[k]]; fixed[[i]] = True, {k, 1, kk}];
   For[k = 1, k ≤ kk, k++, i = pdof[[k]];
    For[j = 1, j ≤ n, j++, If[fixed[[j]], Continue[]];
     rhs[[j]] = rhs[[j]] - Km[[i, j]] * d[[k]];]; rhs[[i]] = d[[k]];];
   Return[rhs];];
```

EJEMPLO DOS TRIANNGULOS

MALLA DOS ELEMENTOS - TRAINGULOS 3 NODOS







MATRIZ RIGIDEZ GLOBAL MODIFICADA CON CONDICIONES CONTORNO

APELLIDOS, NOMBRE: ____E-MAIL(UPV):___________________

001 **PROCESADO DEL MODELO (CONT.)** CURSO 2004-5

3 - OBTENCION DESPLAZAMIENTOS NODALES Y FUERZAS DE REACCION

The unknown node displacements u are then obtained through the built in `LinearSolve` function, as `u=LinearSolve[Khat,fhat]`. This solution strategy is of course restricted to very small systems, but it has the advantages of simplicity.

The function returns arrays `u`, `f` and `p`, which are lists of length 12, 12 and 13, respectively. Array `u` contains the computed node displacements ordered $u_{x1} < u_{y1}, u_{x2}, \ldots u_{y8}$. Array `f` contains the node forces recovered from $\mathbf{f} = \mathbf{K}\mathbf{u}$; this includes the reactions f_{x1}, f_{y1} and f_{y8}.

```
MembraneSolution[nodcoor_, eletyp_, elenod_, elemat_, elefab_, eleopt_,
   doftag_, dofval_] :=
  Module[{K, Kmod, u, f, sig, j, n, ns, supdof, supval, numnod = Length[nodcoor],
    numele = Length[elenod]}, u = f = sig = {};
   K = MembraneMasterStiffness[nodcoor, eletyp, elenod, elemat, elefab, eleopt];
   Print["Master Stiff K"]; Print[MatrixForm[K]]; K = N[K];
 Print["eigs of K=", Chop[Eigenvalues[K]]]; ns = 0;
   Do[Do[If[doftag[[n, j]] > 0, ns++], {j, 1, 2}], {n, 1, numnod}];
   Print["doftag=", doftag]; Print["ns=", ns]; supdof = supval = Table[0, {ns}];
k = 0; Do[Do[If[doftag[[n, j]] > 0, k++; supdof[[k]] = 2 * (n - 1) + j;
      supval[[k]] = dofval[[n, j]]]], {j, 1, 2}], {n, 1, numnod}]; Print["supdof=", supdof];
   f = ModifiedNodeForces[supdof, supval, K, Flatten[dofval]];
   Print["[illegible]"]; Print[MatrixForm[f]];
Kmod = ModifiedMasterStiffness[supdof, K]; Print["Matriz Rigidez Modificada Kmod"];
   Print[MatrixForm[Kmod]];
 Print["eigs of Kmod=", Eigenvalues[Kmod]]; u = Simplify[Inverse[Kmod].f];
   u = Chop[u]; Print["Desplazamientos Nodales u=", MatrixForm[u]];
   f = Simplify[K.u]; f = Chop[f]; Print["[illegible]=", MatrixForm[f]];
   sig = MembraneNodalStresses[nodcoor, eletyp, elenod, elemat, elefab, eleopt, u];
sig = Chop[sig];
Return[{u, f, sig}];
];
```

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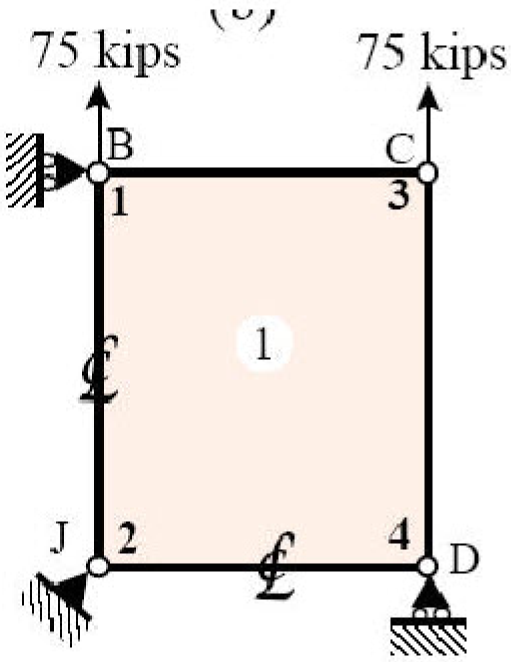
001 **PROCESADO DEL MODELO (CONT.)** **CURSO 2004-5**

EJEMPLO PLACA CON UN ELEMENTO CUADRILATERO 4 NODOS




```
{u, f, sig} = MembraneSolution[NodeCoordinates, ElemTypes, ElemNodeLists,
  ElemMaterial, ElemFabrication, ElemOptions, FreedomTags, FreedomValues];
```

16133.3	-5000.	3066.67	-1000.	-11133.3	1000.	-8066.67	5000.
-5000.	13688.9	1000.	-6488.89	-1000.	-355.556	5000.	-6844.44
3066.67	1000.	16133.3	5000.	-8066.67	-5000.	-11133.3	-1000.
-1000.	-6488.89	5000.	13688.9	-5000.	-6844.44	1000.	-355.556
-11133.3	-1000.	-8066.67	-5000.	16133.3	5000.	3066.67	1000.
1000.	-355.556	-5000.	-6844.44	5000.	13688.9	-1000.	-6488.89
-8066.67	5000.	-11133.3	1000.	3066.67	-1000.	16133.3	-5000.
5000.	-6844.44	-1000.	-355.556	1000.	-6488.89	-5000.	13688.9

={42453.9, 24400., 22612.8, 16133.3, 13688.9, }

MATRIZ RIGIDEZ GLOBAL Y VALORES PROPIOS

doftag={{1, 0}, {1, 1}, {0, 0}, {0, 1}}

ns=4

supdof={1, 3, 4, 8}

GRADOS DE LIBERTAD RESTRINGIDOS – CONDICIONES CONTORNO

Vector Fuerzas f

$$\begin{pmatrix} 0 \\ 75 \\ 0 \\ 0 \\ 0 \\ 75 \\ 0 \\ 0 \end{pmatrix}$$

VECTOR FUERZAS NODALES APLICADAS

APELLIDOS, NOMBRE: ____E-MAIL(UPV):___________________

001 **PROCESADO DEL MODELO (CONT.)** **CURSO 2004-5**

Matriz Rigidez ' Kmod

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 13688.9 & 0 & 0 & -1000. & -355.556 & 5000. & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1000. & 0 & 0 & 16133.3 & 5000. & 3066.67 & 0 \\ 0 & -355.556 & 0 & 0 & 5000. & 13688.9 & -1000. & 0 \\ 0 & 5000. & 0 & 0 & 3066.67 & -1000. & 16133.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

eigs of Kmod={21227., 19575.4, 11306.4, 7535.67, , }

MATRIZ RIGIDEZ GLOBAL MODIFICADAS CON CONDICIONES CONTORNO

$$\mathbf{Ku} = \mathbf{f}$$

PROBLEMA A RESOLVER

$$u = \begin{pmatrix} 0 \\ 0.006 \\ 0 \\ 0 \\ -0.00125 \\ 0.006 \\ -0.00125 \\ 0 \end{pmatrix}$$

SOLUCION DESPLAZAMIENTOS NODALES

$$u_y = qy/E, u_x = -\nu qx/E$$

SOLUCION EXACTA

$$f = \begin{pmatrix} 0 \\ 75. \\ 0 \\ -75. \\ 0 \\ 75. \\ 0 \\ -75. \end{pmatrix}$$

VECTOR FUERZAS APLICADAS Y DE REACCION

APELLIDOS, NOMBRE: ____E-MAIL(UPV):__________________

001	POSPROCEADO DEL MODELO (CONT.)	CURSO 2004-5
	1 - OBTENCION TENSIONES NODALES	

Finally, array sig contains the nodal stresses σ_{xx}, σ_{yy}, σ_{xy} at each node, recovered from the displacement solution. This computation is driven by module MembraneNodeStresses, which is

```
[nodcoor_,eletyp_,elenod_,
  elemat_,elefab_,eleopt_]:=
  Module[{numele=Length[elenod],numnod=Length[nodcoor],numer,
   ne,eNL,eftab,neldof,i,n,Em,v,Emat,th,ncoor,Ke,K},
  K=Table[0,{2*numnod},{2*numnod}];  numer=eleopt[[1]];
  For [ne=1,ne<=numele,ne++,
       {type}=eletyp[[ne]];
       If [type!="Trig3"&&type!="Trig6"&&type!="Trig10"&&
           type!="Quad4"&&type!="Quad9",
           Print["Illegal element type,ne=",ne]; Return[Null]];
       eNL=elenod[[ne]]; n=Length[eNL];
       eftab=Flatten[Table[{2*eNL[[i]]-1,2*eNL[[i]]},{i,n}]];
       ncoor=Table[nodcoor[[eNL[[i]]]],{i,n}];
       {Em,v}=elemat[[ne]];
       Emat=Em/(1-v^2)*{{1,v,0},{v,1,0},{0,0,(1-v)/2}};
       {th}=elefab[[ne]];
       If [type=="Trig3", Ke=Trig3IsoPMembraneStiffness[ncoor,
          {Emat,0,0},{th},{numer}] ];
       If [type=="Quad4", Ke=Quad4IsoPMembraneStiffness[ncoor,
          {Emat,0,0},{th},{numer,2}] ];
       If [type=="Trig6", Ke=Trig6IsoPMembraneStiffness[ncoor,
          {Emat,0,0},{th},{numer,3}] ];
       If [type=="Quad9", Ke=Quad9IsoPMembraneStiffness[ncoor,
          {Emat,0,0},{th},{numer,3}] ];
       If [type=="Trig10",Ke=Trig10IsoPMembraneStiffness[ncoor,
          {Emat,0,0},{th},{numer,3}] ];
       neldof=Length[Ke];
       For [i=1,i<=neldof,i++,ii=eftab[[i]];
         For [j=i,j<=neldof,j++,jj=eftab[[j]];
             K[[jj,ii]]=K[[ii,jj]]+=Ke[[i,j]]
         ];
       ];
     ];
  Return[K];
];
```

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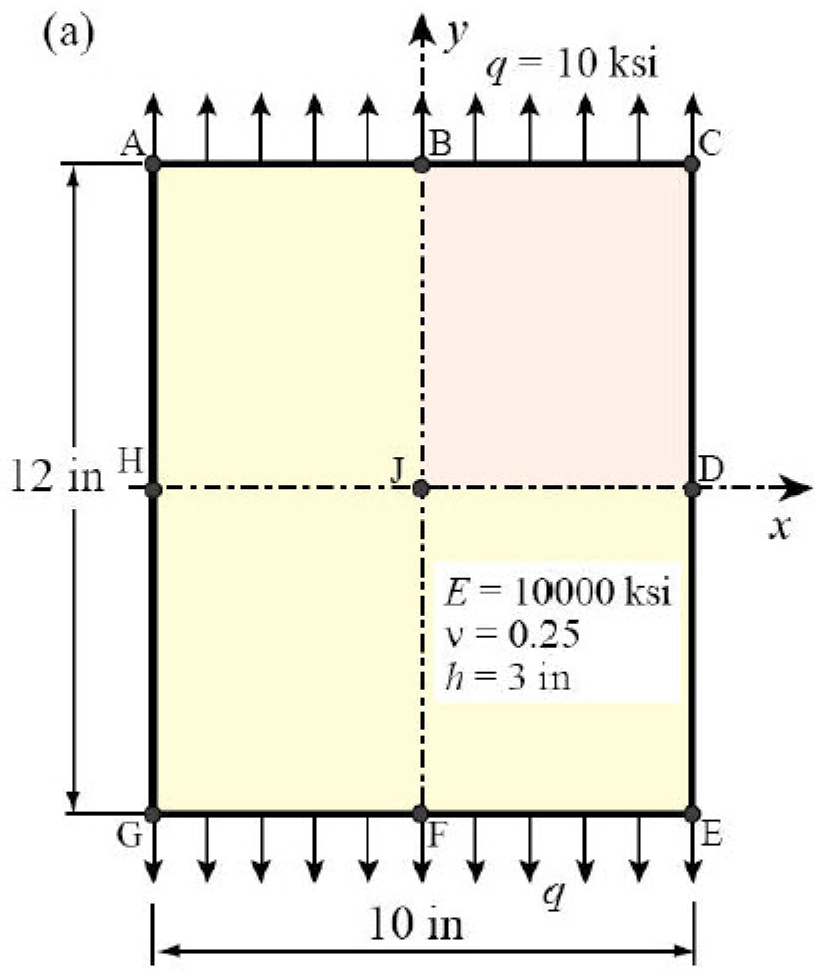
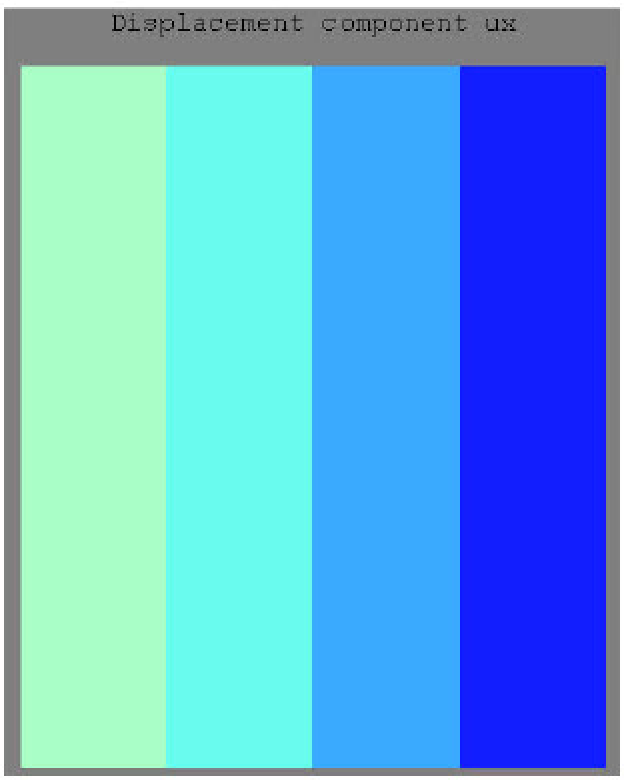
001 **POSPROCEADO DEL MODELO (CONT.)** **CURSO 2004-5**

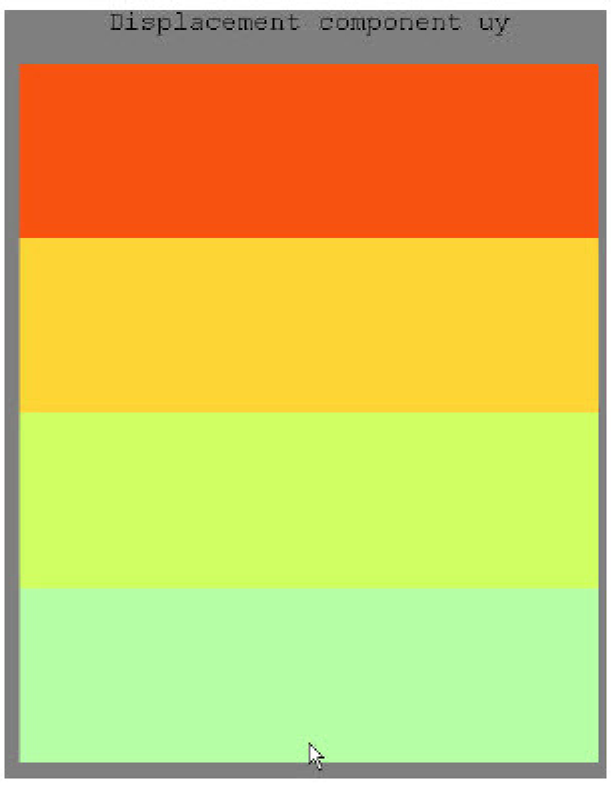
2 VISUALIZACION DESPLAZAMIENTOS

```
ux = uy = Table[0, {numnod}];
Do[ux[[n]] = u[[2*n-1]]; uy[[n]] = u[[2*n]], {n, 1, numnod}];
uxmax = uymax = 0;
Do[uxmax = Max[Abs[ux[[n]]], uxmax]; uymax = Max[Abs[uy[[n]]], uymax], {n, 1, numnod}];
[illegible][NodeCoordinates, ElemNodeLists, ux, uxmax,
  Nsub, aspect, "Displacement component ux"];
ContourPlotNodeFuncOver2DMesh[NodeCoordinates, ElemNodeLists, uy, uymax,
  Nsub, aspect, "Displacement component uy"];
```

EJEMPLO PLACA CON UN ELEMENTO CUADRILATERO 4 NODOS







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001 **POSPROCEADO DEL MODELO (CONT.)** **CURSO 2004-5**

2 VISUALIZACION TENSIONES

```
sxx = syy = sxy = Table[0, {numnod}];
Do[{sxx[[n]], syy[[n]], sxy[[n]]} = sig[[n]], {n, 1, numnod}];
sxxmax = syymax = sxymax = 0;
Do[sxxmax = Max[Abs[sxx[[n]]], sxxmax];
  syymax = Max[Abs[syy[[n]]], syymax];
  sxymax = Max[Abs[sxy[[n]]], sxymax], {n, 1, numnod}];
ContourPlotNodeFuncOver2DMesh[NodeCoordinates, ElemNodeLists, sxx, sxxmax,
  Nsub, aspect, "Nodal stress sig-xx"];
ContourPlotNodeFuncOver2DMesh[NodeCoordinates, ElemNodeLists, syy, syymax,
  Nsub, aspect, "Nodal stress sig-yy"];
ContourPlotNodeFuncOver2DMesh[NodeCoordinates, ElemNodeLists, sxy, sxymax,
  Nsub, aspect, "Nodal stress sig-xy"];
```

EJEMPLO PLACA CON UN ELEMENTO CUADRILATERO 4 NODOS

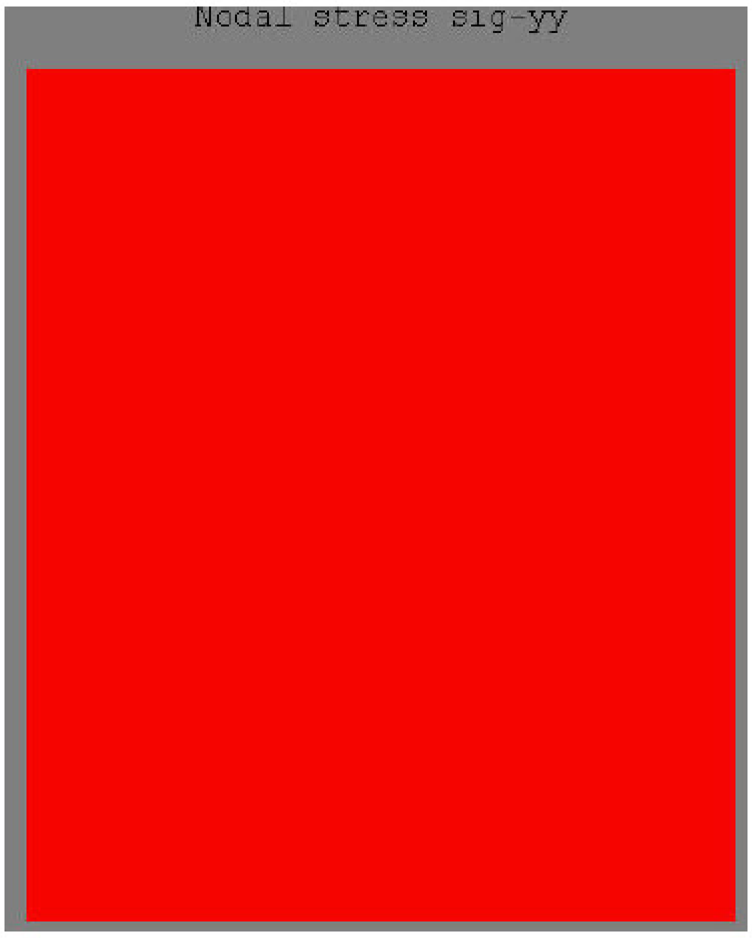
```
sig // MatrixForm
```

$$\begin{pmatrix} 0 & 10. & 0 \\ 0 & 10. & 0 \\ 0 & 10. & 0 \\ 0 & 10. & 0 \end{pmatrix}$$

SOLUCION TENSIONES NODALES PROMEDIADAS

$$\sigma_{yy} = q, \sigma_{xx} = \sigma_{xy} = 0$$

SOLUCION EXACTA



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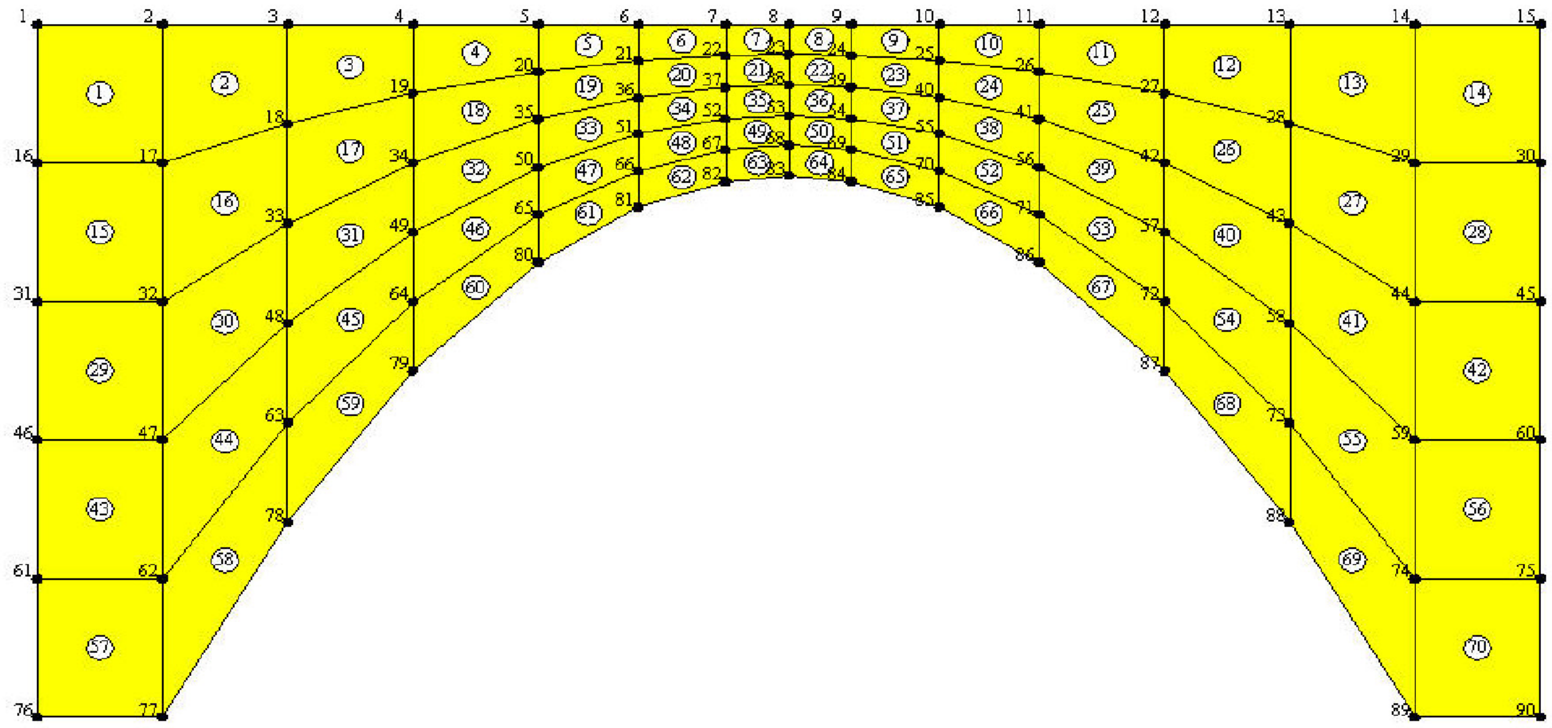
001 EJERCICIO 1 - ANIMACION CURSO 2004-5

Explicar cual es el problema que se plantea y resuelve en el ejemplo “animacion.nb”. Explicar los resultados. Modelarlo con ANSYS, y comparar los resultados.



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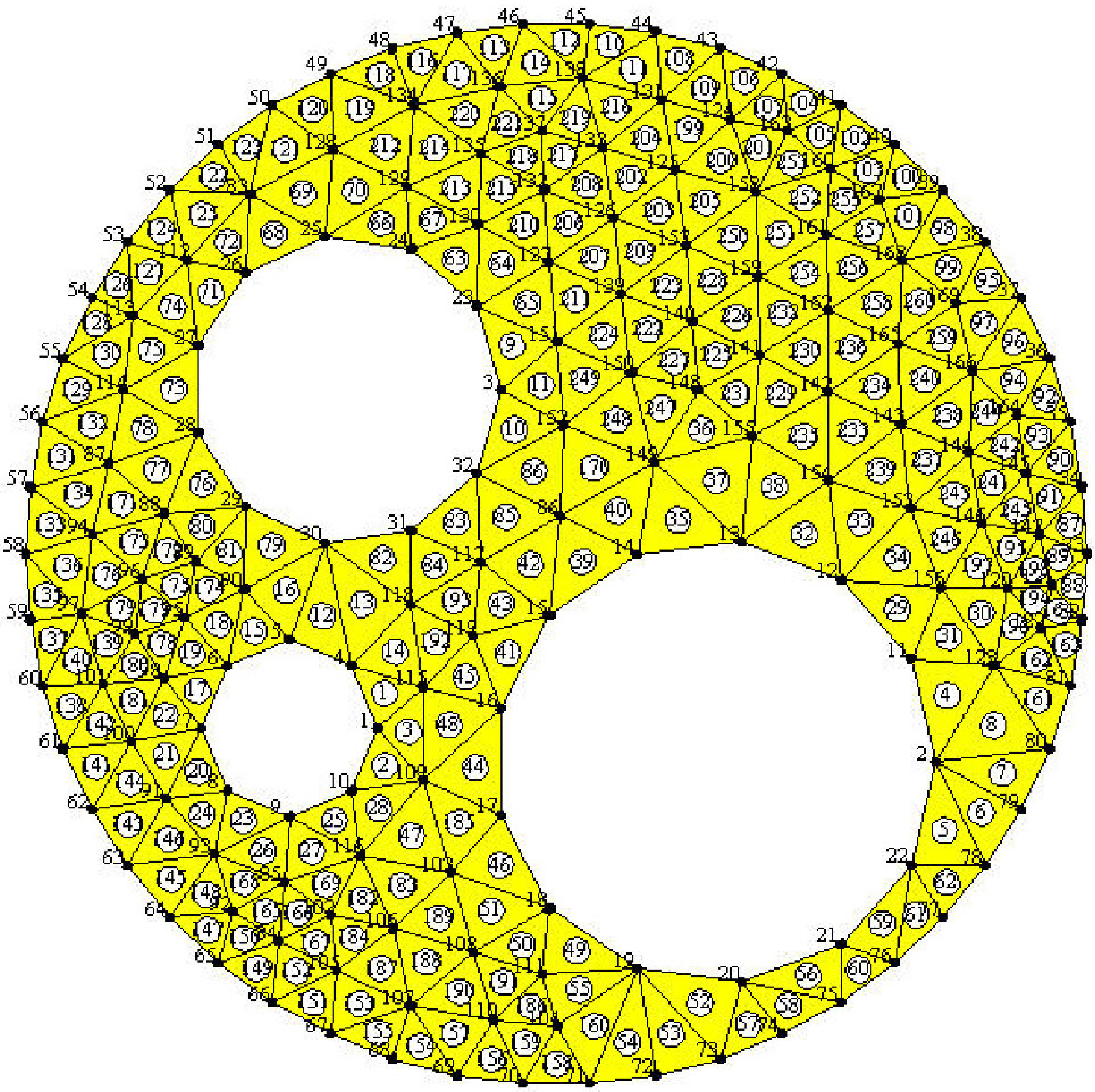
001 EJERCICIO 2 - IMPORTACION MALLA CURSO 2004-5

Explicar cual es el problema que se plantea y resuelve en el ejemplo “importacion.nb”. Explicar los resultados. Modelarlo con ANSYS, y comparar los resultados.



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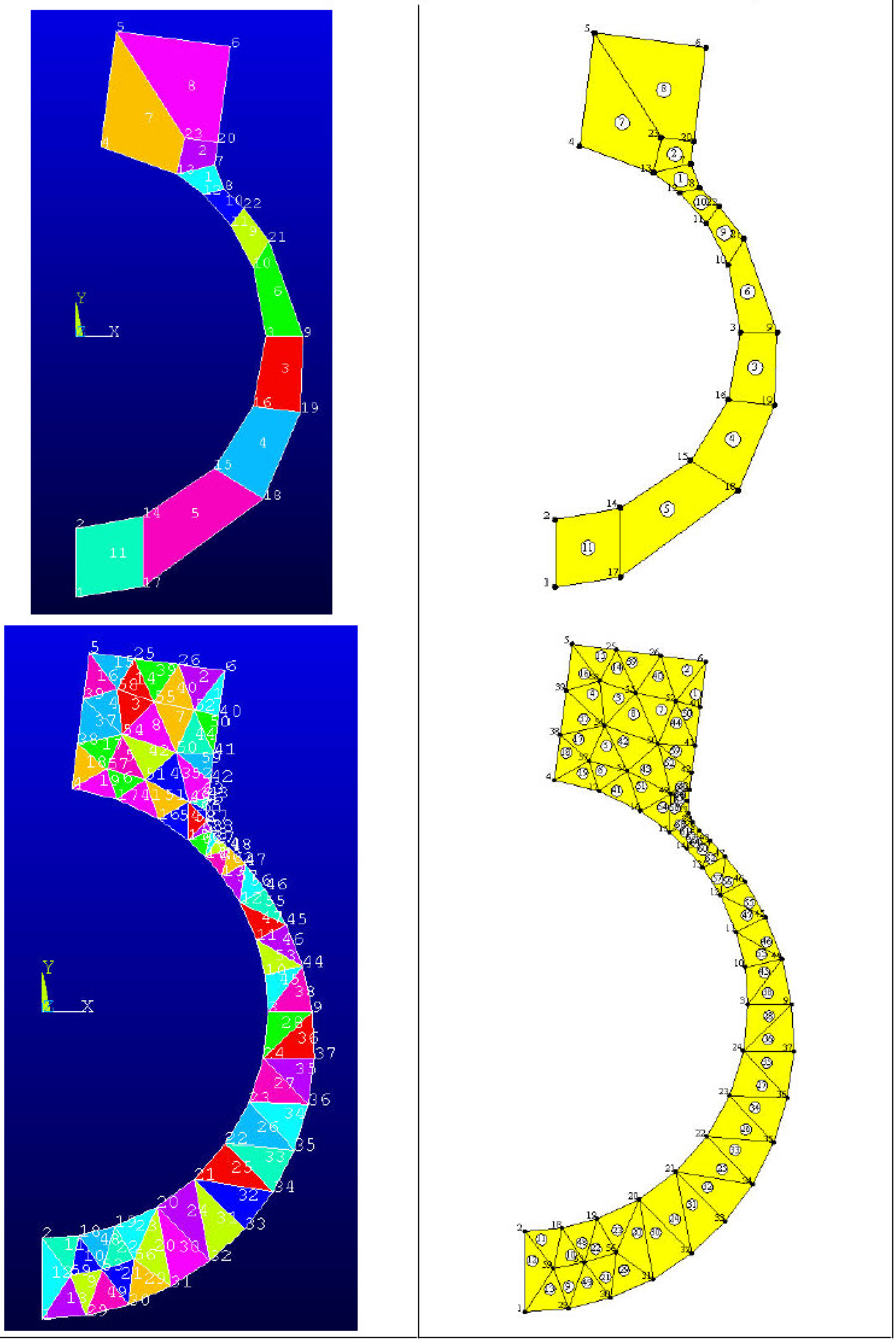
001 EJERCICIO 3 – IMPORTACION MALLA CURSO 2004-5

Utilizando el fichero “importa.nb”, importar varias discretizaciones del ejercicio de la biela bidimensional realizado en la prácticas con ANSYS. Utilizar todos los elementos que sea posible, dentro de los disponibles.



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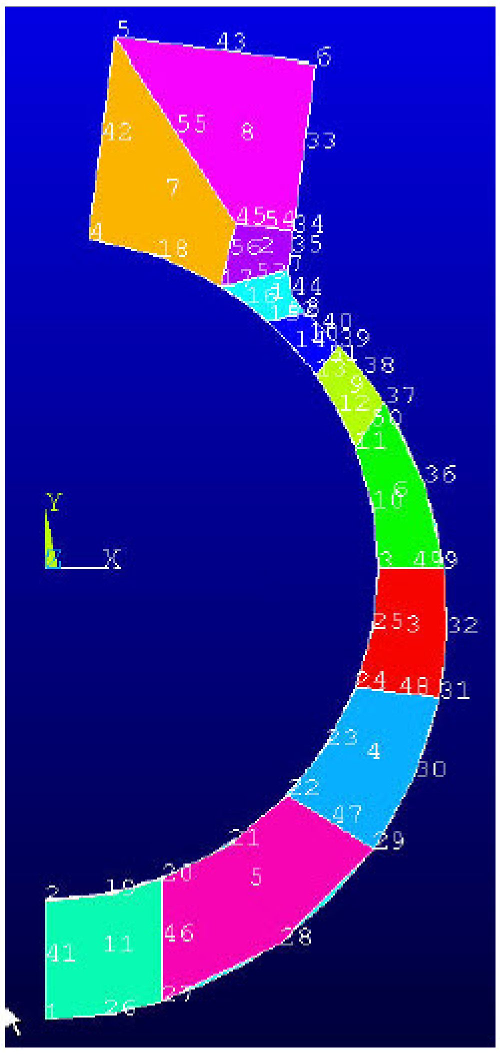
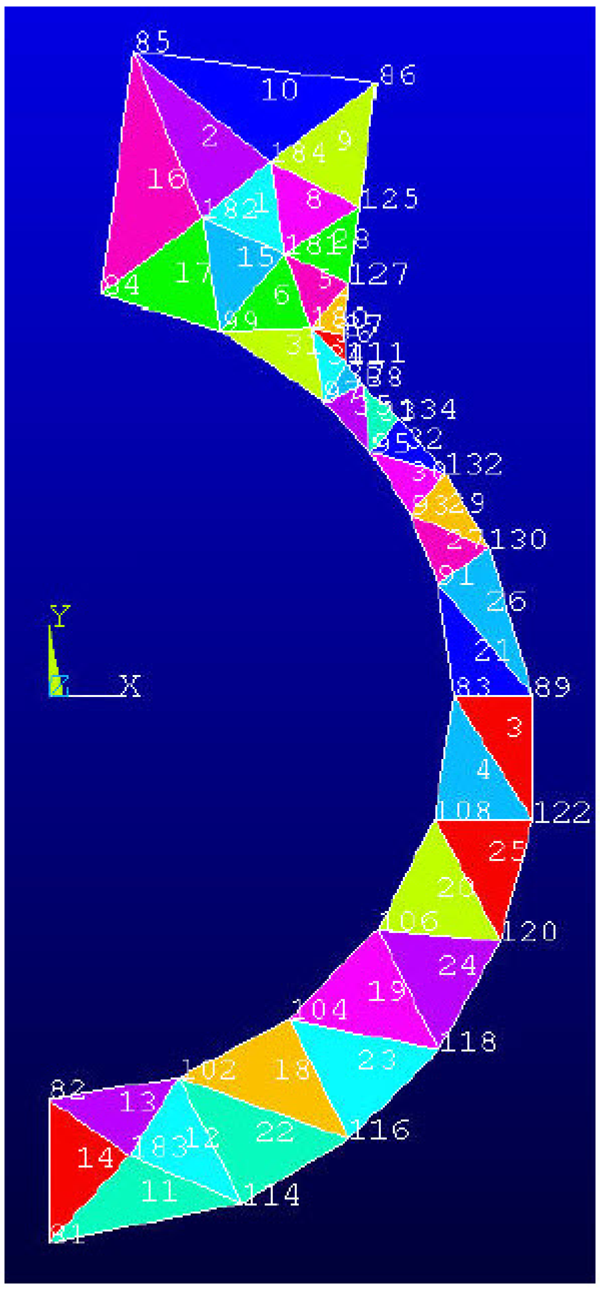
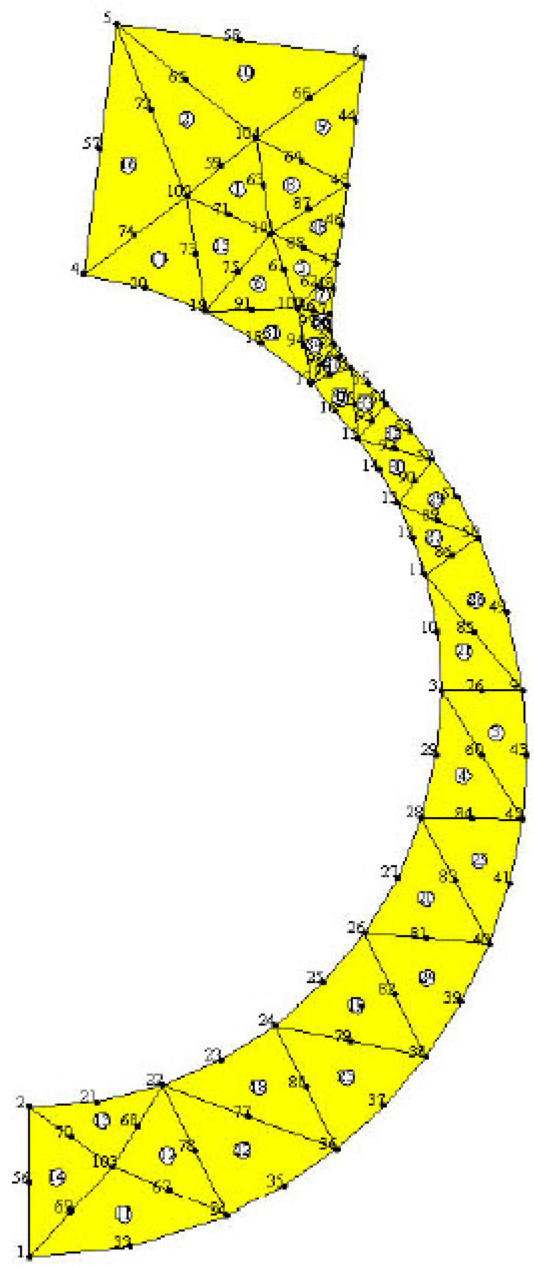
001	EJERCICIO 3 (CONT.)	CURSO 2004-5







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CALCULO DE LAS TENSIONES

001 ¿POR QUE CALCULAR LAS TENSIONES? CURSO 2004-5

001 ¿QUÉ NOMBRE RECIBEN LAS TECNICAS QUE SE UTILIZAN? CURSO 2004-5

These procedures receive the generic name

s in the finite element literature. In the following sections we cover the simplest stress recovery

001 ¿COMO CALCULAR LAS TENSIONES A PARTIR DE LOS DESPLAZAMIENTOS? CURSO 2004-5

SE HA SOLUCIONADO EL PROBLEMA DE LOS DESPLAZAMIENTOS EN LOS NODOS

$$\mathbf{K} \; = \mathbf{f}$$

SE CALCULAN LAS DEFORMACIONES EN CUALQUIER PUNTO DE CUALQUIER ELEMENTO

$$\mathbf{e} =$$

where **B** is the strain-displacement matrix (14.18) assembled with the x and y derivatives of the element shape functions evaluated at the point where we are calculating strains.

Y SE CALCULAN LAS TENSIONES EN CUALQUIER PUNTO DE CUALQUIER ELMENTO COMO

$$\boldsymbol{\sigma} = \;\mathbf{e} = \mathbf{EBu}$$

001 ¿EN QUE PUNTOS SE DEBEN CALCULAR LAS TENSIONES Y QUE SUCEDE CON ELLAS? CURSO 2004-5

NORMALMENTE EN LOS NODOS DE LOS ELEMENTOS

In the applications it is of interest to evaluate and report these stresses at the *element nodal points* located on the corners and possibly midpoints of the element. These are callec

It is important to realize that the stresses computed at the same nodal point from adjacent elements

e, since stresses are not required to be continuous in displacement-assumed finite elements. This suggests some form of stress averaging can be used to improve the stress accuracy, and indeed this is part of the stress recovery technique further discussed in §29.5. The results from this averaging procedure are called

001 ¿COMO CALCULAR LAS TENSIONES EN LOS NODOS? CURSO 2004-5

1. Evaluate directly $\boldsymbol{\sigma}$ at the element node locations by 's of the nodal points as arguments to the shape function modules. These modules return $\mathbf{q}_x$ and $\mathbf{q}_y$ and direct application of (29.2)-(29.4) yields the strains and stresses at the nodes.

2. Evaluate at the s used in the element stiffness integration rule and then to the element node points.

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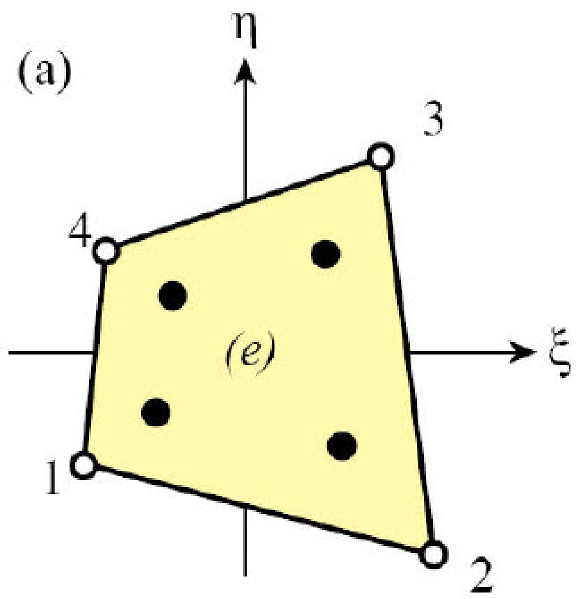
001 **¿CUAL DE LAS DOS TECNICAS ES MAS ES MAS CONVENIENTE Y EN QUE TIPOS DE ELEMENTOS?** **CURSO 2004-5**

Empirical evidence indicates that the second approach generally delivers better stress values for *quadrilateral* elements whose geometry departs substantially from the rectangular shape. This is backed up by “ ” results in finite element approximation theory. For rectangular elements there is no difference.

For isoparametric ... , (identical if the elements are straight sided with midside nodes at midpoints) and so the advantages of the second one are marginal. Both approaches are covered in the sequel.

001 **TECNICA DE EXTRAPOLACION DESDE LOS PUNTOS DE GAUSS** **CURSO 2004-5**

ELEMENTO CUADRILATERO 4 NODOS



Corner node	ξ	η	ξ'	η'	Gauss node	ξ	η	ξ'	η'
1	-1	-1	$-\sqrt{3}$	$-\sqrt{3}$	1’	$-1/\sqrt{3}$	$-1/\sqrt{3}$	-1	-1
2	$+1$	-1	$+\sqrt{3}$	$-\sqrt{3}$	2’	$+1/\sqrt{3}$	$-1/\sqrt{3}$	$+1$	-1
3	$+1$	$+1$	$+\sqrt{3}$	$+\sqrt{3}$	3’	$+1/\sqrt{3}$	$+1/\sqrt{3}$	$+1$	$+1$
4	-1	$+1$	$-\sqrt{3}$	$+\sqrt{3}$	4’	$-1/\sqrt{3}$	$+1/\sqrt{3}$	-1	$+1$

The are ... , which are identified as $1'$, $2'$, $3'$ and $4'$ in Figure 29.1. Point i' is closest to node i so it is seen that Gauss point numbering essentially follows element node numbering in the counterclockwise sense. The natural coordinates of these points are listed in Table 29.1. The stresses are evaluated at these Gauss points by passing these natural coordinates to the shape function subroutine. Then each stress component is “carried” to the corner nodes 1 through 4 through a bilinea based on the computed values at $1'$ through $4'$.

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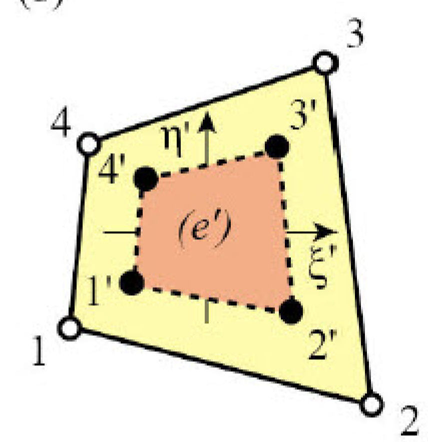
001 **TECNICA DE [illegible] DESDE LOS PUNTOS DE GAUSS** **CURSO 2004-5**

ELEMENTO CUADRILATERO 4 NODOS (CONT.)

(b)



ELEMENTO DE GAUSS

To understand the extrapolation procedure more clearly it is convenient to consider the region bounded by the Gauss points as an "internal element" or "Gauss element". This interpretation is depicted in Figure 29.1(b). The Gauss element, denoted by (e'), is also a four-node quadrilateral. Its quadrilateral (natural) coordinates are denoted by ξ' and η'. These are linked to ξ and η by the simple relations

$$\xi = \xi'/\sqrt{3}, \quad \eta = \eta'/\sqrt{3}, \qquad \xi' = \xi\sqrt{3}, \quad \eta' = \eta\sqrt{3}.$$

Any scalar quantity w whose values w'_i at the Gauss element corners are known can be interpolated through the usual bilinear shape functions now expressed in terms of ξ' and η':

$$w(\xi', \eta') = [\, w'_1 \quad w'_2 \quad w'_3 \quad w'_4 \,] \begin{bmatrix} N_1^{(e')} \\ N_2^{(e')} \\ N_3^{(e')} \\ N_4^{(e')} \end{bmatrix}$$

$$\begin{aligned} N_1^{(e')} &= \tfrac{1}{4}(1 - \xi')(1 - \eta'), \\ N_2^{(e')} &= \tfrac{1}{4}(1 + \xi')(1 - \eta'), \\ N_3^{(e')} &= \tfrac{1}{4}(1 + \xi')(1 + \eta'), \\ N_4^{(e')} &= \tfrac{1}{4}(1 - \xi')(1 + \eta'). \end{aligned}$$

To extrapolate w to corner 1, say, we replace its ξ' and η' coordinates, namely $\xi' = \eta' = -\sqrt{3}$, into the above formula. Doing that for the four corners we obtain

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1 - \frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ -\frac{1}{2} & 1 + \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1 - \frac{1}{2}\sqrt{3} \\ 1 - \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1 + \frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ -\frac{1}{2} & 1 - \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 1 + \frac{1}{2}\sqrt{3} \end{bmatrix} \begin{bmatrix} w'_1 \\ w'_2 \\ w'_3 \\ w'_4 \end{bmatrix}$$

Note that the sum of the coefficients in each row is one, as it should be. For stresses we apply this formula taking w to be each of the three stress components, σ_{xx}, σ_{yy} and τ_{xy}, in turn.

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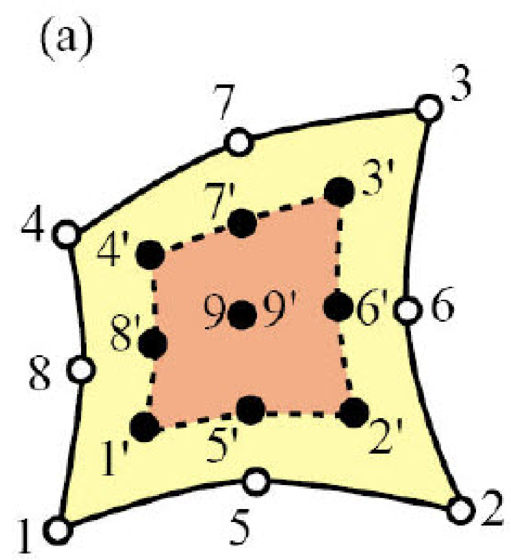
001 **TECNICA DE] DESDE LOS** **CURSO 2004-5**

PUNTOS DE GAUSS

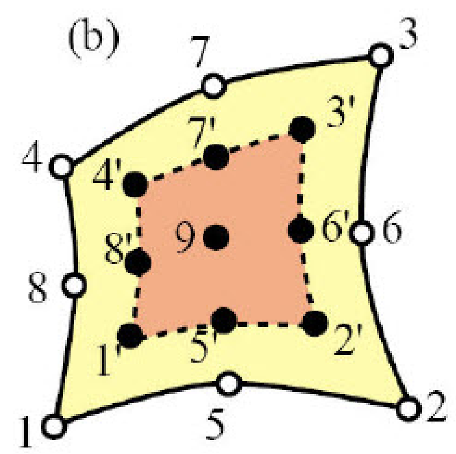
ELEMENTOS DE ORDEN SUPERIOR

For eight-node and nine-node isoparametric quadrilaterals the usual Gauss integration rule is 3×3, and the Gauss elements are nine-noded quadrilaterals that look as in Figure 29.2(a) and (b) above. For six-node triangles the usual quadrature is the 3-point rule with internal sampling points, and the Gauss element is a three-node triangle as shown in Figure 29.2(c).

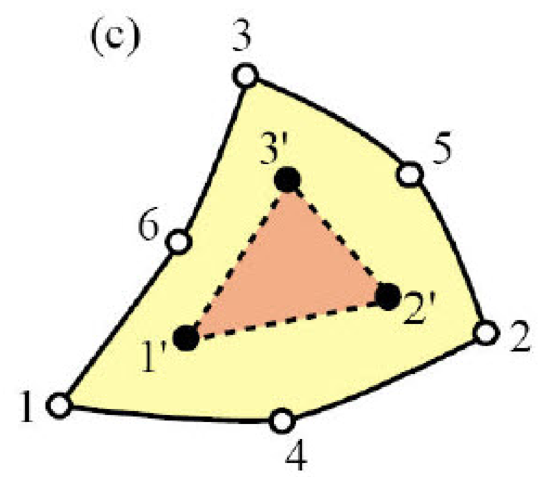
ELEMENTO CUADRILATERO DE 9 NODOS



ELEMENTO CUADRILATERO DE 8 NODOS



ELEMENTO TRIANGULO DE 6 NODOS



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001 ENTRE ELEMENTOS CURSO 2004-5

The stresses computed in element-by-element fashion as discussed above, whether by direct evaluation at the nodes or by extrapolation, will generally exhibit jumps between elements. For printing and plotting purposes it is usually convenient to "smooth out" those jumps by computing . This averaging may be done in two ways:

(I) Unweighted averaging: assign same weight to all elements that meet at a node;

(II) Weighted averaging: the weight assigned to element contributions depends on the stress component and the element geometry and possibly the element type.

Several weighted average schemes have been proposed in the finite element literature, but they do require additional programming.