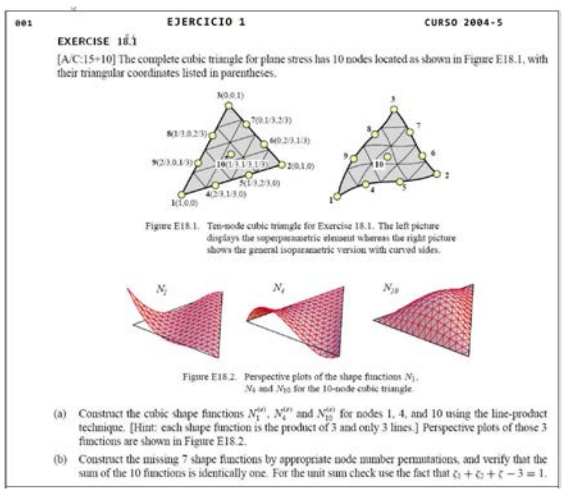
APELLIDOS, NOMBRE: ____E-MAIL(UPV): __________________

Estas ACTIVIDADES DE CLASE deberá realizarse descargando los documentos *NB* disponibles en las páginas web, completandolos adecuadamente, denominandolos de la forma especificada y subiendolos a tu cuenta de entrega personal. En este documento *PDF* habrá que contestar a las *PREGUNTAS* que planteo a lo largo de la grabación en video correspondiente a la clase.

Para familiarizarnos con las ***Funciones de Forma***, su definición, su terminología y su planteamiento; durante las explicaciones en clase habrá que completar este documento PDF.

Estas son imágenes de algunos de los ejercicios considerados en las ACTIVIDADES de esta CLASE:

06-C6-Mathematica-C1



001 EJERCICIO 1 CURSO 2004-5

EXERCISE 18.1

[A/C:15+10] The complete cubic triangle for plane stress has 10 nodes located as shown in Figure E18.1, with their triangular coordinates listed in parentheses.

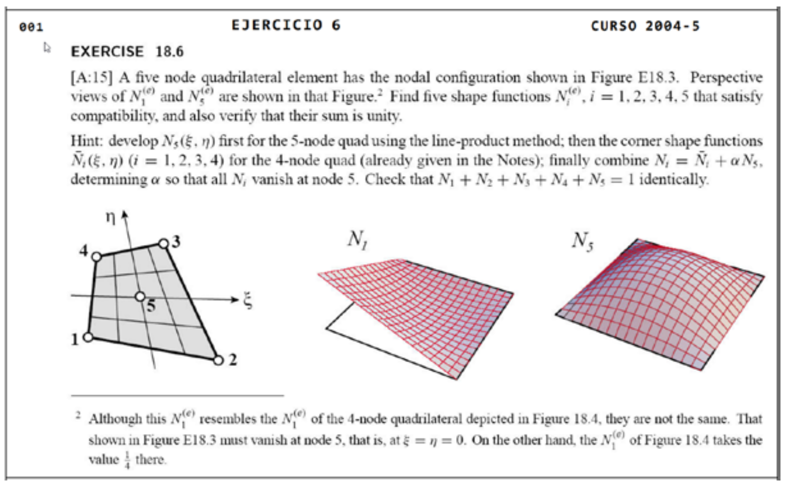
Figure E18.1. Ten-node cubic triangle for Exercise 18.1. The left picture displays the superparametric element whereas the right picture shows the general isoparametric version with curved sides.

Figure E18.2. Perspective plots of the shape functions N_1, N_4 and N_{10} for the 10-node cubic triangle.

(a) Construct the cubic shape functions $N_1^{(e)}$, $N_4^{(e)}$ and $N_{10}^{(e)}$ for nodes 1, 4, and 10 using the line-product technique. [Hint: each shape function is the product of 3 and only 3 lines.] Perspective plots of those 3 functions are shown in Figure E18.2.

(b) Construct the missing 7 shape functions by appropriate node number permutations, and verify that the sum of the 10 functions is identically one. For the unit sum check use the fact that $\zeta_1 + \zeta_2 + \zeta - 3 = 1$.

06-C6-Mathematica-C2



001 EJERCICIO 6 CURSO 2004-5

EXERCISE 18.6

[A:15] A five node quadrilateral element has the nodal configuration shown in Figure E18.3. Perspective views of $N_1^{(e)}$ and $N_5^{(e)}$ are shown in that Figure.[2] Find five shape functions $N_i^{(e)}$, $i = 1, 2, 3, 4, 5$ that satisfy compatibility, and also verify that their sum is unity.

Hint: develop $N_5(\xi, \eta)$ first for the 5-node quad using the line-product method; then the corner shape functions $\bar{N}_i(\xi, \eta)$ $(i = 1, 2, 3, 4)$ for the 4-node quad (already given in the Notes); finally combine $N_i = \bar{N}_i + \alpha N_5$, determining α so that all N_i vanish at node 5. Check that $N_1 + N_2 + N_3 + N_4 + N_5 = 1$ identically.

[2] Although this $N_1^{(e)}$ resembles the $N_1^{(e)}$ of the 4-node quadrilateral depicted in Figure 18.4, they are not the same. That shown in Figure E18.3 must vanish at node 5, that is, at $\xi = \eta = 0$. On the other hand, the $N_1^{(e)}$ of Figure 18.4 takes the value $\frac{1}{4}$ there.

PREGUNTAS Y TUS CONTESTACIONES:

DOCUMENTO PDF A COMPLETAR:

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001 SOBRE LAS FUNCIONES DE FORMA CURSO 2004-5

Means *Direct*

Do in 15 minutes what took smart people several months (and less gifted,)

But ... it looks like magic to the uninitiated

001 CONDICIONES A SATISFACER POR LAS FUNCIONES DE FORMA ISOPARAMETRICAS CURSO 2004-5

Shape Function Requirements

(A)

(B)

(C)

(D)

(A) Takes a unit value at node i, and is zero at all other nodes.

(B) Vanishes along any element boundary (a side in 2D, a face in 3D) that does not include node i.

(C) . Satisfies C^0 continuity between adjacent elements on any element boundary that includes node i.

(D) The interpolation is able to represent exactly any displacement field which is a linear polynomial in x and y; in particular, a constant value.

A statement equivalent to (C) is that the value of the shape function along a side common to two elements must uniquely depend only on its nodal values on that side.

Completeness is a property of *all* element isoparametric shape functions taken together, rather than of an individual one. If the element satisfies (B) and (C), in view of the discussion in §16.6 it is sufficient to check that the *sum of shape functions is identically one*.

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001 CONSTRUCCION DIRECTA DE FUNCIONES DE FORMA CURSO 2004-5

Direct Construction of Shape Functions

as

$$N_i^{(e)} \overset{\text{guess}}{=} c_i \quad \ldots L_m$$

where $L_k = 0$ are equations of "lines" expressed in natural coordinates, that cross except i

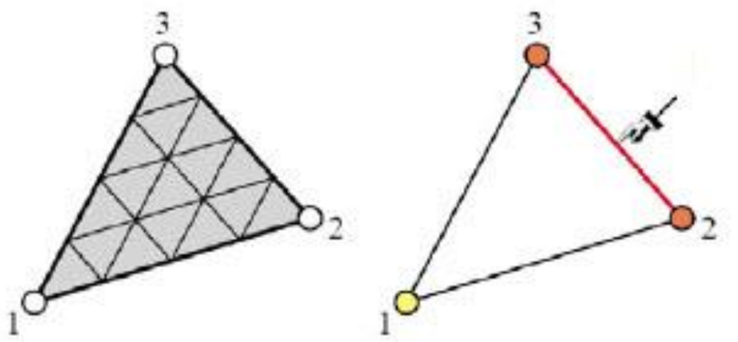
For two-dimensional isoparametric elements, the ingredients in (18.1) are chosen according to the following rules.

1. Select the L_j as the minimal number of lines or curves linear in the natural coordinates that cross all nodes except the i^{th} node. Primary choices are the and . The examples below illustrate how this is done.
2. Set coefficient c_i so that $N_i^{(e)}$ has the value at the node.
3. Check the polynomial order variation over each interelement boundary that contains node i. If this order is n, there must be exactly $n+1$ nodes on the boundary for the compatibility condition to hold.
4. If compatibility is satisfied, check that the of shape functions is identically

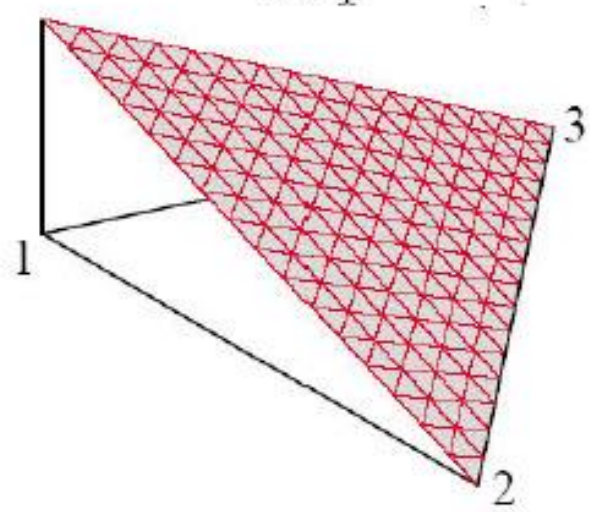
Specific two-dimensional examples in the following subsections show these rules in action. Essentially the same technique is applicable to one- and three-dimensional elements.

001 APLICACIÓN AL TRIANGULO DE TRES NODOS LINEAL CURSO 2004-5

The Three Node Linear Triangle



$N_1 =$



$$N_1 \overset{\text{guess}}{=} c_1 L_1 = c_1$$

At node 1, $N_1 = 1$ whence $c_1 =$
and $N_1 = \zeta_1$ Likewise for N_2 and N_3

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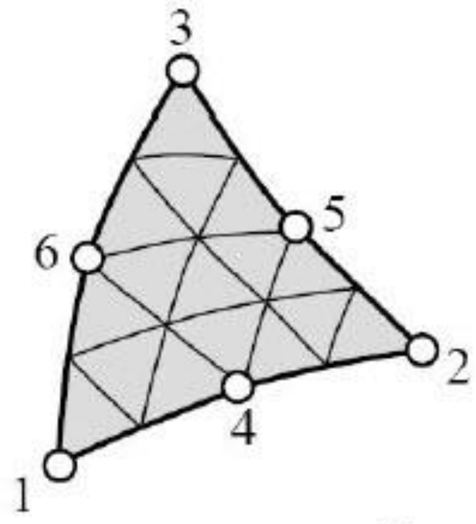
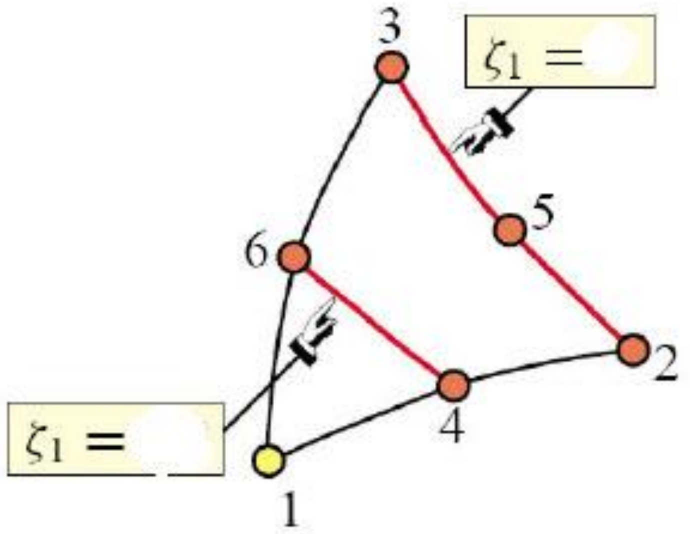
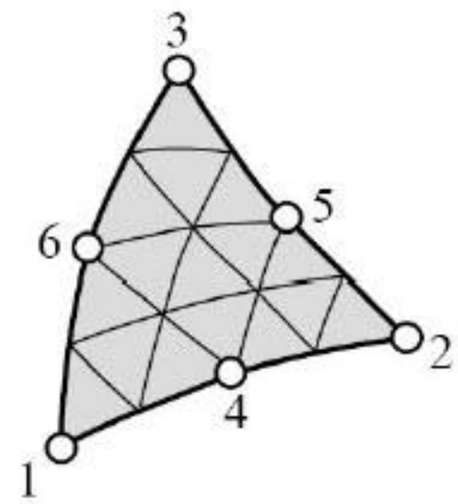
001 **TRIANGULO SEIS NODOS - NODO ESQUINA** CURSO 2004-5

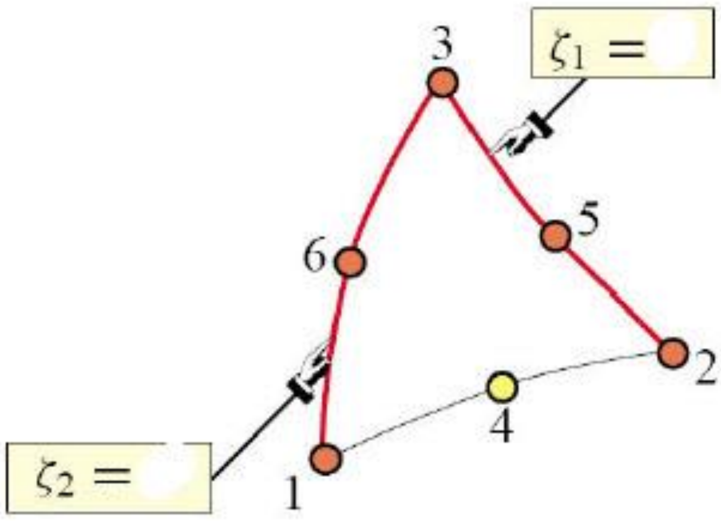




$$N_1^{(e)} \overset{\text{guess}}{=} c_1\, L \qquad L_{4\text{-}6}$$

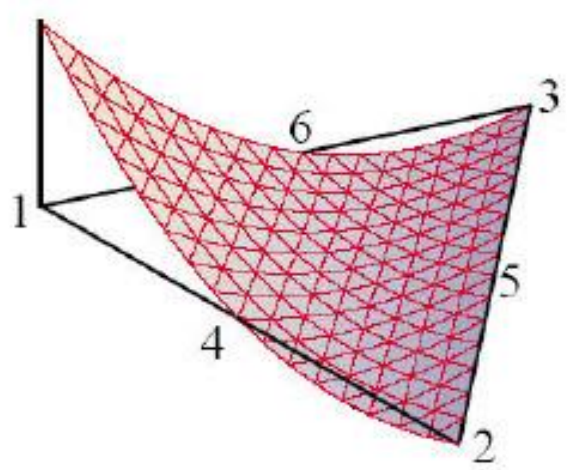
001 **TRIANGULO SEIS NODOS - NODO MEDIO** CURSO 2004-5

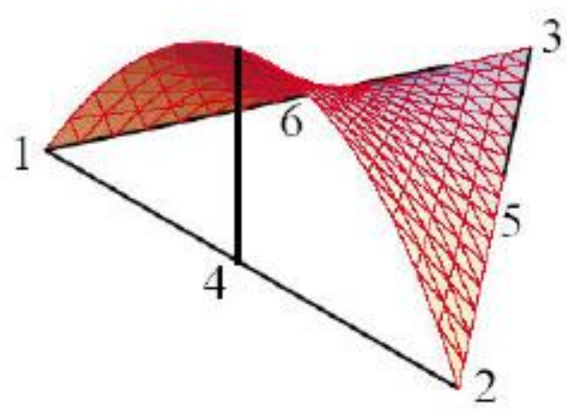




$$N_1^{(e)} \overset{\text{guess}}{=} c_1\, L_{2\text{-}3}\, L$$

001 **GRAFICO FUNCIONES FORMA** CURSO 2004-5





$$N_1^{(e)} = \zeta_1(2\zeta_1 - 1) \qquad\qquad N_4^{(e)} = 4$$

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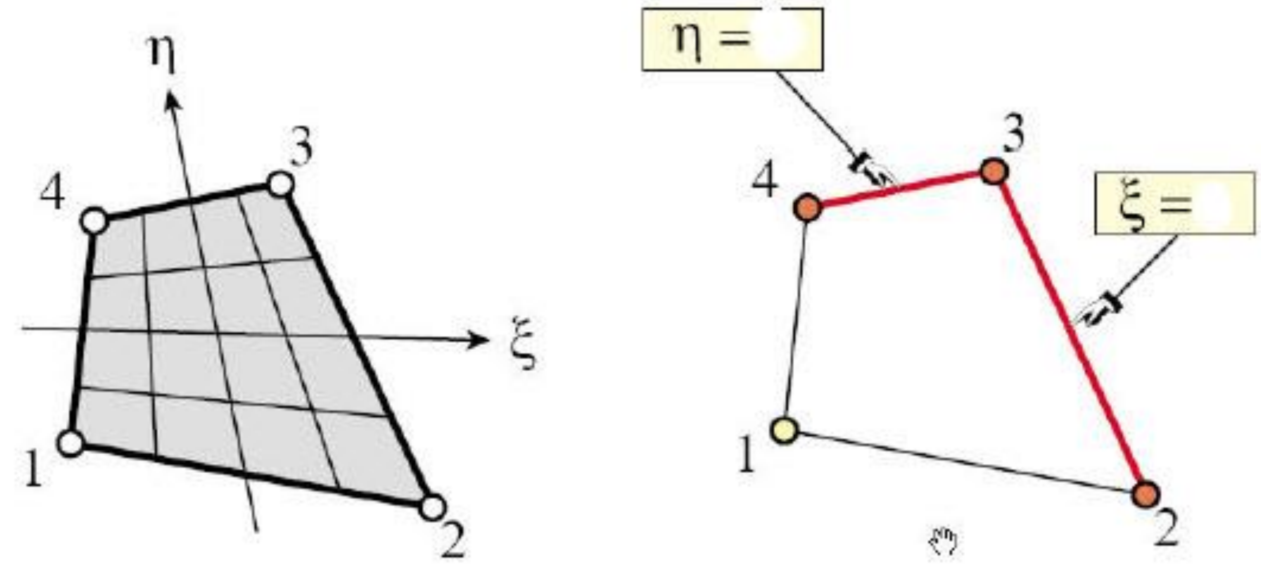
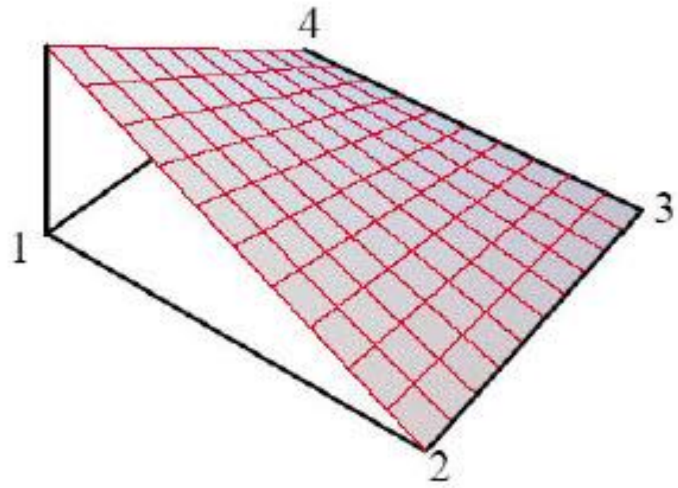
001 **CUADRILATERO BILINEAL DE CUATRO NODOS** CURSO 2004-5



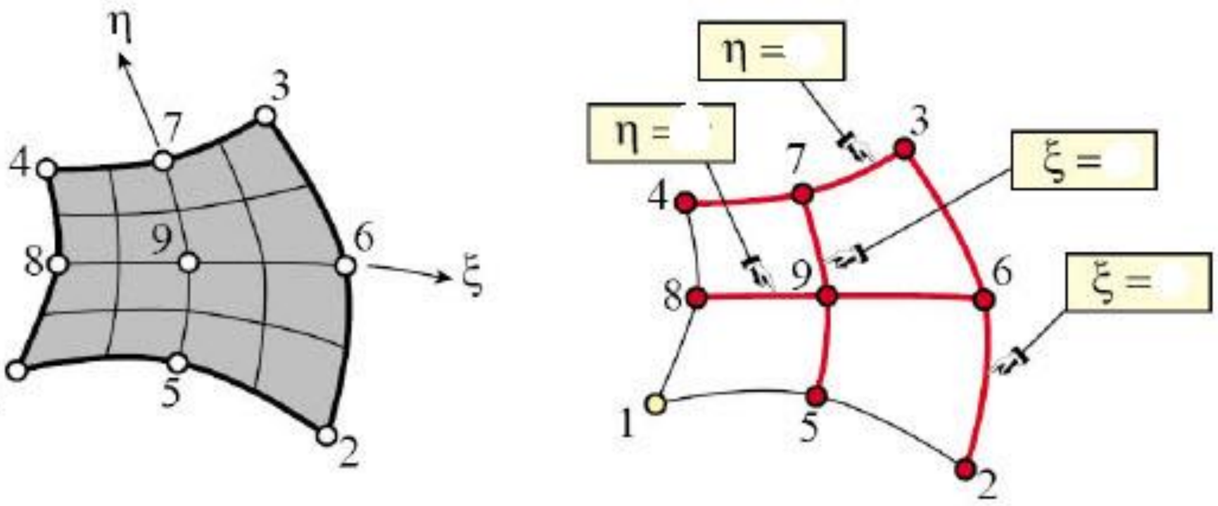
$$N_1^{(e)} \overset{\text{guess}}{=} c_1\, L \qquad L_{3\text{-}4}$$

001 **GRAFICO FUNCIONES FORMA** CURSO 2004-5



$$N_1^{(e)} = \tfrac{1}{4}(1 - \quad)(1 - \quad)$$

001 **CUADRILATERO BICUADRATICO DE NUEVE NODOS - NODO ESQUINA** CURSO 2004-5



$$N_1^{(e)} \overset{\text{guess}}{=} c_1 L_{2\text{-}3} I \quad L_{5\text{-}7} I \quad = c_1(\ \ - 1)(\eta - 1)\ \ \eta$$

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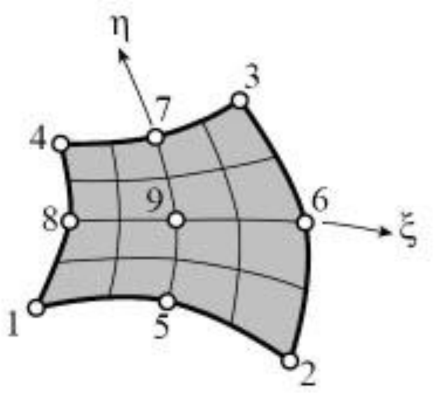
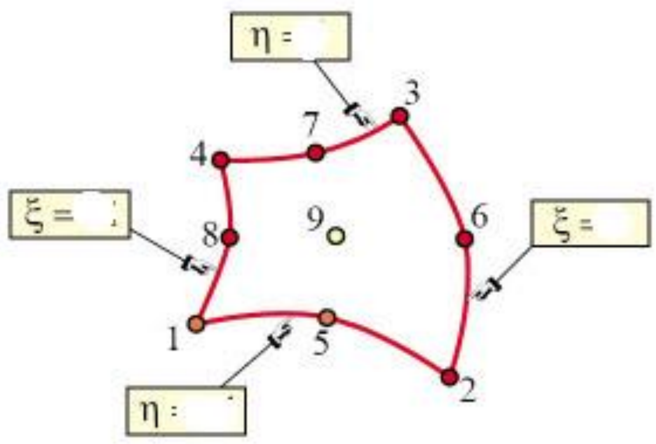
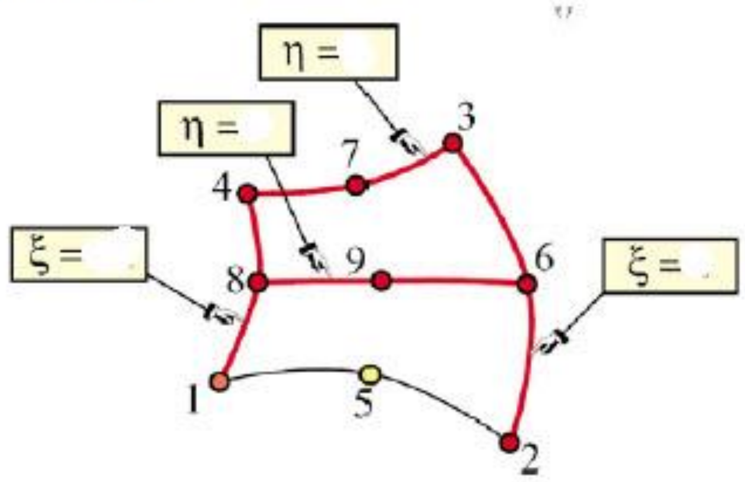
001 **CUADRILATERO BICUADRATICO DE NUEVE NODOS - NODO INTERNO** CURSO 2004-5





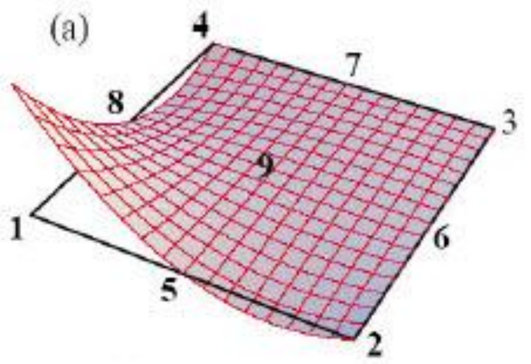
$$N_9^{(e)} = c_9\, L_{1\text{-}2} I \quad {}_3 L_{3\text{-}4} I \quad = c_9\,(\quad -1)(\eta-1)(\quad +1)(\eta+1)$$

001 **CUADRILATERO BICUADRATICO DE NUEVE NODOS - NODO MEDIO** CURSO 2004-5

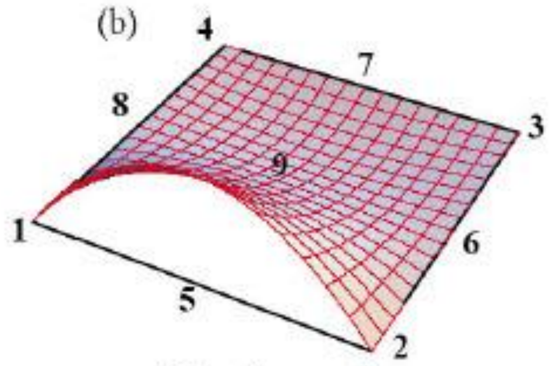


$$N_5^{(e)} = c_5 L_{2\text{-}3} I \quad {}_4 L_{6\text{-}8} I \quad = c_5 \quad -1)(\xi+1) \quad (\eta-1) = c_5\,(1-\xi^2)\eta(1-\eta)$$

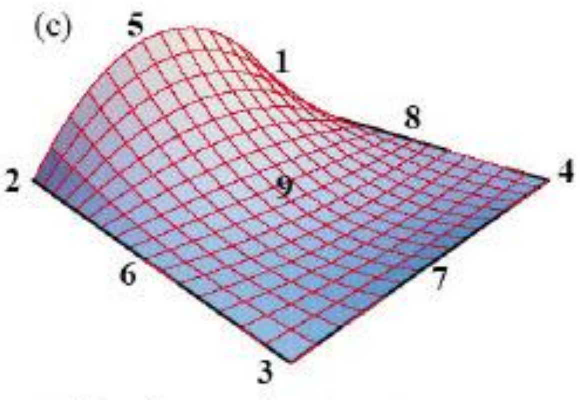
001 **GRAFICO FUNCIONES FORMA** CURSO 2004-5



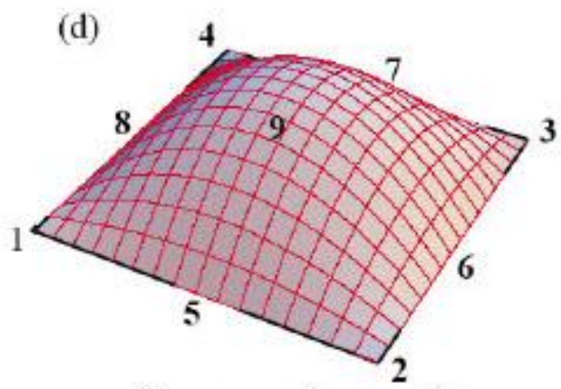
$N_1^{(e)} = \frac{1}{4}(\xi-1)(\eta-1)\xi\eta$



$N_5^{(e)} = \frac{1}{2}(1-\xi^2)\eta(\eta-1)$



$N_5^{(e)} = \frac{1}{2}(1-\xi^2)\eta(\eta-1)$ (back view)



$N_9^{(e)} = (1-\xi^2)(1-\eta^2)$

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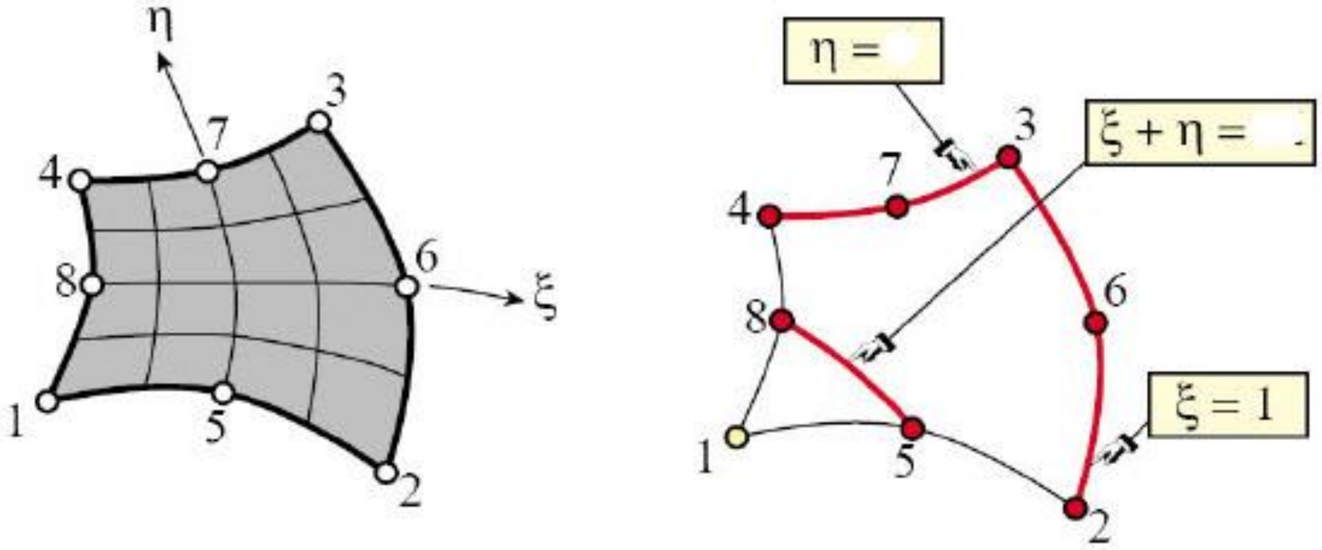
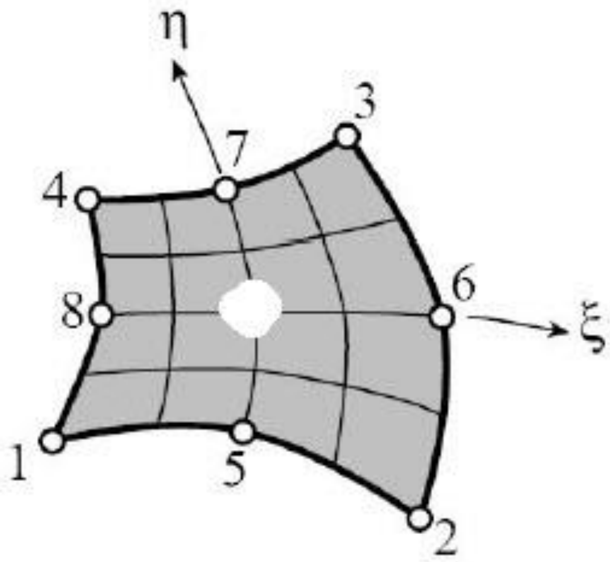
001 **CUADRILATERO "SERENDIPITO" DE OCHO NODOS - NODO** CURSO 2004-5



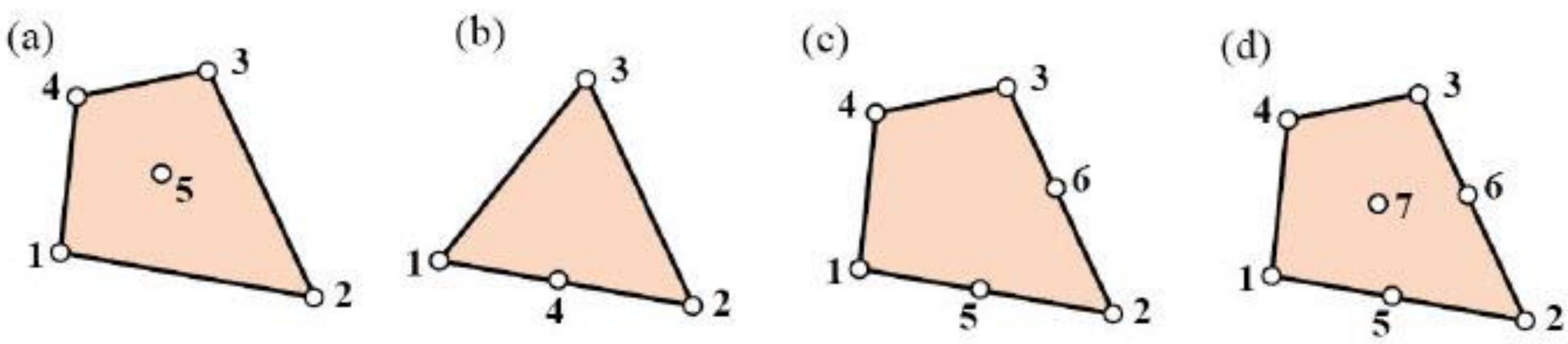
$$N_1^{(e)} = c_1 L \quad L_{3\text{-}4} L \quad = c_1(\xi - 1)(\quad - 1)(1 + \xi + \eta)$$

001 **CUADRILATERO "SERENDIPITO" DE OCHO NODOS - NODO MEDIO** CURSO 2004-5



$$N_5^{(e)} = c_5 L_{2\text{-}3} I \quad {}_1L_{4\text{-}1} = c_5 \quad - 1)(\xi + 1)(\quad - 1) = c_5\,(1 - \xi^2)(1 - \eta).$$

001 **A VECES LA TECNICA FALLA, PERO SIGUE SIENDO UN BUEN PUNTO DE PARTIDA** CURSO 2004-5



$$N_i^{(e)} = c_i\, L_1^c\, L_2^c \ldots L_m^c + d_i\, L_1^d\, L_2^d \ldots L_n^d$$

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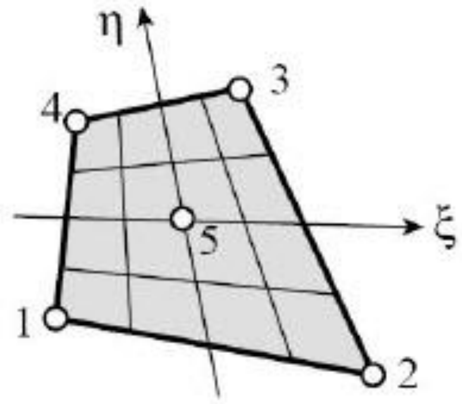
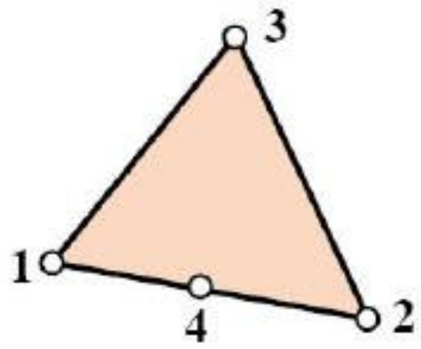
001 **UN EJEMPLO** CURSO 2004-5



$$N_1 = -\tfrac{1}{8}(1-\xi)(1-\eta)(\xi+\eta)$$

001 **OTRO EJEMPLO - ELEMENTOS TRANSICION** CURSO 2004-5



For N_1 try the magic wand: product of side ($\zeta_1 = 0$) and median ($\zeta_1 = \zeta_2$):

$$N_1^{(e)} \overset{\text{guess}}{=} c_1\zeta_1(\zeta_1 - \zeta_2), \quad N_1(1,0,0) = 1 = c_1$$ *fails (C)*

Next, try the shape function of the .
plus a correction: $N_1^{(e)} \overset{\text{guess}}{=} \zeta_1 + c_1\zeta_1\zeta_2$

Coefficient c_1 is determined by requiring this shape function vanish at midside node 4: $N_1^{(e)}(\frac{1}{2}, \frac{1}{2}, 0) = \frac{1}{2} + c_1\frac{1}{4} = 0$, whence $c_1 = -2$ and

$$N_1^{(e)} = \zeta_1 - 2\zeta_1\zeta_2$$ *works*