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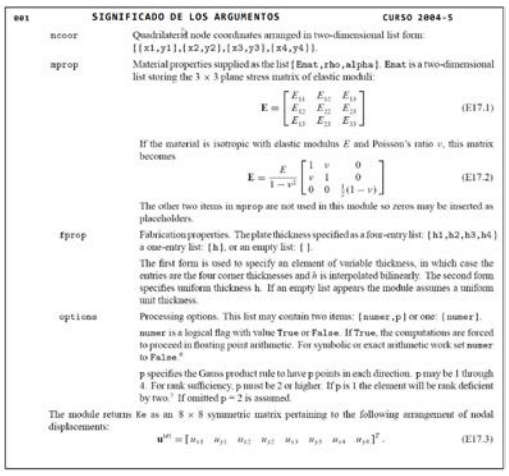
Estas ACTIVIDADES DE CLASE deberá realizarse descargando los documentos *NB* disponibles en las páginas web, completandolos adecuadamente, denominandolos de la forma especificada y subiendolos a tu cuenta de entrega personal. En este documento *PDF* habrá que contestar a las *PREGUNTAS* que planteo a lo largo de la grabación en video correspondiente a la clase.

Para familiarizarnos con la ***Formulacion del Elemento Cuadrilátero***, su definición, su terminología y su planteamiento; durante las explicaciones en clase habrá que completar este documento PDF.

Estas son imágenes de algunos de los ejercicios considerados en las ACTIVIDADES de esta CLASE:

05-C5-Mathematica-C



001 SIGNIFICADO DE LOS ARGUMENTOS CURSO 2004-5

ncoor — Quadrilateral node coordinates arranged in two-dimensional list form: {{x1,y1},{x2,y2},{x3,y3},{x4,y4}}.

mprop — Material properties supplied as the list {Emat,rho,alpha}. Emat is a two-dimensional list storing the 3 × 3 plane stress matrix of elastic moduli:

$$\mathbf{E} = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{12} & E_{22} & E_{23} \\ E_{13} & E_{23} & E_{33} \end{bmatrix} \quad \text{(E17.1)}$$

If the material is isotropic with elastic modulus E and Poisson's ratio ν, this matrix becomes

$$\mathbf{E} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \quad \text{(E17.2)}$$

The other two items in mprop are not used in this module so zeros may be inserted as placeholders.

fprop — Fabrication properties. The plate thickness specified as a four-entry list: {h1,h2,h3,h4} a one-entry list: {h}, or an empty list: { }.

The first form is used to specify an element of variable thickness, in which case the entries are the four corner thicknesses and h is interpolated bilinearly. The second form specifies uniform thickness h. If an empty list appears the module assumes a uniform unit thickness.

options — Processing options. This list may contain two items: {numer,p} or one: {numer}.

numer is a logical flag with value True or False. If True, the computations are forced to proceed in floating point arithmetic. For symbolic or exact arithmetic work set numer to False.[6]

p specifies the Gauss product rule to have p points in each direction. p may be 1 through 4. For rank sufficiency, p must be 2 or higher. If p is 1 the element will be rank deficient by two.[7] If omitted p = 2 is assumed.

The module returns Ke as an 8 × 8 symmetric matrix pertaining to the following arrangement of nodal displacements:

$$\mathbf{u}^{(e)} = [u_{x1} \; u_{y1} \; u_{x2} \; u_{y2} \; u_{x3} \; u_{y3} \; u_{x4} \; u_{y4}]^T. \quad \text{(E17.3)}$$

PREGUNTAS Y TUS CONTESTACIONES:

DOCUMENTO PDF A COMPLETAR:

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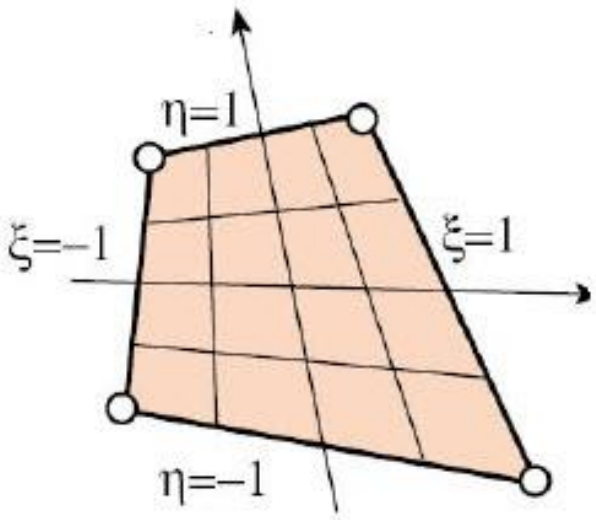
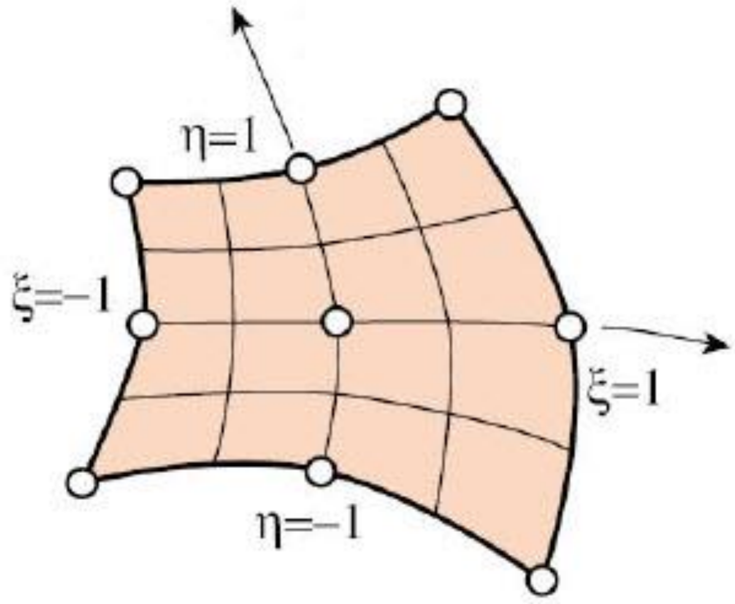
PASOS A SEGUIR PARA PODER FORMULAR UN ELEMENTO CUADRILATERO

CALCULO DE LAS DERIVADAS PARCIALES DE LAS FUNCIONES DE FORMA





NECESITAMOS CALCULAR LAS MATRICES JACOBIANA Y SU INVERSA

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} & \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{bmatrix} d\xi \\ d\eta \end{bmatrix} = \mathbf{J}^T \begin{bmatrix} d\xi \\ d\eta \end{bmatrix}$$

$$\begin{bmatrix} \\ d\eta \end{bmatrix} = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \xi}{\partial x} \\ \frac{\partial \eta}{\partial x} & \end{bmatrix} \begin{bmatrix} dx \\ \end{bmatrix} = \mathbf{J}^{-T} \begin{bmatrix} dx \\ dy \end{bmatrix}$$

in which

$$\mathbf{J} = \frac{\partial(x, y)}{\partial(\xi, \eta)} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ & \frac{\partial y}{\partial \eta} \end{bmatrix} = \begin{bmatrix} J_{11} & \\ J_{21} & J_{22} \end{bmatrix}, \qquad \mathbf{J}^{-1} = \frac{\partial(\xi, \eta)}{\partial(x, y)} = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} \\ & \frac{\partial \eta}{\partial y} \end{bmatrix}$$

$$J = |\mathbf{J}| = \det \mathbf{J}$$

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Y LAS DERIVADAS PARCIALES DE LAS FUNCIONES DE FORMA

$$\frac{\partial N_i^{(e)}}{\partial x} = \frac{\partial N_i^{(e)}}{\quad} \frac{\partial \xi}{\partial x} + \frac{\partial N_i^{(e)}}{\partial \eta} \frac{\quad}{\partial x}$$

$$\frac{\partial N_i^{(e)}}{\quad} = \frac{\partial N_i^{(e)}}{\partial \xi} \frac{\quad}{\partial y} + \frac{\partial N_i^{(e)}}{\partial \eta} \frac{\partial \eta}{\partial y}$$

Main problem is to get $\quad \frac{\partial \xi}{\partial x} \quad \frac{\quad}{\partial x} \quad \frac{\quad}{\partial y} \quad \frac{\partial \eta}{\partial y}$

LOS ELEMENTOS DE LA INVERSA DE LA MATRIZ JACOBIANA SERAN LOS QUE NECESITAMOS

Compute the 2 x 2 Jacobian matrix

$$\mathbf{J} = \frac{\partial(x, y)}{\partial(\xi, \eta)} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

Then invert to get

$$\mathbf{J}^{-1} = \frac{\partial(\xi, \eta)}{\partial(x, y)} = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} \\ \frac{\partial \xi}{\partial y} & \end{bmatrix}$$

These are the quantities we need for the S.F. partials

FORMA DE CALCULAR LOS ELMENTOS DE LA MATRIZ JACOBIANA INVERSA

$$x = \sum_{i=1}^{n} x_i N_i^{(e)} \qquad y = \sum_{i=1}^{n} \quad N_i^{(e)}$$

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$$\frac{\partial x}{\partial \xi} = \sum_{i=1}^{n} x_i \frac{\partial N_i^{(e)}}{\partial \xi}, \qquad \frac{\partial y}{\partial \xi} = \sum_{i=1}^{n} \quad \frac{\partial N_i^{(e)}}{\partial \xi},$$

$$\frac{\partial x}{\partial \eta} = \sum_{i=1}^{n} x_i \frac{\partial N_i^{(e)}}{\quad}, \qquad \frac{\partial y}{\partial \eta} = \sum_{i=1}^{n} y_i \frac{\partial N_i^{(e)}}{\partial \eta}.$$

$$\mathbf{J} = \mathbf{P}\mathbf{X} = \begin{bmatrix} \frac{\partial N_1^{(e)}}{\partial \xi} & \frac{\partial N_2^{(e)}}{\partial \xi} & \dots & \\ \frac{\partial N_1^{(e)}}{\partial \eta} & \quad & \dots & \frac{\partial N_n^{(e)}}{\partial \eta} \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ x_2 & \\ \vdots & \vdots \\ x_n & y_n \end{bmatrix}.$$

SECUENCIA DE PASOS A SEGUIR PARA DETERMINAR LAS DERIVADAS PARCIALES DE LAS FUNCIONES DE FORMA

At a specific point of quad coordinates and η:

Compute $\frac{\partial x}{\partial \xi}\ \frac{\partial y}{\partial \xi}\quad \frac{\partial y}{\partial \eta}$ **from node coordinates and s**

Form J and invert to get and det J

Apply the chain rule to get the x, **partials of the S.F.s**

CALCULO DE LA MATRIZ DEFORMACIONES-DESPLAZAMIENTOS

$$\mathbf{e} = \begin{bmatrix} \frac{\partial N_1^{(e)}}{\partial x} & 0 & \frac{\partial N_2^{(e)}}{\partial x} & 0 & \dots & \frac{\partial N_n^{(e)}}{\partial x} & 0 \\ 0 & \frac{\partial N_1^{(e)}}{\partial y} & 0 & & \dots & 0 & \frac{\partial N_n^{(e)}}{\partial y} \\ \frac{\partial N_1^{(e)}}{\partial y} & & \frac{\partial N_2^{(e)}}{\partial y} & \frac{\partial N_2^{(e)}}{\partial x} & \dots & \frac{\partial N_n^{(e)}}{\partial y} & \end{bmatrix} \mathbf{u}^{(e)} = \ \mathbf{u}^{(e)}$$

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INTEGRACION NUMERICA MEDIANTE REGLAS DE GAUSS

REGLAS UNIDIMENSIONALES

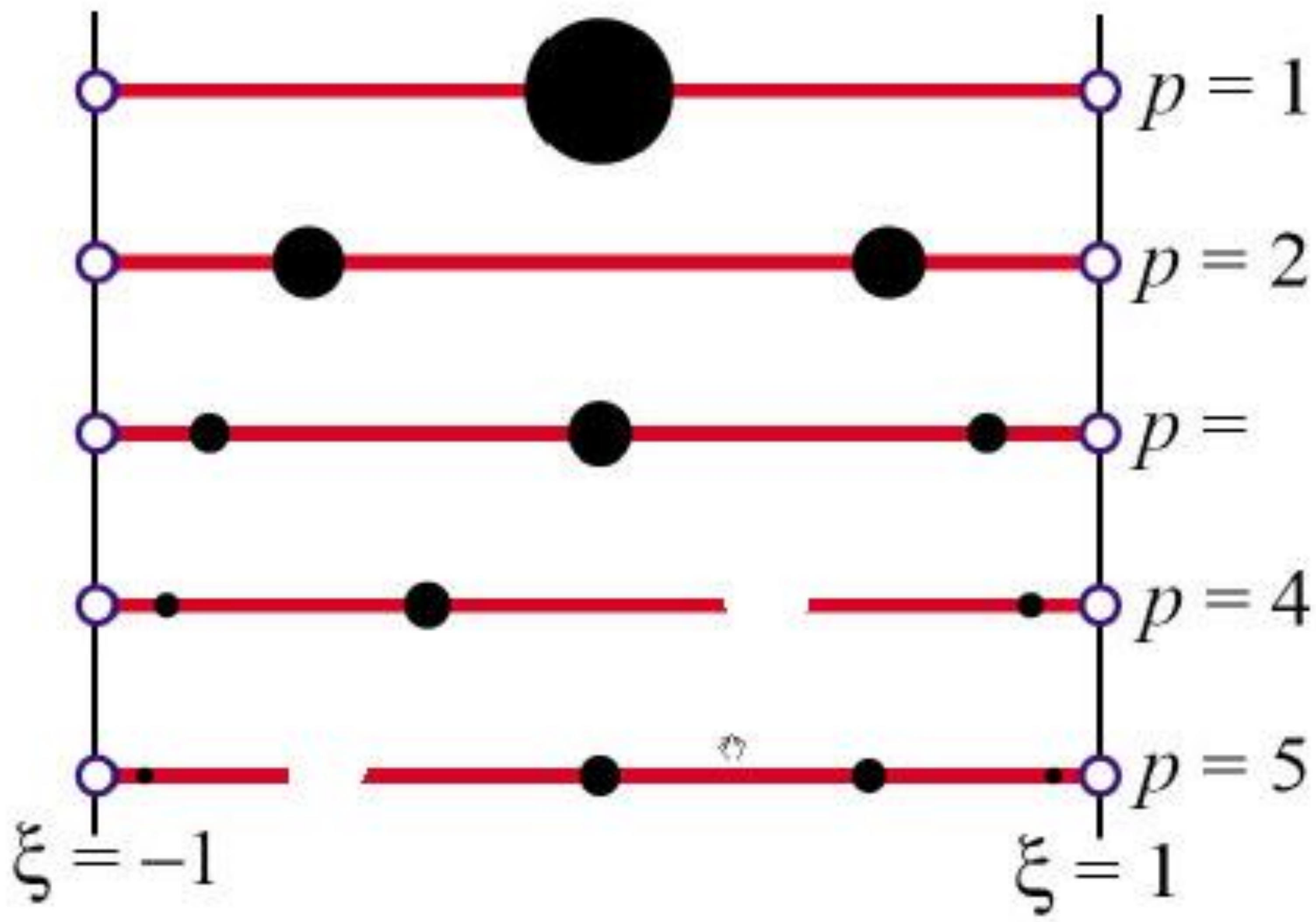
$$\int_{-1}^{1} F(\xi)\, d\xi = \sum_{i=1}^{p} \quad F(\xi_i).$$

One point: $$\int_{-1}^{1} F(\xi)\, d\xi \doteq 2F(0),$$

Two points: $$\int_{-1}^{1} F(\xi)\, d\xi \doteq F(-1/\sqrt{3}) + F(\qquad),$$

Three points: $$\int_{-1}^{1} F(\xi)\, d\xi \doteq \frac{5}{9}F(-\sqrt{3/5}) + \frac{8}{9}F(\ ) + \frac{5}{9}F(\sqrt{3/5})$$

REPRESENTACION GRAFICA DE LAS REGLAS UNIDIMENSIONALES



CONCEPTO DE GRADO DE LA FORMULA DE INTEGRACION DE GAUSS

In general a one-dimensional Gauss rule with p points integrates exactly polynomials of order up to $2p-1$. This is called the ' of the formula.

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REGLAS DE GAUSS PARA PROBLEMAS BIDIMENSIONALES – REGLAS DEL PRODUCTO

Canonical form of integral:

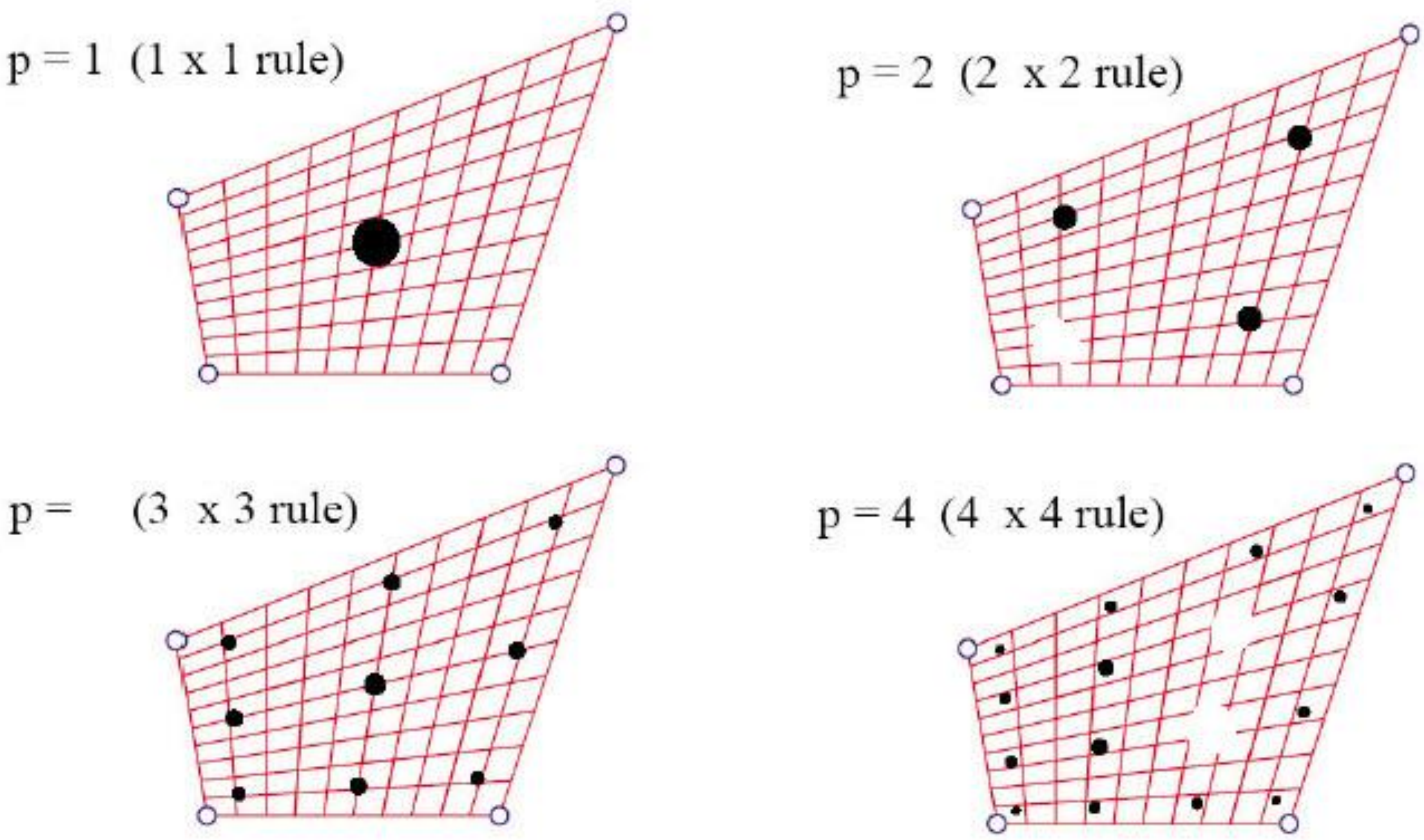
$$\int_{-1}^{1}\int_{-1}^{1} F(\xi,\eta) \quad = \int_{-1}^{1} d\eta \int_{-1}^{1} F(\xi,\eta)\, d\xi.$$

Gauss integration rules with p_1 points in the ξ direction and p_2 points in the η direction:

$$\int_{-1}^{1}\int_{-1}^{1} F(\xi,\eta)\, d\xi\, d\eta = \int_{-1}^{1} d\eta \int_{-1}^{1} \qquad d\xi \approx \sum_{i=1}^{p_1}\sum_{j=1}^{p_2} w_i \quad F(\xi_i,\eta_j).$$

Usually $p_1 = p_2$

REPRESENTACION GRAFICA REGLAS BIDIMENSIONALES



USO DEL MODULO DE MATHEMATICA

In all four forms, logical flag `numer` is set to `True` if numerical information is desired and to if exact information is desired. The module returns ξ_i and η_j in `xii` and `etaj`, respectively, and the weight product $w_i w_j$ in `wij`. This code is used in the Exercises at the end of the chapter. If the

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CALCULO DE LA MATRIZ DE RIGIDEZ

$$\mathbf{K}^{(e)} = \int_{\Omega^{(e)}} h\, \mathbf{B}^T \quad \mathbf{B}\, d\Omega^{(e)}$$

Rewrite in canonical form:

$$\mathbf{K}^{(e)} = \int_{-1}^{1}\int_{-1}^{1} \mathbf{F}(\xi,\eta)\, d\xi\, d\eta.$$

where

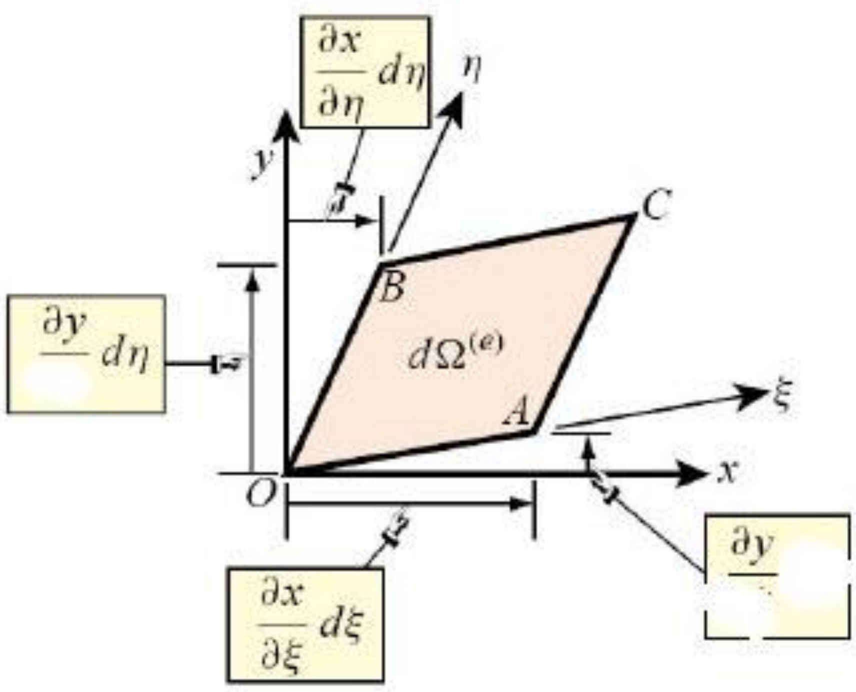
$$d\Omega^{(e)} = dx\, dy = \det \quad d\xi\, d\eta.$$

$$\mathbf{F}(\xi,\eta) = h\, \mathbf{B}^T \mathbf{E} \quad \det \mathbf{J}.$$

Then apply the rule to **F** (a 2n x 2n matrix)

$$\int_{-1}^{1}\int_{-1}^{1} \mathbf{F}(\xi,\eta)\, d\xi\, d\eta = \int_{-1}^{1} d\eta \int_{-1}^{1} \mathbf{F}(\xi,\eta)\, d\xi \approx \sum_{i=1}^{p_1}\sum_{j=1}^{p_2} \quad w_j\, \mathbf{F}(\xi_i,\eta_j).$$

INTERPRETACION GEOMETRICA DEL DETERMINANTE DEL JACOBIANO



$$dA = \vec{OB} \times \vec{OA} = \frac{\partial x}{\partial \xi} d\xi\, \frac{\partial y}{\partial \eta} d\eta - \frac{\partial x}{\partial \eta} d\eta\, \frac{\partial y}{\partial \xi} d\xi = \begin{vmatrix} \frac{\partial x}{\partial \xi} & \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} \end{vmatrix} d\xi\, d\eta = |\mathbf{J}| d\xi\, d\eta.$$

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EXPLICACION MODULOS DISPONIBLES EN MATHEMATICA

001 EJERCICIO 0 – EXPLICACION MODULO "MATHEMATICA" CURSO 2004-5

Quad 4 Element Formation in *Mathematica:* Computation Module

```
Quad4IsoPMembraneStiffness[ncoor_,mprop_,fprop_,options_]:=
  Module[{i,k,p=2,numer=False,Emat,th=1,h,qcoor,c,w,Nf,
   dNx,dNy,Jdet,B,Ke=Table[0,{8},{8}]},  Emat=mprop[[1]];
  If [Length[options]==2, {numer,p}=options, {numer}=options];
  If [Length[fprop]>0, th=fprop[[1]]];
  If [p<1||p>4, Print["p out of range"];Return[Null]];
  For [k=1, k<=p*p, k++,
       {qcoor,w}= QuadGaussRuleInfo[{p,numer},k];
       {Nf,dNx,dNy,Jdet}=Quad4IsoPShapeFunDer[ncoor,qcoor];
        If [Length[th]==0, h=th, h=th.Nf]; c=w*Jdet*h;
        B={ Flatten[Table[{dNx[[i]],       0},{i,4}]],
            Flatten[Table[{0,        dNy[[i]]},{i,4}]],
            Flatten[Table[{dNy[[i]],dNx[[i]]},{i,4}]]};
        Ke+=Simplify[c*Transpose[B].(Emat.B)];
      ]; Return[Ke]
   ];
```

Quad 4 Element Formation in *Mathematica:* Shape . . and Their Derivatives

```
Quad4IsoPShapeFunDer[ncoor_,qcoor_]:= Module[
  {Nf,dNx,dNy,dNξ,dNη,i,J11,J12,J21,J22,Jdet,ξ,η,x1,x2,x3,x4,
   y1,y2,y3,y4,x,y},
  {ξ,η}=qcoor; {{x1,y1},{x2,y2},{x3,y3},{x4,y4}}=ncoor;
   Nf={(1-ξ)*(1-η),(1+ξ)*(1-η),(1+ξ)*(1+η),(1-ξ)*(1+η)}/4;
   dNξ ={-(1-η), (1-η),(1+η),-(1+η)}/4;
   dNη= {-(1-ξ),-(1+ξ),(1+ξ), (1-ξ)}/4;
   x={x1,x2,x3,x4}; y={y1,y2,y3,y4};
   J11=dNξ.x; J12=dNξ.y; J21=dNη.x; J22=dNη.y;
   Jdet=Simplify[J11*J22-J12*J21];
   dNx= ( J22*dNξ-J12*dNη)/Jdet;  dNx=Simplify[dNx];
   dNy= (-J21*dNξ+J11*dNη)/Jdet;  dNy=Simplify[dNy];
   Return[{Nf,dNx,dNy,Jdet}]
];
```

001 EJERCICIO 0 – EXPLICACION MODULO "MATHEMATICA" (CONT.) CURSO 2004-5

Quad 4 Element Formation in *Mathematica:* 2D - Quadrature Rule Information

```
QuadGaussRuleInfo[{rule_,numer_},point_]:= Module[
 {xi,eta,p1,p2,i1,i2,w1,w2,k,info=Null},
  If [Length[rule]==2,  {p1,p2}=rule, p1=p2=rule];
  If [Length[point]==2, {i1,i2}=point,
      k=point; i2=Floor[(k-1)/p1]+1; i1=k-p1*(i2-1) ];
  {xi, w1}=  LineGaussRuleInfo[{p1,numer},i1];
  {eta,w2}=  LineGaussRuleInfo[{p2,numer},i2];
  info={{xi,eta},w1*w2};
  If [numer, Return[N[info]], Return[Simplify[info]]];
];
```

Works for any combination of $p_1 = 1,2,3,4,5$ and $p_2 = 1,2,3,4,5$

Calls 1D Gauss rule module of next slide twice

Quad 4 Element Formation in *Mathematica:* 1D Gauss Rule Information

```
LineGaussRuleInfo[{rule_,numer_},point_]:= Module[
  {g2={-1,1}/Sqrt[3],w3={5/9,8/9,5/9},
   g3={-Sqrt[3/5],0,Sqrt[3/5]},
   w4={(1/2)-Sqrt[5/6]/6, (1/2)+Sqrt[5/6]/6,
       (1/2)+Sqrt[5/6]/6, (1/2)-Sqrt[5/6]/6},
   g4={-Sqrt[(3+2*Sqrt[6/5])/7],-Sqrt[(3-2*Sqrt[6/5])/7],
        Sqrt[(3-2*Sqrt[6/5])/7], Sqrt[(3+2*Sqrt[6/5])/7]},
   g5={-Sqrt[5+2*Sqrt[10/7]],-Sqrt[5-2*Sqrt[10/7]],0,
        Sqrt[5-2*Sqrt[10/7]], Sqrt[5+2*Sqrt[10/7]]}/3,
   w5={322-13*Sqrt[70],322+13*Sqrt[70],512,
       322+13*Sqrt[70],322-13*Sqrt[70]}/900,
   i=point,p=rule,info={Null,0}},
  If [p==1, info={0,2}];
  If [p==2, info={g2[[i]],1}];
  If [p==3, info={g3[[i]],w3[[i]]}];
  If [p==4, info={g4[[i]],w4[[i]]}];
  If [p==5, info={g5[[i]],w5[[i]]}];
  If [numer, Return[N[info]], Return[Simplify[info]]];
];
```

Works for $p = 1,2,3,4,5$

EXPLICACION ARGUMENTOS

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The other two items in `mprop` are not used in this module so zeros may be inserted as placeholders.

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The module returns `Ke` as an 8 × 8 symmetric matrix pertaining to the following arrangement of nodal displacements:

$$\mathbf{u}^{(e)} = [\, u_{x1} \quad u_{y1} \quad u_{x2} \qquad u_{x3} \qquad u_{x4} \quad u_{y4} \,]^T . \qquad \text{(E17.3)}$$